Fluctuations in the UK Equity Market: What drives stock returns?¹

Abstract:

Present value parameters from a state-space model are estimated for the UK FT All-Share Index. The estimated parameters are used to construct a time series of expected future returns and expected future values of dividend growth, both of which are found to be time-varying with persistent components. Variations in the price-dividend ratio appear to be driven primarily by the variance in expected returns. A comparison with the findings from a present value-constrained vector autoregression (VAR) model indicates that the latter forecasts future realized returns and dividend growth better than the series constructed using a state-space approach. Furthermore, when the model is estimated for monthly and quarterly data, expected dividend growth is found to be more persistent.

Keywords: State-Space model, UK Equity Market, Present value

JEL Code: G12, C32, C58

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1. Introduction

According to the present value model for equities, movements in stock prices and, therefore, returns are explained by changes in the discount rate and variations in future cash flows (dividends). Such variations are usually captured by the price-dividend ratio, which is used extensively in the predictability literature with differing levels of success. According to the literature, a high price-dividend ratio implies low future returns, while a low price-dividend ratio suggests high future returns. Explaining the volatility of equity markets based on the price-dividend ratio is a consequence of the present value identity, which asserts that the ratio can be decomposed into expected returns and expected dividend growth rate components. One of the key empirical challenges faced by researchers with respect to this concept has been the measurement of these variables over time given that they are inherently latent or unobservable.

One approach to overcome this problem is the use of the vector autoregression (VAR) technique popularized by Campbell and Shiller (1988). In this setting, the present value can be linearized and estimated by a constrained VAR. Advantages of this approach include the possibility of estimating variance decompositions, determining long-run relationships and the testing of joint nulls of no predictability. However, the model assumes that parameters are constant throughout, without accounting for shocks that enter the market. An alternative technique that has recently been considered in the literature is the state-space approach of Binsbergen and Koijen (2010).

The central tenet of Binsbergen and Koijen (2010) is that expected returns and expected dividend growth can be estimated from the price-dividend ratio using present value identity via a Kalman filter. Using cash and market reinvested dividends, they find strong results for the predictability of realized returns. They also document that discount rate news is the most important factor behind movements in stock returns. The state-space
model and the VAR are similar in the sense that under conditions of stationarity, the VAR is a special case of the state-space approach with extremely long lags. However, the state-space approach has lower computational requirements. In addition, the problem of drawing inferences from many lags is avoided. The main advantage of the state-space model is, perhaps, that expected returns and expected dividend growth rate are allowed to vary over time.

There are several papers that have developed and applied the state-space approach in the context of the present value of stock prices. Closely linked to Binsbergen and Koijen (2010), Rytchkov (2012) finds that expected returns are time varying and can predict realized returns better than the price-dividend ratio. Golinski et al. (2015) exploit the fact that the price-dividend ratio may be fractionally integrated to demonstrate that the conventional returns equation in prior investigations can be unbalanced\(^2\). They also document a marginal improvement in predictability after considering fractional integration in expected returns. Piatti and Trojani (2015) consider inference problems within this set-up by employing nonparametric bootstrap procedures when testing for predictability, where they indicate that the role of dividend growth predictability is still important.

With respect to empirical applications, Su, Ma and Wohar (2012) find that expected returns are more important for the stock price decomposition in the case of Chinese ‘A’ stocks. With evidence of counter-cyclicality in expected returns noted, they argue that interest rate policies may have heavily influenced the fluctuations that they uncovered in expected returns. Ma and Wohar (2014) estimate the present value

\(^2\) An unbalanced regression occurs when returns and a predictor variable have different orders of integration, which may invalidate conventional inference procedures (Phillips and Lee, 2013).
relationship for one of the leading equity indices in the UK by considering various specifications of expected returns and expected dividend growth and highlight a possible problem of inference. Interestingly, they find that expected returns are persistent but that the expected growth in dividend cash flows is negatively serially correlated over time. In this paper, we compare the state-space and structural VAR frameworks in the case of the UK FT All-Share index for the period 1973 to 2014. This time span is slightly shorter than that employed by Ma and Wohar (2014), but it includes more up-to-date data points and considers issues of parameter stability as well as the minimization of measurement errors. Quarterly and monthly frequencies are also considered in the current paper, whereas previous investigations have concentrated on a one return interval.

Whereas the present value identity plays an important role in the state-space framework, changes in the latent variables imply changes in the price-dividend ratio. For example, if expected returns were to increase from one period to another, it would likely lead to a fall in the price-dividend ratio. One important assumption of this approach is that both expected returns and expected dividend growth are first-order autoregressive series. This assumption is intuitive as it assumes that agents update their expectations for both dividends and expected returns in every period. Such an assumption can be traced back to the macroeconomic literature on learning (Evans and Honkapohja (2001) and Timmermann (1996)). The rational learning literature assumes that agents update the parameters of their models on the basis of recent outcomes.

Once the state-space model is optimized, it yields a series of parameters that can be used to construct a time series of expected returns and a time series of expected dividend growth. The parameters are used to estimate the variance of the expected discount rate and the volatility of expected future cash flows in the context of the price-dividend ratio and unexpected return decomposition. Tests of time variation, persistence, and equality of
persistence for the expected returns and the expected dividend growth are also performed and indicate that both latent series are time varying, although the expected returns series is more persistent than the expected dividend growth series. A Monte Carlo exercise is implemented to assess the predictability of returns and dividend growth in a VAR model where a present value constraint is introduced. Contrary to some of the US findings in this area, the results in this paper suggest that the constrained VAR model predicts both returns and dividend growth jointly. Moreover, the VAR model performs better than the filtered series. We also report on estimation issues for the state-space approach in the presence of weak signal-noise ratios. Finally, a reduced form equation is considered that connects the state variables to the structural form of the VAR. The results from connecting the state-space approach to the structural VAR suggest that estimation differences, with the exception of the correlation coefficients, are relatively small.

The remainder of this paper is structured as follows. Section 2 illustrates the present value model and outlines the derivation of the state-space version of the price-dividend relationship. Section 3 reports the results from the state-space model based on annual data. Tests of hypotheses from the present value model and variance decompositions of the price-dividend ratio are then analysed. A Monte Carlo simulation is performed for a present value VAR model to evaluate the predictive ability of the state-space approach employed. The state-space version of the present value model is applied to monthly and quarterly data to determine whether data frequency affects the findings. As a robustness check, we also report Monte Carlo estimates for the results yielded by connecting the structural VAR with the state-space model. Section 4 concludes the paper.
2. Methodology

The dynamic present value model for stock prices can be traced back to Campbell and Shiller (1988), who found that the price-dividend ratio can be log-linearized into an expected dividend growth and an expected discount rate component. In this context, the price-dividend ratio moves as a result of changes in the anticipated discount rate and variations in expected future dividend growth. Similarly, the Campbell-Shiller equation demonstrates that unexpected stock returns are driven by shocks to expected future cash flows and shocks to discount rates.

2.1 Present Value.

In this section, we illustrate the log-linearized present value model before introducing the state-space model of the present value relationship. Denoting $D_t$ and $P_t$ as the dividends from the stock market index and the stock price, respectively, at time $t$, the log returns on the index from $t$ to $t + 1$ ($r_{t+1}$), the dividend growth from $t$ to $t + 1$ ($\Delta d_{t+1}$) and the logarithm of the price-dividend ($pd_t$) is defined as follows:

\[
 r_{t+1} = \ln \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right) \tag{1}
\]

\[
 pd_t = \ln \left( \frac{P_t}{D_t} \right) \tag{2}
\]

\[
 \Delta d_{t+1} = \ln \left( \frac{D_{t+1}}{D_t} \right) \tag{3}
\]

Equations (1) to (3) can be measured directly from the data as they are based on realized relationships. An important assumption presented by Binsbergen and Koijen (2010) is that the expected returns and dividend series follow an AR(1) process as follows:
\[
\begin{align*}
\mu_{t+1} - \delta_0 &= \delta_1(\mu_t - \delta_0) + \epsilon_{\mu,t+1}, \quad (4) \\
g_{t+1} - \gamma_0 &= \gamma_1(g_t - \gamma_0) + \epsilon_{g,t+1}, \quad (5)
\end{align*}
\]

where \(\mu_{t+1} = E_t(r_{t+1})\) and \(g_{t+1} = E_t(\Delta d_{t+1})\); \(\mu_{t+1}\) and \(g_{t+1}\) are market expectations of future realized returns and dividend growth, respectively; \(\delta_0\) and \(\gamma_0\) represent the unconditional mean of the expected returns and dividend growth, respectively; and \(\delta_1\) and \(\gamma_1\) are the autoregressive parameters and are usually assumed to be less than one. Shocks to the expected returns and growth process are assumed to be normally distributed \(\epsilon_{\mu,t+1} \sim N(0, \sigma_{\mu}^2)\) and \(\epsilon_{g,t+1} \sim N(0, \sigma_g^2)\). The correlation between the expected returns and dividend growth is estimated in the model and is denoted by \(\rho_{g,\mu}\).

The future realized growth rate in dividends is equal to the expected growth rate \((g_t)\) and an unobserved shock \((\epsilon_{d,t+1})\) as follows:

\[
\Delta d_{t+1} = g_t + \epsilon_{d,t+1}. \quad (6)
\]

where \(\epsilon_{d,t+1}\) and \(g_t\) are assumed to be orthogonal to each other. Equation (6) is one equation linking a measured variable \(\Delta d_{t+1}\) to an expected variable \(g_t\). Because we have two unobservable expected variables, we must examine another relationship between the unobserved and the observed variable. This represents the present value approximate identity.

The Campbell-Shiller (1988) equation is written as follows:

\[
pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}, \quad (7)
\]

where \(\kappa = \ln(1 + e^{\bar{pd}}) - \rho pd\) and \(\rho = \frac{e^{\bar{pd}}}{1 + e^{\bar{pd}}}\) and \(\bar{pd}\) is the mean of the price-dividend ratio. By applying the conditional expectations operator to equation (7) and iterating \(pd_{t+1}\) to infinity,
\[ pd_t = \frac{\kappa}{1-\rho} + \frac{1}{1-\rho} g_t - \frac{1}{1-\rho} \mu_t \]  

Equation (8) states that the current payoff should reflect discounted expectations of returns and payoff growth. At this stage, it is emphasized that \(pd_t\) and \(\Delta d_{t+1}\) are measurable and observable at time \(t+1\), unlike \(g_t\) and \(\mu_t\). Before calculating equations (4) and (5), it is important to note that any movement in \(\mu_t\) and \(g_t\) will cause \(pd_t\) to change.

3.2 State-space Equation

The basic state-space model\(^3\) comprises a state equation model and a measurement equation. The state equation defines the structure of the non-measurable variable, whereas the measurement equation defines the dynamics of the measurable or observed variable and the relationship between the non-measured variable and an observed variable. In the context of the present value model, the non-measurable variables are expected returns and expected dividend growth. The observed variables are the price-dividend ratio and the realized dividend growth. Beginning with the transition equation, equations (4) and (5) can be rewritten in demeaned form as expected dividend growth (9) and conditional expected returns (10) as follows:

\[ \hat{\mu}_{t+1} = \delta_1 \hat{\mu}_t + \varepsilon_{\mu,t+1}, \]  
\[ \hat{g}_{t+1} = \gamma_1 \hat{g}_t + \varepsilon_{g,t+1}, \]  

where \(\hat{g}_{t+1}\) and \(\hat{\mu}_{t+1}\) are demeaned expected dividend growth and returns, respectively. In other words, \(\hat{g}_t = g_t - \gamma_0\) and \(\hat{\mu}_t = \mu_t - \delta_0\).

The measurement equations are given by the following:

\(^3\) For a detailed explanation on these models, see Durbin and Koopman (2012).
\[ \Delta d_{t+1} = \gamma_0 + \hat{\gamma}_t + \epsilon_{d,t+1}, \quad (11) \]
\[ pd_t = B_0 - B_1 \mu_t + B_2 \hat{\gamma}_t \quad (12) \]

Equation (11) states that the realized dividend growth is equal to its expected counterpart plus the unobserved shock \( \epsilon_{d,t+1} \). Equation (12) is the Campbell-Shiller (1988) present value form, which relates the price–dividend ratio to the expected dividend growth and expected returns. The terms \( B_0, B_1 \) and \( B_2 \) are defined as follows:

\[ B_0 = \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho}, \quad (13) \]
\[ B_1 = \frac{1}{1 - \rho \delta_1}, \quad (14) \]
\[ B_2 = \frac{1}{1 - \rho \gamma_1}. \quad (15) \]

The Kalman filter can be applied to the model by optimizing the log-likelihood function from the Kalman filter on the data. The objective of such a procedure is to yield the autoregressive terms \( (\gamma_1, \delta_1) \), the intercept terms \( (\gamma_0, \delta_0) \), the shock terms \( (\sigma_\mu, \sigma_g, \sigma_d) \) and the correlation parameters \( (\rho_{g\mu}, \rho_{d\mu}) \). The vector of parameters to be estimated from the model is given by the following:

\[ \Phi = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_\mu, \sigma_g, \sigma_d, \rho_{g\mu}, \rho_{d\mu}) \]

Sequentially, once the optimal values are solved, it is possible to derive a time series of expected returns and expected dividend growth values. The implied present value parameters \( B_0, B_1 \) and \( B_2 \) can also be determined. The last two parameters depend on the autoregressive parameters \( \delta_1 \) and \( \gamma_1 \). High levels of persistence, which imply high values for the autoregressive parameters, give greater weight in the decomposition to a particular series. For instance, if expected returns are more persistent \( (\delta_1 > \gamma_1) \), then most of the variation in the price-dividend ratio is due to expected returns. However, this will also
depend on the variance of the noise terms $\sigma_{\mu}$ and $\sigma_{g}$. For identification purposes in the current paper, we impose the condition that $\rho_{gd} = 0$.

At this stage, some practical aspects of the state-space procedure must be identified. Different aspects of the measurement-state equations can be considered and estimated. For instance, instead of linking the realized growth to unobserved growth, one of the measurement equations can use realized returns as an observed variable and link it to expected returns. Similarly, realized returns and realized dividend growth can be linked to expected returns and expected dividend growth, rather than using the present value identity. The latter is important as it imposes a theoretical limit as to the degree to which expected returns and expected dividend growth can vary together.

3. Results

Data on monthly dividends and the dividend yield were collected from Thompson Reuters DataStream for the period January 1973 until December 2014. Dividends were geometrically compounded to obtain the annual growth rate. The price-dividend ratio was an average of annualized values over these years. For this dataset, the mean annual dividend growth was 7.6%, with a standard deviation of 8.25%. Over the entire period, though the mean return was 8.3%, it also exhibited a higher volatility of 24.6%. The dividend yield was approximately equal to 4.27%, with a standard deviation of 1.52%. The results from optimizing equations (12) to (15) are illustrated in Table 1.

The results indicate that the mean expected dividend growth rate and expected returns are 6.85 % and 10.52 %, respectively. The autoregressive parameter for expected dividend growth is relatively high at 0.553, which is in contrast to Ma and Wohar (2014), who find $\gamma_1 = 0.289$. This may be due to the different sample sizes employed and the
more recent dataset used in the current paper\textsuperscript{4}. Dividend payments in the UK tended to be higher after the 1960s. Such a finding for expected dividend growth is not surprising as evidence from Lintner (1956), Fama and Babick (1968) and Baker et al. (1983) indicates that firms tend to consider past dividends when setting future dividend levels and gradually increase dividends to new higher levels over a period of years. Additionally, firms are slow to cut dividends and seek to maintain disbursements to shareholders (Chowdhury and Miles, 1989).

Expected returns, however, tend to display high persistence ($\delta = 0.9$). The high level of persistence implicitly implies that shocks to the expectations process of investors will impact over longer time periods. Expected returns have become more persistent over recent years, compared with the earlier sample of Ma and Wohar (2014). Shocks to expected returns ($\sigma_\mu = 0.02$) and expected dividend growth ($\sigma_g = 0.07$) are also typically smaller than those reported by these authors. We evidence a high correlation between expected dividend growth and expected returns, implying a slow moving price-dividend ratio. The graphical plots of realized dividend growth against expected dividend growth and realized against expected returns are illustrated in Figures 1 and 2, respectively.

Figure 1 displays the plot of realized dividend growth against expected dividend growth. Dividend growth appears to follow a cyclical pattern. Realized and expected dividend growth values were both particularly high in the late 1970s and before the 2007 to 2009 crisis. During the crisis, both expected dividend growth and realized dividend growth plummeted as companies struggled to maintain cash payments to shareholders.

\textsuperscript{4} Setting different identification conditions do impact on some of the estimates. For instance, assuming $\rho_{ud}=0$ ($\rho_{ug}$) leads to an increase in the persistence levels to 0.65 (0.72).
and investors’ expectations about receiving a cash payout were lowered. Generally, expectations of dividend growth follow from realized dividend growth.

In contrast to the dividend growth values, Figure 2 indicates that expected returns are less volatile than realized. According to the present value literature, prices are high when expected returns are low, which implies future lower returns. Figure 2 indicates that the discount rate is low prior to high returns on the FT All-Share.

We also report the summary statistics of the filtered series in Table 2. The mean growth for realized and expected dividends is 7.6% and 7.3%, respectively, with only a marginal difference between the two. It is worth noting that expected returns have a relatively higher mean (11.6%) than their realized counterparts (8.3%). Realized returns are negatively skewed with a high level of kurtosis compared to their expected returns counterpart. A number of time series implications emerge from an analysis of Table 2. The null hypothesis of no unit root (I(0)) for realized and expected returns is rejected by the KPSS test, and the null hypothesis of a unit root (I(1)) is rejected by the ADF test. Such mixed findings are not uncommon if dividend growth is fractionally integrated as Golinksi et al. (2015) suggest. Moreover, the presence of structural breaks in the realized growth series, as demonstrated by the Nyblom-Hansen break test, may account for these mixed results. Structural breaks distort the choice when selecting between stationary and nonstationary series. Conversely, realized returns are strictly stationary. Expected returns display a near unit root behaviour, which is in stark contrast with the realized returns series. For the expected returns series, the KPSS test rejects the null of stationarity at 1%.

**Likelihood Ratio Tests**

The results illustrated in Tables 1 and 2 reveal that both expected returns and expected
dividend growth are persistent and statistically significant with autoregressive coefficients less than one. Bootstrapped likelihood ratio tests are performed to establish the significance of these findings. The unconstrained model is denoted by $L_0$, and the constrained model is given by $L_1$. The likelihood ratio test statistic is computed as follows:

$$LR = 2(L_1 - L_0)$$

The likelihood ratio is asymptotically distributed as $\chi^2(k)$ where $k$ represents the number of restrictions or constrained parameters. The first test performed examines predictability and time variation in expected returns and expected dividend growth. The null hypotheses of no predictability in returns and dividend growth are given, respectively, as follows:

$H_0: \delta_1 = \sigma = \rho_{\mu} = \rho_{\rho} = 0$

$H_0: \gamma_1 = \sigma = \rho_{g\mu} = 0$

Having established that the autoregressive coefficient for expected returns is close to unity, a test to determine whether expected dividend growth is persistent is conducted.

$H_0: \gamma_1 = 0$

The final test examines whether persistence in expected dividend growth is equal to persistence in expected returns, which involves setting the null hypothesis as follows:

$H_0: \gamma_1 = \delta_1$

The results of these tests are illustrated in Table 3.

The test results are interesting as all of the null hypotheses are rejected. The null hypotheses of no predictability and no time variation are rejected, with computed likelihood ratios of 23.69 and 17.12, respectively. Persistence in expected returns is rejected at the 0.1 % level. Equal persistence between expected returns and expected dividend growth is rejected at the 0.9 % level. Thus, the evidence from the state-space approach to estimating the present value relationship for the UK’s FT All-Share index
confirms previous findings that expected dividend growth is predictable from past values. More importantly, the results suggest that recent findings of persistence in actual returns (or momentum) may be due to persistence in expected returns rather than investor irrationality (Rouwenhurst 1998).

**Variance Decomposition**

The state-space model allows the variance of the price-dividend ratio to be decomposed into an expected dividend growth component and an expected returns component. The variance of the price-dividend ratio is written as follows:

\[ \sigma_{pd}^2 = B_1^2 \sigma_{\mu}^2 + B_2^2 \sigma_g^2 - 2B_1 B_2 \sigma_{\mu g}, \]  

(16)

where \( B_1^2 \sigma_{\mu}^2 \) refers to the proportion of the variance of the price-dividend ratio, which is attributable to the variance of expected returns (discount rate); \( B_2^2 \sigma_g^2 \) is that part of the variance due to a variation in expected dividend growth; \( 2B_1 B_2 \sigma_{\mu g} \) measures the covariation between both components. The percentage of contribution from each component is reported in Table 4.

Table 4 indicates that most of the variation in the price-dividend ratio is derived from expected returns. Furthermore, discount rates do move stock prices, which is consistent with the most recent evidence in the literature from the United States. An examination of Table 4 reveals that 148% of the movement in the price-dividend ratio is strictly associated with changes in expected returns. This contribution is similar to previous findings of Ma and Wohar (2015) but for a larger sample size. Dividend growth contributes only a small portion to the volatility of the price-dividend ratio (19%). The contribution of the covariance is negative as expected because expected dividend growth and expected returns move in opposite directions.
Predictability

The in-sample predictability of expected returns from the state space model is compared to the results of the present value VAR. The VAR approach for stock returns is estimated as follows:

\[
\begin{align*}
    r_{t+1} &= a_r + b_rpd_t + u_{r,t+1}, \\
    \Delta d_{t+1} &= a_d + b_dpd_t + u_{d,t+1}, \\
    pd_{t+1} &= a_{pd} + b_{pd}pd_t + u_{pd,t+1}.
\end{align*}
\]

Equation 17 implies that one-year future returns can be forecast from the price-dividend ratio. Equation 18 forecasts dividend growth from the price-dividend series. Equation 19 imposes an autoregressive structure on the price-dividend ratio, which seeks to capture any persistence that may be present in the series. According to the Campbell-Shiller present value identity, the price-dividend ratio can predict either returns or dividend growth, or both. If returns are unpredictable ($\hat{b}_r = 0$), it means that the variation in the price-dividend ratio is matched by the variation in the expected dividend growth rate. Similarly, if $\hat{b}_d = 0$ (meaning that dividend growth is unpredictable), returns should be predictable. The present value imposes the constraint $b_r = 1 - \rho b_{pd} + b_d$ on the parameters of the VAR.

Following Engsted and Pedersen (2010), joint predictability is tackled through a Monte Carlo approach where respective nulls of no predictability in either returns or dividend growth are imposed. When returns are unpredictable, $u_{r,t+1}$ and $u_{pd,t+1}$ are drawn from the likelihood model to reconstruct the series of the realized returns and the
price-dividend ratio according to the parameters that were initially extracted. The VAR is then estimated, and the number of replications is 20,000. This procedure provides a series of estimated coefficients under respective nulls to which the state-space sample estimate (from Table 1) can be compared. Such a procedure tackles the problem associated with our lack of knowledge regarding the underlying distribution of returns and dividend growth under the present value identity. The results from the VAR are illustrated in Table 5.

Results from Panel A of Table 5 illustrate that both returns and dividend growth are highly predictable using the unconstrained VAR model. The t-values (2.689 and 2.209) are statistically significant, and the $R^2$ statistics are 27.9% and 10.9% for returns and dividend growth equations, respectively. However, the sign of the dividend growth coefficient is positive, not negative. Panels A and B report evidence of predictability after taking into account the signs and the present value restriction. Interpreting $P(b_r > \hat{b}_r)$ provides strong evidence rejecting the null hypothesis of return unpredictability. The evidence also strongly rejects the null of no dividend growth predictability $P(b_d > \hat{b}_d)$. It is further evident that the null of no predictability of returns and dividend growth when the null of no dividend growth is imposed is rejected. In this case, the marginal probabilities reported are close to zero for $P(b_r < \hat{b}_r)$ and $P(b_d < \hat{b}_d)$. The rejection probabilities are similar to those under the null of no predictability. Figure 3 reports the joint distribution of the simulated parameters (panel A) and their corresponding t-statistics (Panel B). The lines from the y-axis represent the OLS estimate (point (Panel A) and t-statistic (Panel B)) for returns. The x-axis presents the corresponding estimate for dividend growth. The point of intersection refers to the case of taking joint nulls into account. Both graphs reveal that the null of no-predictability for both dividend growth
and returns is rejected, as both points of intersection are located away from the cluster of points that are illustrative of the joint distribution under the null hypothesis.

Long horizon predictability in the series is also considered and rejected. The long-run parameters for dividend growth \((b_{d}^{lr})\) and expected returns \((b_{r}^{lr})\) are computed as \(b_{d} / (1 - \rho b_{pd})\) and \(b_{r} / (1 - \rho b_{pd})\). The parameters \((b_{d}^{lr} = 0.494\) and \(b_{r}^{lr} = 2.283\)) and their corresponding distributions are displayed in Figure 4. Figure 4 presents strong evidence against the long run unpredictability of returns and dividend growth. From panel A of Figure 4, the point estimate \(b_{r}^{lr}\) lies outside the distribution of the \(b_{r}^{lr}\) when there is assumed to be no predictability in long run returns. Similarly, there is evidence of unpredictability with respect to dividend growth (Panel B of Figure 4). However, the sign for the dividend growth variable is incorrect since a positive coefficient is estimated.

We also test for the predictability of the filtered series. The predictive regression considered is as follows:

\[
\begin{align*}
    r_{t+1} &= a_r + b_r \mu_t + u_{r,t+1}, \\
    \Delta d_{t+1} &= a_d + b_d g_t + u_{d,t+1}.
\end{align*}
\]

(20)  
(21)

It is found that the filtered series has weak predictability for dividend growth. The slope coefficient is weak, and the Newey-West p-value rejects the null hypothesis at the 12.5% level of significance. Given the sample size, this can be interpreted as extremely weak predictability rather than no predictability at all. However, there appears to be quite good predictability for returns with an \(R^2\) of approximately 23.87%.
**Alternative Frequencies**

The state-space model is also estimated for monthly and quarterly data. At monthly frequencies, dividend growth is defined as the percentage change from the previous month’s reported dividends to the current month’s dividend. The price-dividend ratio is the closing price-dividend ratio. With respect to quarterly data, dividend growth is computed as the percentage change in dividends paid out at the end of one quarter compared to the dividends paid out in the next quarter. The price-dividend ratio is the price-dividend ratio measured out at the end of the quarter. The estimations are reported in Table 7.

There are considerable differences in the findings across the different frequencies. The persistence in expected returns is still high, and it rises as the frequency level increases. Moreover, there is a much higher level of persistence in expected dividend growth at the monthly (0.928) and quarterly (0.941) frequencies. This finding has important implications for predictability as it means that dividend growth should be predictable for longer horizons when higher frequency data are employed. In turn, the high persistence in expected returns also implies better predictability at the higher frequencies. These results provide some strong conclusions for the present value literature. Expected returns still tend to be highly persistent regardless of the frequency over which they are measured. Expected dividend growth, on the contrary, indicates an increase in persistence levels when frequencies are higher although persistence is greater with quarterly rather than monthly data.

The implied present value parameters $B_1, B_2$ demonstrate the levels of persistence in the economy. If both $B_1$ and $\sigma_\mu$ are high, then most of the variation in the price-dividend ratio is derived from the discount rate. This effect is further enhanced when $\sigma_g$ is small. According to the estimated values, most of the variation in the price-dividend
ratio is derived from the discount rate (104 % monthly and 61.14 % quarterly). At the monthly level, dividend growth contributes a meagre 0.85 % towards the variance of the price-dividend ratio. This percentage is somewhat higher for quarterly frequencies at approximately 6.44 %.

**Confidence Intervals of estimates**

One of the main issues in the state-space model, as highlighted by Ma and Wohar (2014), is the low signal-to-noise ratio. A low ratio would suggest that there is “a large amount of uncertainty around the parameter estimates” (Ma and Wohar, 2014, p. 2467); in the current paper, it would imply weak identification of returns and dividend growth, therefore posing an inference issue (Nelson and Startz, 2007; Ma and Nelson 2010). The signal-to-noise ratios, defined as \(\frac{\sigma_g}{\sigma_d}\) and \(\frac{\sigma_u}{\sigma_r}\), respectively, for dividend growth and returns, are computed from the time series of each pair of variables. The ratios indicate that dividend growth has a better ratio (0.636) than returns (0.174). While the signal-to-noise ratio for returns is less than 20% in the case of annual data, there is an improvement in the ratio for returns when quarterly (0.837) and monthly (0.378) observations are employed. This contrasts with relatively low values of the ratio for dividend growth when quarterly (0.090) and monthly (0.141) data are examined.

Given the small signal-to-noise ratio, standard errors may not be correctly estimated, especially in the case of the zero-information limit condition\(^5\). This poses an inference problem, especially in the case of \(\delta_1\), which is close to a unit root. The extent to which this is an issue is elaborated through a reduced-form test in Ma and Wohar (pp.

\(^5\) The zero-information-limit condition is the case where the signal or the precision of the parameter is overestimated.
This involves a 2 stage process where a restricted VARMA is estimated in the first stage and the residuals are extracted. In the second stage, a first-order Taylor expansion of the price-dividend ratio is regressed against the extracted residuals in order to test for any remaining serial correlation. The associated t-values for a given range of $\delta_1$ can then be numerically inverted to produce valid confidence intervals. We illustrate the confidence intervals based on the associated reduced form test for the different frequencies of our data in Figure 5.

The results from this Ma and Wohar test suggest that our earlier conclusion that the contribution of expected returns to prices should be interpreted with caution. The figures plot the corresponding t-statistic for the range of nulls for $\delta_1$. It can be seen that for a range of values for the persistence parameter between -1 and 1, the test does not reject the null hypothesis since the confidence intervals are essentially wider than in the standard inference case. However, it is worth noting that the dividend growth (for annual data) does not suffer from this weak identification problem since the signal-to-noise ratio is relatively large.

**Robustness Check**

Outputs from the state-space approach and the VAR procedure can be connected. Consider equations (20) to (22), which demonstrate how the present value relationship can be estimated as a VAR. Under the assumption of stationarity and using the Wold decomposition, the VAR can be rewritten in matrix notation as a moving average as follows:

$$Y_t = (I - A)^{-1}u_t$$  \hspace{1cm} (22)
where $Y_t = [r_t \Delta d_t - pd_t]'$, $A = \begin{bmatrix} 0 & 0 & b_r \\ 0 & 0 & b_r + \rho b_{pd} - 1 \\ 0 & 0 & b_{pd} \end{bmatrix}$ and $u_t = [u_{r,t} u_{d,t} u_{pd,t}]'$. 

$I$ is an identity matrix. The state variables, expected returns and expected dividend growth must be expressed in the form of the measured variables $pd_{t+1}$ and $\Delta d_{t+1}$. Similar to Binsbergen and Koijen (2010), the original state-space model is rewritten in matrix notation as follows:

Transition equation:

$$S_{t+1} = FS_t + \Gamma \varepsilon_{s,t+1} \quad (23)$$

Measurement equation:

$$W_t = Z_0 + Z_1 W_{t-1} + Z_2 S_t \quad (17)$$

where

$$S_t = [\bar{\varepsilon}_{d,t} \varepsilon_{g,t} \varepsilon_{\mu,t}]'$$

$$W_t = [\Delta d_t \ pd_t]'$$

$$F_1 = \begin{bmatrix} \gamma_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Gamma = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Z_0 = \begin{bmatrix} \gamma_0 \\ 1 - \delta_1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} 0 \\ 0 \delta_1 \end{bmatrix} \quad Z_2 = \begin{bmatrix} 1 \\ B_2 (\gamma_1 - \delta_1) \quad 1 \\ B_2 \quad 0 \quad 0 \quad B_1 \end{bmatrix}$$

Under conditions of stationarity, the reduced form of (24) is equal to the following:

$$W_t = (I - Z_1)^{-1}(Z_0 + Z_2 S_t)$$

It is further noted that $W_t = MY_t$

where $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Matrix $M$ ensures that the constraint is binding. Expected returns is a reduced form equation derived by linking the unobserved state variables to the structural shocks of the VAR:

$$S_t = \tilde{Z}_2 [\tilde{Z}_1 M \tilde{A}^{-1} u_t - Z_0]$$

(25)

where $\tilde{Z}_2$ is the matrix containing the inverse of individual elements in $Z_2$. $\tilde{Z}_1$ and $\tilde{A}$ are defined, respectively, as $I - Z_1$ and $I - A$.

Equation (25) links the unobserved state variables to the structural shocks from the constrained VAR. The sample structural residuals $\tilde{u}_t$ can be used to construct the implied state vector. The structural shocks from the state space and those of the VAR are henceforth related given the present value approximation condition and the first order autoregression of expected returns and expected dividend growth rate. It is further noted that the present value restriction can also be reconciled with simulated values of $\Delta d_{t+1}$ and $pd_{t+1}$. Table 8 illustrates the case where random draws of $\tilde{u}_t$ are used to reconstruct the measurement variables $W_t$ according to the present value restriction and the state space applied therein.

Table 8 reports the median estimate of the state-space parameters from simulating the $Y_t$ through the VAR using the empirical distribution of the residuals ($u_t$). The mean squared error is computed as the average squared difference between the state-space estimate and the median estimate from the Monte Carlo simulations. A high mean squared error indicates that the sample estimate reported in Tables 1 and 7 deviates considerably
from the median estimate of the Monte Carlo experiments. Based on the reported mean squared error, most of the state-space estimates lie close to the median estimates.

The good fit of the model is evident, especially in the case of the standard deviations ($\sigma_\mu, \sigma_g, \sigma_d$) and the intercept terms ($\gamma_0, \delta_0$) of the latent variables. These parameters exhibit a mean squared error of less than 0.01. The correlation between expected returns and expected dividend growth is imprecisely estimated, however, and this tends to be exacerbated in the case of quarterly and monthly observations. With respect to the autoregressive parameter estimates, we find evidence of a moderate mean squared error ranging between 0.95 and 0.295. Though the persistence parameter is poorly estimated at annual frequencies, this is offset by a better and closer estimate of the VAR. At higher frequencies, persistence estimates of expected returns perform better at the cost of expected dividend growth.

Concluding Remarks.

Novel findings are reported regarding the sources of fluctuations in equity prices in the UK using both VAR models and state-space models. Both models conclude that discount rates play a more prominent role in movements in the price-dividend ratio and that expected returns, which are time-varying, are more persistent than expected return growth. Extending the analysis to higher frequencies indicates that dividend growth tends to become as persistent as the discount rate.

The expected return performs slightly worse than the present value constrained VAR framework, and there is no evidence of dividend growth predictability. The decomposition of the price-dividend ratio indicates that the expected dividend growth is as important as the discount rate in moving the price-dividend ratio. However, at higher
frequencies, we find that the discount rate becomes more important than the expected dividend growth, though the latter displays higher persistence.

It is important to note certain limitations of this study. One such limitation is the modest sample size. However, the new sample reflects better recent developments in stock market operations, which perhaps a larger sample size may fail to capture. Additionally, it is important to consider problems with inference when parameters are close to unit roots, such as in the case of expected returns. Robustness checks indicate that considerable care must be taken when considering the correlation coefficients.
Bibliography:


