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Affirmative Action Policy and Effort Levels. Sequential-Move Contest Game Argument

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Affirmative Action Policy and Effort Levels. Sequential-Move Contest Game Argument*

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Abstract

In this paper we analyse a simple two-person sequential-move contest game with heterogeneous players. Assuming that the heterogeneity could be the consequence of past discrimination, we study the effects of implementation of affirmative action policy, which tackles this heterogeneity by compensating discriminated players, and compare them with the situation in which the heterogeneity is ignored and the contestants are treated equally. In our analysis we consider different orders of moves. We show that the order of moves of contestants is a very important factor in determination of the effects of the implementation of the affirmative action policy. We also prove that in such cases a significant role is played by the level of the heterogeneity of individuals. In particular, in contrast to the present-in-the-literature predictions, we demonstrate that as a consequence of the interplay of these two factors, the response to the implementation of the affirmative action policy option may be the decrease in the total equilibrium effort level of the contestants in comparison to the unbiased contest game.

Keywords: Asymmetric contest; sequential-move contest; affirmative action; discrimination;

JEL classification: C72; D63; I38; J78

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1 Introduction

Affirmative action is a public policy instrument that has as its object the amelioration of the adverse effects of discrimination on affected groups of individuals. Practical implementation of affirmative action programs, however, is the source of intense public discussion. One of the main issues in this discussion is potential consequences of affirmative action policy with respect to effort incentives and how this affects the effort levels of both – the discriminated and non-discriminated individual. Economic theory that addresses the problem of potential effects of affirmative action programs with respect to effort provision is very scarce and provides little guidance. The main aim of this paper is an attempt to bridge this gap in theoretical analysis by addressing the question as regards possible effort level effects of affirmative action policy.

The problem of effort provision under equal treatment and affirmative action policy was studied in Franke (2007). The author develops a simple model which is a version of a rent-seeking game in the style of Tullock (1980) with heterogeneous players. In his work two potential ethical interpretations, that hold contestants ethically responsible or not responsible for the source of their heterogeneity, lead to two policy options: equal treatment policy and affirmative action policy. Both policies are defined formally as restrictions on the contest rule which, depending on the implemented policy option, imply different effort incentives for the individuals. With this formulation, the key question studied by the author is how individuals react to the changes in incentives that are induced by the two policies. He shows that, as the response to the implementation of affirmative action policy, the total effort level of players always increases.

One of the main assumptions in Franke (2007) is that players always move simultaneously. This assumption is typical in the contest game literature and may hold in some contexts, especially when the competitors do not have possibility to observe each other. In many other contexts, however, agents do not decide about their effort levels at the same time and may observe each other[1]. Therefore, the objective of this paper

[1]Real contests (e.g., promotion tournaments or tournaments between salesmen) show that agents often act sequentially and may be able to observe their competitors' efforts when deciding on their own effort. Hence, the individuals may get some information during the game, which will influence their succeeding effort choices. It is obvious that these features cannot be discussed within a simultaneous-move game. In addition, some contests are even organized sequentially in practice, which holds for diverse sport contests. This sequential-move approach finds justification also on microeconomic theory grounds (for instance in Leininger (1993), Morgan (2003) and Baik and Shogren (1992)).

is to study the problem of effort provision under equal treatment and affirmative action policy in a sequential-move case. Using similar setting as in Franke (2007), we consider a contest in which individuals are assumed to choose their efforts sequentially and can observe their opponent's choices. In our two-player contest, one of the agents first chooses his effort. After that, the other agent observes this effort and then has to decide about his own effort level. Using the formal definitions of the policies as in Franke (2007), we study how individuals react to the changes in incentives induced by the two policy options and how those reactions depend on the order of their moves[2].

Using our model we show that the order of moves of contestants is a very important factor in determination of the effects of the implementation of affirmative action policy. Moreover, we prove that in such cases a significant role is played by the level of the heterogeneity of individuals. In particular, contrary to Franke (2007), it is shown that in a two-player contest game, as the response to the implementation of the affirmative action policy option, the total equilibrium effort level of the contestants may decrease in comparison to the unbiased contest game. This happens when the non-discriminated individual moves before the discriminated one and the underlying heterogeneity of individuals is not too severe. However, when the underlying heterogeneity of individuals is very severe or both individuals move in the reversed order, then the effect of the affirmative action policy option on the total equilibrium effort level

[2]The economic literature on sequential-move contests suggests that, when the leader is a "non-discriminated" player (with lower marginal cost of effort), and the follower is a "discriminated" one (with higher marginal cost of effort), then higher effort levels exerted by the first agent are met by lower effort levels exerted by the second agent. As a consequence, the first agent can credibly commit to increase his effort expenditures knowing that this will be profitable in increased probability of winning owing to the second agent's reduction in expenditures. Intuitively, the implementation of affirmative action policy, which favors a discriminated contestant, should increase his effort expenditure at the cost of decreasing the effort of a non-discriminated contestant. The net effect on the sum of equilibrium effort levels is therefore unclear and would depend on the size of individuals' reactions to the changes in the incentives. This is one of our key questions that we study in this paper. In the opposite case, as the literature suggests, when the leader is a discriminated player and the follower is a non-discriminated one, the first agent can afford to reduce his exerted effort level, knowing that the second agent will follow this behavior and likewise reduce. In this case, the possibility to commit to a lower level of effort enables the discriminated player to credibly reduce the "aggressiveness" of the contest. Again, intuitively, the implementation of affirmative action policy, which favors a discriminated contestant, should increase his effort expenditure and also of his opponent - of a non-discriminated contestant. The net effect on the total effort seems here to be positive. Checking this prediction is another key question of our paper.

is always positive, as in Franke (2007).

In our analysis we also compare the games with different orders of moves, including a simultaneous-move one, given the policy option: the equal treatment or the affirmative action. This part of our analysis reveals that under the affirmative action policy option, the order of moves is irrelevant. In this situation, independently of the order of moves – when the non-discriminated individual moves before the discriminated one, the other way round or when they move simultaneously, each individual exerts his effort at the same level and the total effort level is the same.

As a part of our analysis, we also study preferences of the contest organizer for the two policy options, assuming that he is purely interested in maximizing the total effort level exerted by the contestants. We show that if the contest organizer cannot specify the order of moves of the contestants in a game, that is this order is determined endogenously by the players, then he always prefers the implementation of affirmative action policy to equal treatment policy. These preferences are somehow changed, if specified order of moves is exogenous for the players, and the contest organizer is able to make them move according to this order. In such a situation, in general in all cases of the order of moves the contest organizer prefers the implementation of affirmative action policy to equal treatment policy. The exception is the case in which the non-discriminated individual moves before the discriminated one. In this particular situation – independently of whether this order of moves is exogenous for the contest organizer or whether it is selected by him as the one which maximizes his utility – he may sometimes prefer the implementation of equal treatment policy to affirmative action policy. This happens when the underlying heterogeneity of the individuals is not too severe.

Our work is related to some other models in the economic theory literature. It builds directly on the paper by Franke (2007), mentioned earlier, who investigates the problem of effort provision under equal treatment and affirmative action policy in a simultaneous-move setting. It is also related to numerous papers that study the effects of affirmative action policy in various settings. Most closely related to our paper are studies of the effects of affirmative action policy in competitive situations. For instance Fryer and Loury (2005) consider a simple model of pair-wise tournament competition to investigate group-sighted and group-blind forms of affirmative action in winner-take-all-markets. Fu (2006) addresses the problem of affirmative action policy in admissions to a college using a two-player all-pay auction model. Schotterand, Weigelt (1992) using the tournament-game framework study experimentally whether affirmative action programs and equal opportunity laws affect the output of economic agents. Our work is also related to the papers that study sequential-move contest games. For instance, Leininger (1993) shows that in a two-player contest with heterogenous contestants sequential play arises endogenously. When contestants are homogenous, either sequential or simultaneous play can arise as equilibrium. The heterogeneity is understood there as the fact that agents have asymmetric valuations of the prize[3]. In Leininger (1993) the timing decision occurs following the realization of valuations for each of the contestants. This is what makes the Leininger (1993)'s model different from the one in Morgan (2003), where the timing decision is determined prior to the realization of specific valuations. In Morgan (2003) contestants are ex ante homogenous and always move sequentially in equilibrium. Baik and Shogren (1992) analyzed a contest with heterogenous contestants in a very similar but a little more general framework than Leininger (1993). They also show that sequential play arises endogenously as in Leininger (1993). Ludwig (2006) compares sequential and simultaneous contests under different informational settings: contestants' types are either public or private information. Glazer and Hassin (2000) study a rent-seeking contest in which n firms choose their expenditures sequentially, one after the other one. Dixit (1987) considers the value of precommitment of effort by heterogenous players, which can occur for instance in a sequential-move contest. The problem of precommitment of effort appears also in Wärneryd (2000), who studies strategic delegation in two-player contests. Baik, Cherry, Kroll and Shogren (1992), Shogren and Baik (1992) and Weimann, Yang and Vogt (2000) study experimentally the theoretical predictions of the sequential-move contest game theory. Other papers with similar focus as the ones about sequential-move contest games are those related to the theory of tournaments. For instance, Jost and Kräkel (2005, 2006 and 2007) compare contestants' strategic behavior and study the principal's optimal choice of the prize spread in sequential-move and simultaneous-move tournaments.

We formulated our contest model in general terms, which allows reflecting in a stylized way a variety of situations in which the implementation of affirmative action can have consequences on the incentive structure of effort provision. Possible real world examples of contest-like environments in which the effect of the implementation of the affirmation action policy on the levels of exerted effort may be an important

[3] Leininger (1993) uses a slightly different form of a utility function, which is an affine transformation of ours. In terms of our current discussion a disadvantaged agent is the one with lower valuation of the prize.

issue cover for instance real corporate tournaments, such as sales contests in which salesmen compete for a bonus. Discriminated workers might get some type of limited advantage to guarantee a more balanced competition. Moreover, in these contests often workers do not decide perfectly simultaneously in a one-shot game. On the contrary, workers can observe each other during a certain period in which the tournament takes place and choose their efforts in sequence. Another example covers sport contests. In some of them, mainly amateur ones, advantaged competitors are artificially handicapped by the contest rule to guarantee a level playing field. In such cases there exist special systems of calculation of scores based on past player's performance which allows players of different proficiency to play against each other on somewhat equal terms. A more experienced player is disadvantaged in order to make it possible for a less experienced player to participate in the game or sport while maintaining fairness. Examples of this we can find for instance in golfing, bowling or track and field sports like showjumping, shotput, broad jump, high jump and other. Moreover, all these sport contests in practice are organized as sequential ones. A little different example that comes from sports, but which also fits our discussion is the one of ski-jumping. Here again, the competition takes place in a sequential way. Although by the contest rule of this sport all players are treated equally in terms of calculation of scores, clearly there exists some limited advantage of skiers who jump earlier than later. This advantage comes from the fact that the condition of the in-run tracks gets worse as more skiers use it, so that the later players are handicapped. Those who are handicapped are typically more experienced players, as by the contest rules they jump later.

This paper is organized as follows: In Section 2 we introduce our model and present formal definitions of equal treatment and affirmative action policy. In Section 3 we perform the analysis of the total effort level for the two policy options under different scenarios of the order of moves. Section 4 concludes.

2 The model

Affirmative action instruments are typically employed in situations of competitive social interaction. A convenient tool that can be used to capture the competitive structure of these situations is a contest game in which contestants compete for an indivisible prize. In this game, by exerting more effort the contestants can increase their respective probability of winning the contested prize, which reflects the basic structure of many situations of competitive social interaction. Moreover, there exists a relatively high grade of discretion on the side of the competition organizer, which implies that contestants face a probabilistic outcome. This property is also reflected by a contest game model.

A distinguishing feature of our model relative to those already existing in the literature lies in the assumption on the timing of effort level decisions of contestants. In our version of a contest game model effort level decisions are made sequentially: a contestant (the follower) makes his effort level decision after his opponent (the leader) has already made his.

To guarantee analytical tractability and closed form solutions, our model is formulated under complete information, i.e. the only element of uncertainty is the final winner of the contest. In the paper we will use the standard notion of Subgame Perfect Equilibrium (SPE).

2.1 Primitives

Let $N = \{1, 2\}$ denote the set of risk-neutral individuals who compete against each other in a sequential-move contest game. To win the contest, each contestant $i \in N$ exerts an effort level $e_i \in R^+$, while his opponent – a contestant $j \in N, i \neq j$ – exerts an effort level $e_j \in R^+$. It is assumed that both contestants have the same positive valuation V for the contested prize. Apart from the timing of an effort level decision, the contestants differ in the respective "cost function" that captures the disutility of exerting effort e_i. This function depends on a parameter β_i that (potentially) reflects the degree of discrimination of a contestant i. It is assumed that for all $i \in N$ this cost function is linear in e_i and multiplicative in β_i, such that:

$$c_i(e_i) = \beta_i e_i. \tag{1}$$

We assume also that the contestants are heterogenous in terms of their marginal cost parameter and are ordered, such that $\beta_1 < \beta_2$, with normalization $\beta_1 = 1$. We denote $\beta_2 = \beta$.

The contestants perceive the outcome of the contest game as probabilistic. However, they can influence the probability of winning by exerting effort, which means that the outcome depends on the vector of effort levels exerted by both individuals. In our model we will employ the following Contest Success Function (CSF) $p_i : R^2_+ \rightarrow [0, 1]$:

$$p_i(e_i, e_j) = \frac{\alpha_i^P e_i}{\alpha_i^P e_i + \alpha_j^P e_j}, \text{ for all } i \in N, \tag{2}$$

with $\alpha_i^P > 0$ for all $i \in N$. This function maps the vector of effort levels (e_i, e_j) into win probabilities for each contestant. This is a restricted version of a CSF axiomatized in Clark and Riis (1998)[4]. This function possesses a very convenient feature that allows an asymmetric treatment of the contestants that can be interpreted as affirmative action policy. This is done by appropriate setting values of positive weights α_i^P, that depend on the policy P, which will be defined formally in the next section. If no contestant exerts positive effort, it is assumed that none of the individuals receives the prize, i.e. $p_i(0,0) = 0$ for all $i \in N$[5].

A contestant $i \in N$ aims to maximize his expected utility, which, given the cost function (1) and the contest mechanism (2), takes the following (additive separable) form:

$$u_i\left(e_i, e_j\right) = p_i(e_i, e_j)V - \beta_i e_i. \tag{3}$$

The contestants make their respective effort decisions sequentially. With respect to timing of these decisions we study two cases: one in which the contestant who is less (or not) discriminated (the one with lower marginal cost β_i, the stronger contestant) is the leader (moves first) (Case 1) and the other one in which he is the follower (moves second) (Case 2). In both cases, the implemented policy option P is announced to both contestants before the leader's move.

2.2 The Policy Options

In this section we describe two policy options P which later will be compared in terms of total effort levels that they generate[6].

We assume that the choice of the policy P is based on the ethical perception of the heterogeneity of the contestants (i.e. the different marginal cost functions). This directly implies what is the normative objective of the respective policy option and therefore determines the individual effort weights (α_i^P, α_j^P).

Two potential ethical interpretations that hold contestants ethically responsible or not responsible for the source of the heterogeneity, lead to

[4]In Clark and Riis (1998) the CSF has the form $p_i(e_i, e_j) = \frac{\alpha_i^P e_i^r}{\alpha_i^P e_i^r + \alpha_j^P e_j^r}$, for all $i \in N$, with $r > 0$. The parameter r measures the sensitivity of the outcome of the contest game with respect to differences in effort. The assumption about r is needed because for a non-linear CSF with a general parameter $r > 0$ it is not possible to derive closed form solutions. As the existence of closed form solutions is crucial for the comparative analysis of the policy alternatives, it is assumed that the CSF is linear with $r = 1$. Also with a general parameter $r > 0$ the existence of pure strategy equilibria cannot be guaranteed (see Baye, Kovenock and de Vries (1994) for details). With the restriction $r = 1$ all our equilibria are in pure strategies.

[5]Another convention in the contest-game literature is that $p_i(0,0) = \frac{1}{2}$ for all $i \in N$. The choice of either definition is not important in terms of the results that we obtain in this paper.

[6]This section follows Franke (2007).

two policy options: equal treatment policy (ET) and affirmative action policy (AA). Equal treatment policy follows the interpretation that the contestants are held ethically responsible for their respective cost function, and in this case the probability to win the contest game (i.e. the CSF) should only depend on the vector of exerted effort. This means that if a contestant i exerts the same effort level as a contestant j, then both contestants should win the contest game with the same probability. Hence this policy option treats the contestants equally with respect to their exerted effort level.

Definition 1 *A policy is called equal treatment approach (ET) if:*

$$e_i = e_j \Rightarrow p_i(e_i, e_j) = p_j(e_i, e_j) \text{ for all } i \neq j.$$

For our CSF defined in (2) this definition implies that policy weights must be equal for all players, that is

$$\alpha_i^{ET} = \alpha^{ET} \text{ for all } i \in N.$$

Note that this policy postulates that the contest success function neither depends on the specific names nor on the exogenous characteristics of the players, therefore it could also be interpreted as an anonymity principle. However, the outcome, i.e. expected equilibrium utility, of the contest game will be indirectly determined by the characteristics of the players, as the weaker player will exert less effort in equilibrium.

In turn, affirmative action policy follows the interpretation that contestants cannot be held ethically responsible for their heterogeneity. This can be justified, for instance, because it is the consequence of past discrimination. In this case fairness requires that two contestants who face equal disutility induced by the chosen effort level (that could be different) should have the same probability to win the contest game.

Definition 2 *A policy is called affirmative action (AA) if:*

$$c_i(e_i) = c_j(e_j) \Rightarrow p_i(e_i, e_j) = p_j(e_i, e_j) \text{ for all } i \neq j.$$

For our CSF defined in (2) this definition implies that policy weights must satisfy for all players the following relation

$$\frac{\alpha_i^{AA}}{\beta_i} = \frac{\alpha_j^{AA}}{\beta_j} \text{ for all } i \neq j.$$

Given that the CSF is homogenous of degree zero, without loss of generality these weights can be normalized such that

$$\alpha_i^{AA} = \beta_i \text{ for all } i \in N.$$

The AA policy generates thus a bias of the CSF in favor of discriminated contestants in such a way that both contestants have the same probability of winning the contest whenever they face the same disutility of effort[7].

In the following section we will carry out an analysis of the two normative policy options by comparing an aggregated measure of total effort, i.e. the sum of equilibrium effort that each policy induces. In situations in which affirmative action is potentially implemented, using the total equilibrium effort as the standard of comparison seems to be appropriate because it captures the notion of social loss (or gain)[8]. Hence, the total equilibrium effort can be interpreted here as a measure of "social efficiency". For instance, in sport competitions it can be argued that spectators are interested in the overall performance of all players because ex-ante predictable sport competitions are usually perceived as boring. Also in the corporate tournaments, where this kind of tournament acts as an incentive device, the employer is obviously interested in high effort levels by all employees, irrespective of the identity of the final winner.

The equilibrium effort level of each contestant will depend on the ex-ante announced policy parameter P and the standard of comparison will therefore be expressed and denoted in the following way: $E_P^* = \sum_{i \in N} e_i^*(P)$ for $P \in \{ET, AA\}$. Additionally, this specification also allows the analysis of individual choices of the contestants by comparing equilibrium effort for individual contestants.

3 Analysis

We start by solving our model by backward induction in a general case, with a contestant i being the leader and a contestant j being the follower.

We first look at optimal effort level decision of a contestant j. Using the CSF in eq. (2), the expected utility function in eq. (3) for a

[7] It is worth noting that the results in terms of the effects of AA policy may depend on the interplay between the specification of the CSF in eq. (2) and how the AA policy is implemented. Notice that if we use the CSF in a general form $p_i(e_i, e_j) = \frac{\alpha_i^P e_i^r}{\alpha_i^P e_i^r + \alpha_j^P e_j^r}$ with $r \to 0$, then a limiting CSF is $p_i(e_i, e_j) = \frac{\beta_i}{\beta_i + \beta_j}$ under the AA policy, and $p_i(e_i, e_j) = \frac{1}{\#N}$ (the fair lottery) under the ET policy. With such CSFs the total effort level will be zero and the results on the AA will be different.

[8] Comparing the effect of the implementation of ET and AA policy in terms of winning probabilities given by the CSF in eq. (2), we may notice that AA policy aims at choosing the discriminated agent (with the higher marginal cost parameter β_i) more often than ET policy. Therefore, other standard of comparison that we can think of here could relate to the equilibrium win probabilities.

contestant j given the policy option $P \in \{ET, AA\}$ can be written as

$$u_j(e_i, e_j) = \frac{\alpha_i^P e_j}{\alpha_i^P e_i + \alpha_j^P e_j} V - \beta_j e_j \text{ for } j \in N, i \neq j. \tag{4}$$

The problem of a contestant j is to maximize this function with respect to his non-negative effort level e_j, given a non-negative effort level of a contestant i, e_i, and a policy parameter P. The first order condition yields

$$\frac{\alpha_j^P \alpha_i^P e_i}{\left(\alpha_i^P e_i + \alpha_j^P e_j\right)^2} V - \beta_j = 0,$$

which after considering a non-negativity constraint on the effort level of a contestant j produces

$$e_j = \frac{\sqrt{\alpha_j^P \alpha_i^P e_i V} - \alpha_i^P e_i \sqrt{\beta_j}}{\alpha_j^P \sqrt{\beta_j}}.$$

So the effort level candidate of a contestant j given an effort level of a contestant i can be written as:

$$\begin{aligned} e_j(e_i) &= \frac{\sqrt{\alpha_j^P \alpha_i^P e_i V} - \alpha_i^P e_i \sqrt{\beta_j}}{\alpha_j^P \sqrt{\beta_j}} \\ &= \sqrt{\frac{\alpha_i^P}{\alpha_j^P} \frac{V}{\beta_j} e_i} - \frac{\alpha_i^P}{\alpha_j^P} e_i. \end{aligned}$$

In some situations this equation can produce non-positive effort levels. This happens if

$$\sqrt{\alpha_j^P \alpha_i^P e_i V} - \alpha_i^P e_i \sqrt{\beta_j} \leq 0,$$

that is when

$$e_i = 0 \text{ or } e_i \geq \frac{\alpha_j^P}{\alpha_i^P} \frac{V}{\beta_j}. \tag{5}$$

As the effort level of a contestant j is constrained to be non-negative, it follows that in all such cases in which the condition (5) is satisfied, his effort level is zero. Note however, that a situation in which both contestants exert effort at a zero level cannot be equilibrium. Then a very small increase in an effort level of one of the contestants makes him win the game with certainty with positive pay-off. Therefore we may omit the case of $e_i = 0$ in our further analysis, without losing generality.

With this, the solution of maximization problem of a contestant j given an effort level of a contestant i becomes

$$e_j(e_i) = \begin{cases} \sqrt{\frac{\alpha_i^P}{\alpha_j^P}\frac{V}{\beta_j}e_i} - \frac{\alpha_i^P}{\alpha_j^P}e_i, \text{if } e_i < \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}, \\ 0, \qquad \text{if } e_i \geq \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}. \end{cases} \tag{6}$$

The second order condition for a contestant j yields

$$\frac{\partial^2 u_j(e_i, e_j)}{\partial e_j^2} = -\frac{2\alpha_i^P e_i \left(\alpha_j^P\right)^2}{\left(\alpha_i^P e_i + \alpha_j^P e_j\right)^3} V < 0,$$

which proves concavity, as long as an effort level of a contestant i is strictly positive. As we will show later in our analysis, this is in fact true. Thus the maximum exists and is unique.

Now we turn to the maximization problem of the leader, a contestant i. Using the CSF in eq. (2), the expected utility function in eq. (3) for a contestant i given the policy option $P \in \{ET, AA\}$ can be written as

$$u_i(e_i, e_j(e_i)) = \frac{\alpha_i^P e_i}{\alpha_i^P e_i + \alpha_j^P e_j(e_i)} V - \beta_i e_i \text{ for } i \in N, i \neq j, \tag{7}$$

where $e_j(e_i)$ denotes a best reply function of a contestant j given in (6).

Consider first a case of $e_i < \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}$. A contestant j exerts then an effort at a positive level, and his best reply function $e_j(e_i)$ is given by the first-line expression in (6). With this the first order condition for a contestant i yields

$$\frac{1}{2}\sqrt{\frac{\alpha_i^P}{\alpha_j^P}\frac{\beta_j V}{e_i}} - \beta_i = 0,$$

whose only solution is

$$e_i = \frac{\alpha_i^P}{\alpha_j^P}\frac{\beta_j V}{4\beta_i^2}. \tag{8}$$

Notice, that an effort level given by this solution is always strictly positive. It follows, that in this case both contestants: a contestant i being the leader and a contestant j being the follower are active in a game.

In a case of $e_i \geq \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}$, by (6) a contestant j exerts an effort at a zero level and his best reply function is $e_j(e_i) = 0$. Then a contestant i wins the game with certainty. As exerting effort causes disutility, by eq. (7) a maximization problem of a contestant i just requires to minimize a level of his exerted effort over its domain, which produces

$$e_i = \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}, \tag{9}$$

and which is of course always positive. It follows, that in this case a contestant i – the leader is active in a game and a contestant j – the follower is not.

Note, that by plugging an effort level given in eq. (8) into the condition $e_i < \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}$, we obtain that this effort level satisfies the condition whenever

$$2\alpha_j^P\beta_i - \alpha_i^P\beta_j > 0.$$

This result implies that the condition $e_i \geq \frac{\alpha_j^P}{\alpha_i^P}\frac{V}{\beta_j}$ may be expressed as

$$2\alpha_j^P\beta_i - \alpha_i^P\beta_j \leq 0.$$

As our discussion showed, these two last conditions, jointly written as

$$2\alpha_j^P\beta_i - \alpha_i^P\beta_j \gtreqqless 0, \tag{10}$$

determine whether contestants are playing a game in which both – the leader and the follower – exert effort at a positive level – that is both are active or whether they are playing a game in which only the leader is active and the follower is not. To understand better their meaning we rewrite (10) in relative terms, which produces

$$2\frac{\alpha_j^P}{\alpha_i^P} \gtreqqless \frac{\beta_j}{\beta_i}, \tag{11}$$

where $\frac{\alpha_j^P}{\alpha_i^P}$ is the follower's relative policy weight – and $\frac{\beta_j}{\beta_i}$ is the follower's relative marginal cost – with respect to the leader. The expression (11) reveals that what really matters in terms of the equilibrium type – with one or two contestants active – is the relation between the follower's relative policy weight and his relative marginal cost. To interpret, fix the values of the leader's policy weight α_i^P and marginal cost parameter β_i. Fix also for a moment the follower's cost parameter β_j. If the follower's relative policy weight is big enough, such that the LHS of the expression (11) is strictly greater that the its RHS, then $2\alpha_j^P\beta_i - \alpha_i^P\beta_j > 0$ holds and by our previous discussion contestants are playing a game in which both are active. This requires α_j^P be big enough. However, when the opposite holds – that is the follower's relative policy weight is small enough, such that the LHS of the expression (11) is less than or equal to its RHS, then $2\alpha_j^P\beta_i - \alpha_i^P\beta_j \leq 0$ holds and, as our previous analysis showed, contestants are playing a game in which the follower is not active. This requires α_j^P be small enough. Note that increasing the value of the follower's cost parameter β_j, shifts upwards his relative marginal cost. As a result of this increase, the minimal level of the leader's policy weight

α_i^P, which is necessary for the LHS of the expression (11) to be greater than its LHS and which guarantees that both agents are active in a game, goes up.

It follows from our analysis, that the solution of the maximization problem of a contestant i is

$$e_i = \begin{cases} \frac{\alpha_i^P \beta_j}{4\alpha_j^P \beta_i^2} V, \text{if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j > 0, \\ \frac{\alpha_j^P}{\alpha_i^P \beta_j} V, \text{ if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j \leq 0. \end{cases} \quad (12)$$

The second order condition for a contestant i yields

$$\frac{\partial^2 u_i(e_i, e_j)}{\partial e_i^2} = -\frac{\sqrt{\alpha_i^P \beta_j V}}{4\sqrt{\alpha_j^P e_i^3}} < 0,$$

which proves concavity. Hence the maximum exists and is unique.

It follows from our analysis, that there exists a unique equilibrium, in which a contestant i – the leader – always exerts positive effort, and a contestant j – the follower– exerts effort at a non-negative level. By eq. (12), and by eq. (6) after plugging into it corresponding values of eq. (12), those equilibrium effort levels given the policy option P are

$$\begin{cases} e_i^*(P) = \frac{\alpha_i^P \beta_j}{4\alpha_j^P \beta_i^2} V \\ e_j^*(P) = \frac{\alpha_i^P \left(2\alpha_j^P \beta_i - \alpha_i^P \beta_j\right)}{4\left(\alpha_j^P\right)^2 \beta_i^2} V \end{cases}, \text{ if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j > 0, \quad (13)$$

and

$$\begin{cases} e_i^*(P) = \frac{\alpha_j^P}{\alpha_i^P \beta_j} V \\ e_j^*(P) = 0 \end{cases}, \text{ if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j \leq 0. \quad (14)$$

Using eq. (13) and (14) the equilibrium sum of effort levels given the policy option P admits

$$E_P^* = \begin{cases} \frac{\alpha_i^P \left(\alpha_j^P (2\beta_i + \beta_j) - \alpha_i^P \beta_j\right)}{4\left(\alpha_j^P\right)^2 \beta_i^2} V, \text{if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j > 0, \\ \frac{\alpha_j^P}{\alpha_i^P \beta_j} V, \qquad \text{if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j \leq 0. \end{cases} \quad (15)$$

Similarly, the equilibrium expected utility levels given the policy option P admit

$$\begin{cases} u_i\left(e_i^*(P), e_j^*(P)\right) = \frac{\alpha_i^P \beta_j}{4\alpha_j^P \beta_i} V \\ u_j\left(e_i^*(P), e_j^*(P)\right) = \frac{\left(2\alpha_j^P \beta_i - \alpha_i^P \beta_j\right)^2}{4\left(\alpha_j^P\right)^2 \beta_i^2} V \end{cases}, \text{ if } 2\alpha_j^P \beta_i - \alpha_i^P \beta_j > 0, \quad (16)$$

and

$$\begin{cases} u_i\left(e_i^*(P), e_j^*(P)\right) = \frac{\alpha_i^P\beta_j - \alpha_j^P\beta_i}{\alpha_i^P\beta_j}V \\ u_j\left(e_i^*(P), e_j^*(P)\right) = 0 \end{cases}, \text{ if } 2\alpha_j^P\beta_i - \alpha_i^P\beta_j \leq 0. \quad (17)$$

Now we are going to compare the two policy options using as a standard of comparison the sum of equilibrium effort levels. We will also look at individual effort decisions of contestants, which will help us to understand better mechanisms that lead to changes in the sum of equilibrium effort levels under different policy options.

Contestant 1 is assumed to be the one with a lower marginal cost parameter such that $\beta_1 = 1$ (we will call him "the stronger contestant"), and contestant 2 the one with a marginal cost parameter $\beta_2 = \beta > 1$ ("the weaker contestant"). Note that Definitions 1 and 2 imply that the bias for contestant 1 is normalized to $\alpha_1^P = 1$ for $P \in \{ET, AA\}$, and for contestant 2 is $\alpha_2^{ET} = 1$ and $\alpha_2^{AA} = \beta$. In our comparisons we consider two cases: Case 1 in which contestant 1 is the leader and contestant 2 is the follower ($i = 1, j = 2$ in our general solution), and Case 2 in which this order of moves is reversed ($i = 2, j = 1$ in the solution).

In the following we present our results related to the comparison of the two policy options using as a standard of comparison the sum of equilibrium effort levels. For clarity of presentation, the detailed analysis and comparison of the individual effort levels are delegated to Appendix.

3.1 Case 1

We begin our analysis with the case, in which contestant 1 is the leader.

Proposition 1 *In a sequential-move game in which the stronger contestant is the leader, if $\beta < 3$ ($\beta > 3$) the total equilibrium effort level of contestants is lower (higher) under the affirmative action policy than under the equal treatment policy option, and if $\beta = 3$, both policy options produce the same level of the total equilibrium effort. This can be summarized as*

$$E_{ET}^* \gtreqless E_{AA}^*, \textit{ if } \beta \lesseqgtr 3, \textit{ respectively.}$$

Proof. Setting $i = 1$, $j = 2$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i ($i \in N$) in eq. (15) we find that

$$E_{ET}^* = \begin{cases} \frac{1}{2}V, \text{ if } \beta < 2, \\ \frac{1}{\beta}V, \text{ if } \beta \geq 2. \end{cases}$$

and

$$E_{AA}^* = \frac{\beta + 1}{4\beta}V.$$

To prove the proposition we need to show that for $\beta < 2$ the inequality

$$\frac{1}{2}V > \frac{\beta+1}{4\beta}V, \tag{18}$$

and for $\beta \geq 2$ the inequality

$$\frac{1}{\beta}V \gtreqqless \frac{\beta+1}{4\beta}V, \tag{19}$$

if $\beta < 3$, $\beta = 3$ and $\beta > 3$, respectively, are satisfied.

Consider first the case of case of $\beta < 2$, given by the inequality (18). Using some algebra, within the domain of $V > 0$ and $\beta > 0$, we obtain that this inequality reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. It follows, that the inequality (18) is always true for $\beta \in (1, 2)$.

In the second case, if $\beta \geq 2$, within the domain of $V > 0$ and $\beta > 0$, the inequality given in (19) reduces to

$$\beta \lesseqqgtr 3,$$

which is exactly the conditions on β in our claim. ■

It follows from Proposition 1 that in a sequential-move game in which the stronger player is the leader, the implementation of the affirmative action policy option may have detrimental effect on the total equilibrium effort level of players, as compared to the level under the equal treatment policy option. In a particular case, the existence of this effect depends on a marginal cost level of the follower. If players are rather homogenous in terms of their marginal cost of effort, then the implementation of the affirmative action policy option leads in equilibrium to drop in the total effort level of contestants with respect to the level under the equal treatment policy option. The opposite happens if players are very heterogenous - in such a case the total equilibrium effort level of contestants increases. This result is in contrast to the ones that we can find in the economic literature. In particular, in a similar study but assuming that players move simultaneously, Franke (2007) showed that the implementation of the affirmative action policy option is always enhancing the total equilibrium effort level. We show that this doesn't have to be true in a more general case. When players are allowed to move sequentially, then it may happen that the total equilibrium effort level will be lower under the affirmative action policy option than under the equal treatment policy option. Moreover, Franke (2007)'s result is not dependent on the marginal cost parameter of players. We show that if we consider possibility of sequential moves, then levels of marginal cost of players become an important factor in assessing the effects of the policy change.

Apart from studying the total equilibrium effort level, it is interesting to look at its ingredients, that is individual effort decisions of contestants. It allows us to understand deeper various mechanisms that govern the behavior of players under different policy options and lead to the changes in the total equilibrium effort level. Proposition 7 and Proposition 8 in Appendix provide results on comparison of individual effort levels of both players in Case 1 under the two policy options. Using those propositions we may try to explain the mechanisms that lead to the result given in Proposition 1. By Proposition 8, the follower always benefits from the change of the policy from the equal treatment to the affirmative action option and increases his equilibrium effort level. However, the leader's reaction to the policy change is more complex. The leader exerts in equilibrium effort at a higher level under affirmative action policy if $\beta > 4$, and at a lower one if $\beta < 4$. If $\beta = 4$ his equilibrium effort level is the same in both cases. Hence, under affirmative action policy for $\beta > 4$ in equilibrium both contestants exert effort at a higher level and for $\beta = 4$ the follower exerts effort at a higher level and the leader at the same level as under the equal treatment policy option. This implies that if $\beta \geq 4$ the total equilibrium effort level is higher under affirmative action policy than under equal treatment policy. In turn, for $\beta < 4$ under affirmative action policy the follower exerts in equilibrium effort at a higher level and the leader at a lower level. Then if $\beta < 3$ and the policy option changes, in equilibrium the size of the drop in an effort level of the leader is bigger that the size of the increase in an effort level of the follower, which results in the total equilibrium effort level lower under affirmative action policy than under equal treatment policy. In turn, for $\beta \in (3, 4)$ in equilibrium the size of the drop in an effort level of the leader is lower that the size of the increase in an effort level of the follower, which results in the total equilibrium effort level higher under affirmative action policy than under equal treatment policy. For $\beta = 3$, in equilibrium the drop in the effort level of the leader is exactly compensated by the increase in the effort level of the follower, so that the total equilibrium effort level of contestants doesn't change.

3.2 Case 2

Now we study the case in which contestant 1 is the follower.

Proposition 2 *In a sequential-move game in which the stronger contestant is the follower, the total equilibrium effort level of contestants is* *higher under the affirmative action policy than under the equal treatment policy option, that is*

$$E^*_{ET} < E^*_{AA}.$$

Proof. Setting $i = 2$, $j = 1$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i ($i \in N$) in eq. (15) we find that

$$E^*_{ET} = \frac{1}{2\beta}V$$

and

$$E^*_{AA} = \frac{\beta+1}{4\beta}V.$$

We need to show that $E^*_{ET} < E^*_{AA}$ is satisfied in our model, that is that for all $\beta > 1$ the inequality

$$\frac{1}{2\beta}V < \frac{\beta+1}{4\beta}V \tag{20}$$

is true.

Using some algebra, within the domain of $V > 0$ and $\beta > 0$, we obtain that the inequality given in (20) reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. ■

It follows from Proposition 2 that in a sequential-move game in which the weaker player is the leader, the implementation of the affirmative action policy option, has always positive effect on the total equilibrium effort level of players, as compared to the level under the equal treatment policy option. Moreover, this result shows again that the order of moves of contestants in a game matters. When we reverse the order of moves with respect to the one in Case 1, the effect of the policy change in terms of the total equilibrium effort level of contestants is no longer dependent on the value of the marginal cost parameter of the weaker player β. For any admissible value of this parameter this effect is positive.

As in Case 1, in order to be able to understand which mechanisms lead to the changes in the total equilibrium effort level, it is useful to look at individual effort decisions of contestants. Proposition 9 and Proposition 10 in Appendix provide results on comparison of individual effort levels of both players in Case 2 under the two policy options. Using those propositions, it is easy to explain the mechanisms that lead to the result given in Proposition 2. By Proposition 9 and Proposition 10, both contestants in equilibrium benefit from the change of the policy from the equal treatment to the affirmative action option, and increase their effort levels. This implies that the total equilibrium effort rises, which is the result stated in Proposition 2.

	Case 1 (1→2)	**Case 2 (2→1)**	**Case 3 (1↔2)**
$e_1^*(ET)$ $e_2^*(ET)$	$\begin{cases}\frac{\beta}{4}V, \text{if } \beta < 2,\\ \frac{1}{\beta}V, \text{if } \beta \geq 2,\end{cases}$ $\begin{cases}\frac{2-\beta}{4}V, \text{if } \beta < 2,\\ 0, \quad \text{if } \beta \geq 2,\end{cases}$	$\frac{2\beta-1}{4\beta^2}V$ $\frac{1}{4\beta^2}V$	$\frac{\beta}{(\beta+1)^2}V$ $\frac{1}{(\beta+1)^2}V$
$e_1^*(AA)$ $e_2^*(AA)$	$\frac{1}{4}V$ $\frac{1}{4\beta}V$	$\frac{1}{4}V$ $\frac{1}{4\beta}V$	$\frac{1}{4}V$ $\frac{1}{4\beta}V$
E_{ET}^*	$\begin{cases}\frac{1}{2}V, \text{if } \beta < 2,\\ \frac{1}{\beta}V, \text{if } \beta \geq 2,\end{cases}$	$\frac{1}{2\beta}V$	$\frac{1}{\beta+1}V$
E_{AA}^*	$\frac{\beta+1}{4\beta}V$	$\frac{\beta+1}{4\beta}V$	$\frac{\beta+1}{4\beta}V$

Table 1: Summary of the results on the equilibrium effort levels

	Case 1 (1→2)	**Case 2 (2→1)**	**Case 3 (1↔2)**
$u_1\left(e_1^*(ET), e_2^*(ET)\right)$ $u_2\left(e_1^*(ET), e_2^*(ET)\right)$	$\begin{cases}\frac{\beta}{4}V, \quad \text{if } \beta < 2,\\ \frac{\beta-1}{\beta}V, \text{if } \beta \geq 2,\end{cases}$ $\begin{cases}\frac{(2-\beta)^2}{4}V, \text{if } \beta < 2,\\ 0, \quad \text{if } \beta \geq 2,\end{cases}$	$\frac{(2\beta-1)^2}{4\beta^2}V$ $\frac{1}{4\beta}V$	$\frac{\beta^2}{(\beta+1)^2}V$ $\frac{1}{(\beta+1)^2}V$
$u_1\left(e_1^*(AA), e_2^*(AA)\right)$ $u_2\left(e_1^*(AA), e_2^*(AA)\right)$	$\frac{1}{4}V$ $\frac{1}{4}V$	$\frac{1}{4}V$ $\frac{1}{4}V$	$\frac{1}{4}V$ $\frac{1}{4}V$

Table 2: Summary of the results on the equilibrium utility levels

3.3 Case 1 vs Case 2

In this section our objective is to compare Case 1 and Case 2 in terms of the total effort levels that they generate. Additionally, to highlight relations between sequential-move and simultaneous-move games, we consider in our analysis Case 3, in which contestants make their effort level decisions simultaneously. To make our current analysis easier, we summarized all the results relevant to this section in Table 1 and Table 2. For a sequential-move game, formulas in Table 1 were derived using eq. (13), (14) and (15), by setting $i = 1$ and $j = 2$ for Case 1, and $i = 2$ and $j = 1$ for Case 2, and using corresponding values of policy weights α_i^P and marginal cost parameter β_i ($i \in N$). The same procedure was used to find formulas in Table 2, using eq. (16) and (17). Additionally, in both tables we computed corresponding formulas related to Case 3 – a simultaneous-move contest (see Franke (2007) for details).

We begin our analysis in this section by comparing the three Cases in terms of the total equilibrium effort level of contestants that they generate under the equal treatment policy option.

Proposition 3 *Under equal treatment policy, the highest level of the total equilibrium effort is generated when the stronger contestant is the leader (Case 1), and the lowest level – when he is the follower (Case 2) in a sequential-move game. The level of the total equilibrium effort when the contestants move simultaneously (Case 3) is (strictly) in-between those produced by Case 1 and 2. It may be summarized as*[9]

$$E^*_{ET,2} < E^*_{ET,3} < E^*_{ET,1}.$$

Proof. According to Table 1, under equal treatment policy, the level of the total equilibrium effort in each Case is

$$E^*_{ET,1} = \begin{cases} \frac{1}{2}V, \text{if } \beta < 2, \\ \frac{1}{\beta}V, \text{if } \beta \geq 2, \end{cases}$$
$$E^*_{ET,2} = \frac{1}{2\beta}V,$$
$$E^*_{ET,3} = \frac{1}{\beta+1}V.$$

So to prove the proposition we need to show that for $\beta > 1$ the inequality

$$\frac{1}{2\beta}V < \frac{1}{\beta+1}V, \tag{21}$$

and for $\beta < 2$ the inequality

$$\frac{1}{\beta+1}V < \frac{1}{2}V, \tag{22}$$

and for $\beta \geq 2$ the inequality

$$\frac{1}{\beta+1}V < \frac{1}{\beta}V, \tag{23}$$

are satisfied.

Consider first the inequality (21). Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. It follows, that the inequality (21) is always true for $\beta > 1$.

[9] To distinguish the total effort levels related to different Cases in our notation we add additional subscript which denotes the number of Case (1,2 or 3). $E^*_{P,k}$ denotes the total equilibrium effort level of contestants in Case k given a policy option P.

Consider now the inequality (22), for $\beta < 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. It follows, that the inequality (22) is always true for $\beta \in (1, 2)$.

Finally consider the inequality (23), for $\beta \geq 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$1 > 0,$$

which is always true. It follows, that the inequality (23) is always satisfied for $\beta \geq 2$. ∎

It follows from Proposition 3 that the highest level of the total equilibrium effort is generated when the stronger contestant is the leader and the lowest – when he is the follower in a sequential-move game. The total equilibrium effort level in a simultaneous-move game is exactly in-between those of a sequential-move game.

Now, we are going to consider the total effort levels that are generated under affirmative action policy.

Proposition 4 *Under affirmative action policy, in a sequential-move game the level of the equilibrium effort for each contestant when the stronger contestant is the leader (Case 1) is the same as when he is the follower (Case 2). Moreover, those common levels of the contestant's equilibrium effort in a sequential-move game are equal to the levels of the contestant's equilibrium effort in a simultaneous-move game (Case 3). This result also implies that in all three cases the level of the total equilibrium effort is the same. Hence,*

$$e^*_{i,1}(AA) = e^*_{i,2}(AA) = e^*_{i,3}(AA) := e^*_{i,1-3}(AA) \text{ for } i \in N^{10},$$

and

$$E^*_{AA,1} = E^*_{AA,2} = E^*_{AA,3} := E^*_{AA,1-3}.$$

Proof. According to Table 1, under affirmative action policy, the levels of the equilibrium effort for each contestant in each Case are

$$e^*_{i,1}(AA) = \begin{cases} e^*_1 = \frac{1}{4}V, \\ e^*_2 = \frac{1}{4\beta}V, \end{cases}$$

$$e^*_{i,2}(AA) = \begin{cases} e^*_1 = \frac{1}{4}V, \\ e^*_2 = \frac{1}{4\beta}V, \end{cases}$$

$$e^*_{i,3}(AA) = \begin{cases} e^*_1 = \frac{1}{4}V, \\ e^*_2 = \frac{1}{4\beta}V. \end{cases}$$

We see that for each contestant they are equal. This result also implies that the level of the total equilibrium effort is the same in each Case, that is

$$E^*_{AA,1} = \frac{\beta+1}{4\beta}V,$$
$$E^*_{AA,2} = \frac{\beta+1}{4\beta}V,$$
$$E^*_{AA,3} = \frac{\beta+1}{4\beta}V,$$

which is also confirmed by data in Table 1. ■

Proposition 4 highlights the essence of affirmative action policy in terms of effort levels. As a result of implementation of affirmative action policy, the order of moves in a game is no longer important. Across the three Cases an equilibrium effort level of a given contestant and consequently – the total equilibrium effort level are the same. This result also implies that under affirmative action policy in all three Cases a given player has in equilibrium the same expected utility level. However, after more detailed analysis, it turns out that equilibrium expected utility levels are equal not only across Cases for a given contestant but also across contestants (see Table 2). This means that affirmative action policy, which formally is equalizing probability to win the contest game if contestants face the same disutility induced by the chosen effort level (that could be different), induces in equilibrium equalization of the expected utility level across contestants. So, although contestants are different in terms of their marginal cost parameter, their expected payoff is the same.

The intuition behind the results in Proposition 4 is the following: the change from equal treatment policy to the affirmative action policy option makes an "asymmetric" ex ante game "symmetric" in the sense that under affirmative action policy two heterogenous players are treated as if they were homogenous. Dixit (1987), Leininger (1993) and Wärneryd (2000) show that the homogeneity of players leads to the symmetric simultaneous-move outcomes in a game where players choose their efforts sequentially. Homogeneity is understood there as the fact that players have the same valuations of the prize[11] and the same level of the marginal cost of effort. In our model, before and after the implementation of affirmative action policy players are heterogenous because they differ

[11] In Leininger (1993) the same valuation of the prize is understood as the fact that the valuation of one agent is equal to the valuation of other agent multiplied by some constant. This constant accounts for some differences in the way agents are treated by the contest rule, which is similar to what we have in our model.

in terms of their marginal cost of effort. However, as a result of the policy implemented they are treated by the contest rules as if they were homogenous in its terms.

Using our results we may compare the total equilibrium effort levels generated under the two policy options and in different Cases. However, before proceeding we need to state a supplementary result, related to the simultaneous-move case (Case 3).

Lemma 1 *In a simultaneous-move contest game (Case 3), the total equilibrium effort level is always higher under the affirmative action policy option than under the equal treatment policy option. This result can be summarized as*

$$E^*_{ET,3} < E^*_{AA,3}.$$

Proof. See Franke (2007). ■

Lemma 1 means that in a simultaneous-move contest game (Case 3), the total equilibrium effort level is always higher under the affirmative action policy option than under the equal treatment policy option.

Now, using Propositions 1, 2, 3, 4 and Lemma 1 we may rank the total equilibrium effort levels exerted by contestants in different situations.

Conclusion 1 *The total equilibrium effort level of the contestants satisfies the following relations:*

$$E^*_{ET,2} < E^*_{ET,3} < \left\{E^*_{ET,1}, E^*_{AA,1-3}\right\}$$

and

$$E^*_{ET,1} \gtreqless E^*_{AA,1-3}, \text{ if } \beta \lesseqgtr 3, \text{ respectively.}$$

3.4 Optimality of Affirmative Action Policy

As our analysis reveals, given the order of moves of contestants and their marginal cost of effort, the implementation of affirmative action policy may have positive or negative effect in terms of the total effort level of contestants. We showed in particular, that in a two-player contest game, as the response to the implementation of the affirmative action policy option, the total effort level of individuals may decrease in comparison with the unbiased contest game. This happens when the non-discriminated individual moves before the discriminated one and the underlying heterogeneity of individuals is not too severe. However, when the underlying heterogeneity of individuals is very severe or both individuals move in the reversed order, then the effect of the affirmative action policy option on the total equilibrium effort level is always positive.

It follows that the interplay of the order of moves of contestants, of their marginal cost of effort and of the type of the policy option that is implemented may have various effects on the total effort level. As it was assumed earlier, the contest organizer aims only at maximizing the total equilibrium effort level exerted by the contestants, therefore it is interesting to study this interplay from his point of view. In this section, using the total effort level as a measure of the contest organizer's utility, we will try to examine in particular under which circumstances he prefers the implementation of the affirmative action policy option to the equal treatment policy option.

While studying the contest organizer's problem of the choice of the policy we will consider two cases – with the exogenously and endogenously determined order of moves of the individuals. In both cases the contest organizer can decide whether to implement the affirmative action policy option or not. As far as the exogenous order of moves is concerned, the contest organizer can always make the contestants move in specified order. In terms of the choice of this order we will consider here two mechanisms: one in which the order is given to the contest organizer exogenously[12] and the other in which the contest organizer can specify this order on his own[13], that is can decide whether the game will be a simultaneous or sequential-move contest and determine the order of moves of contestants in the latter one. As far as the endogenous order of moves is concerned, the contest organizer is not able to specify the order of moves in detail but can design the contest conditions such that moving sequentially or simultaneously might be possible[14].

[12] Real life examples of situations in which the order of moves is exogenous for both the players and the contest organizer can be found for instance in many professional sports, where it is specified by the contest rules. In professional ski-jumping, more experienced players jump by the contest rules later. This makes them handicapped in comparison to the earlier players as the in-run tracks gets worse as more skiers use it.

[13] For instance in some more official amateur sport events, such as some amateur golfing competition, the order of player's moves may be determined by the event organizers and therefore be exogenous for the players. As we noted in Introduction, in golfing the affirmative action policy is an issue, since typically to guarantee a level playing field advantaged competitors are artificially handicapped by the contest rule.

[14] Real life examples of situations in which players choose their order of moves can be found for instance in amateur sports such as amateur golfing. Here, before he actual play, players may agree on this order between themselves. As we noted earlier, in this example the affirmative action policy is an issue, since typically to guarantee a level playing field advantaged competitors are artificially handicapped by the contest rule.

3.4.1 Exogenous Order of Moves

The case in which the individuals' moves are given exogenously, that is are determined by the contest organizer, coincides with all our analysis which we have performed so far. This analysis is based on an implicit assumption that the order of moves was predetermined. In this section we want to supplement it by explicitly stating under which conditions the contest organizer prefers implementation of the affirmative action policy option to the equal treatment policy option.

In this section we will assume that the contest organizer can make contestants move in specified order according to Case 1, 2 or 3. However, in terms of the choice of a given Case to implement, we will consider here two situations in which the contest organizer can select a specific Case to apply. We will call it respectively "full specification" and "partial specification" of the order of moves. In the "full specification" case the number of a Case to implement is chosen by the contest organizer, whereas in the "partial specification" this number is given to the contest organizer exogenously. That is, in the "full specification" case the contest organizer selects one of the three Cases, which would maximize his utility, given that he will make decision about whether to apply affirmative action policy or not. In the case of the "partial specification", he cannot select any of the Cases on his own – the case to implement is given to him exogenously and the only decision that he can make now is the one about whether to employ affirmative action policy or not.

We look first at the preferences of the contest organizer, given a Case – that is when the order of moves is specified partially.

Proposition 5 *If the contest organizer is able to specify the order of moves in a contest partially, then*

(i) in a case in which the stronger contestant is the follower (Case 2) in a sequential-move game, or when the contestants move simultaneously (Case 3), he always prefers the implementation of the affirmative action policy option to the equal treatment policy option; and

(ii) in a case in which the stronger contestant is the leader (Case 1) in a sequential-move game, if $\beta > 3$ ($\beta < 3$) the contest organizer prefers (doesn't prefer) the implementation of the affirmative action policy option to the equal treatment policy option, and $\beta = 3$ he is indifferent between the two policy options.

Proof. Note first, that given that the total equilibrium effort level of the contestants is a measure of utility of the contest organizer, to prove the proposition it is enough to compare corresponding levels of the total equilibrium effort.

Let's start with the part (i) of the proposition. By Proposition 2 for a game in which the stronger contestant is the follower (Case 2), and by Lemma 1 for a simultaneous-move contest game (Case 3), under the affirmative action policy option the total equilibrium effort level of contestants is always higher than under the equal treatment policy option. This implies that in both cases the contest organizer always prefers the implementation of the affirmative action policy option to the equal treatment policy option.

Consider now the part (ii) of the proposition. By Proposition 1, if the stronger contestant is the leader in a sequential-move game (Case 1), for $\beta < 3$ ($\beta > 3$) under the affirmative action policy option the total equilibrium effort level of contestants is lower (higher) than under the equal treatment policy option, and for $\beta = 3$, both policy options produce the same level of the total equilibrium effort. This implies that if $\beta > 3$ ($\beta < 3$) then the contest organizer prefers (doesn't prefer) the implementation of the affirmative action policy option to the equal treatment policy option, and if $\beta = 3$ he is indifferent between the two policy options. ■

Proposition 5 states that when the weaker contestant is the leader (Case 2) in a sequential-move game, or when the contestants move simultaneously (Case 3), then the contest organizer always prefers the implementation of affirmative action policy to equal treatment policy. In Case 1, in which the stronger contestant is the leader, the preference for affirmative action policy depends on the marginal cost parameter of the weaker contestant β, which is a direct consequence of Proposition 1.

Now, we will be assuming that the contest organizer can specify the order of moves fully, that is he is able both – to make contestants move in specified order according to Case 1, 2 or 3, and to choose the number of a Case.

Conclusion 2 *If the contest organizer is able to specify fully the order of moves in a contest, then he will maximize his utility by setting a contest in which the stronger contestant is the leader (Case 1). In this case if $\beta > 3$ ($\beta < 3$) the contest organizer prefers (doesn't prefer) the implementation of the affirmative action policy option to the equal treatment policy option, and $\beta = 3$ he is indifferent between the two policy options.*

This result is a direct consequence of Conclusion 1. In a situation, in which the order of moves can be determined fully, the contest organizer will always choose Case 1. Moreover, he will implement the affirmative policy only if contestants are heterogenous enough in terms of their marginal cost of effort.

3.4.2 Endogenous Order of Moves

In the following we will consider a situation in which the contest organizer is not able to specify the order of moves in a contest in detail but can design the contest conditions such that moving sequentially or simultaneously might be possible. The individuals then decide whether or not to move sequentially and in which order. The central question then is under which circumstances the contest organizer prefers implementation of the affirmative action policy option to the equal treatment policy option.

To start the analysis we first investigate how the individuals coordinate on the order of moves in case the contest organizer allows the possibility for a simultaneous or sequential-move contest. We will assume that the following game is played: Both agents choose and publicly announce the stage in which they will exert effort. Hence, each agent can either choose stage 1 to be the leader or stage 2 to act as the follower. These choices are done simultaneously. Moreover, the agents are strictly committed to their choices. There are three possible outcomes. Either the stronger contestant announces stage 1 and his opponent stage 2 so that we have a sequential-move contest with the leader being the stronger contestant, or he announces stage 2 and his opponent stage 1 so that we have a sequential-move game with reversed order, or both individuals announce the same stage so that we have a simultaneous-move contest.

Lemma 2 *If the contest organizer is not able to specify the order of moves in a contest, under the equal treatment policy option the equilibrium order of moves is the one in which the leader is the weaker contestant (Case 2).*

Proof. See Leininger (1993)[15]. ■

So, if the contest organizer is not able to specify the order of moves in a contest, then contestants will move according to the order as in Case 2, in which the leader is the weaker contestant. With this result we can state now our result about the optimality of affirmative action policy for the contest organizer.

[15] Leininger (1993) uses a slightly different form of a utility function, which is an affine transformation of ours. His and our model are equivalent for the Proof by setting in the Leininger (1993)'s model $a = 1, V_x = V$ and $V_y = \frac{V}{\beta}$.

Proposition 6 *If the contest organizer is not able to specify the order of moves in a contest, then he always prefers the implementation of the affirmative action policy option to the equal treatment policy option.*

Proof. Note first, that given that the total equilibrium effort level of the contestants is a measure of utility of the contest organizer, to prove the proposition it is enough to show that the total equilibrium effort level is always higher under the affirmative action policy option than under the equal treatment policy option.

By Lemma 2, if the contest organizer is not able to specify the order of moves in a contest, then under the equal treatment policy option the equilibrium order of moves is the one in which the leader is the weaker contestant, as in Case 2. By Proposition 2, in Case 2 under the affirmative action policy option the total equilibrium effort level of the contestants is higher than under the equal treatment policy option. Moreover, by Proposition 4, under this policy option the total equilibrium effort level in Case 2 is the same as the one in Case 1 and Case 3. All this implies that independently of the order of moves, that the contestants will choose under the affirmative action policy option, always this policy yields a higher total equilibrium effort level than the equal treatment policy option. ■

By the result given in Proposition 6 the implementation of affirmative action policy is the most preferred option for the contest organizer, whenever he is not able to specify the order of moves in a game.

4 Conclusions

In this paper, using a version of a rent-seeking sequential-move game in the style of Tullock (1980) with heterogeneous players, we studied how individuals react to the changes in incentives that are induced by the two policies - equal treatment policy and affirmative action policy. Our main point of interest was the effect of the changes on effort provision, both at the individual and total level.

Using our model we showed that the order of moves of contestants is a very important factor in the determination of the effects of the implementation of affirmative action policy. We also proved that in such cases a significant role is played by the level of the heterogeneity of individuals. In particular, contrary to Franke (2007), it is shown that in a two-player contest game, as the response to the implementation of the affirmative action policy option, the total equilibrium effort level of the contestants may decrease in comparison to the unbiased contest game. This happens when the non-discriminated individual moves before the discriminated one and the underlying heterogeneity of individuals is not too severe. In such a case the optimal response of the leader to the implementation of the affirmative action policy option is to decrease his optimal effort level, and although the effect of the policy change for the discriminated player is in this case always positive, as a result, the sum of the equilibrium effort levels of both individuals decreases. However, when the underlying heterogeneity of individuals is very severe or both individuals move in the reversed order, then the effect of the affirmative action policy option on the total equilibrium effort level is always positive, as in Franke (2007). In such a case, as the optimal response to the implementation of the affirmative action policy option, both the leader and the follower increase their respective effort levels, which produces the rise in the total equilibrium effort level.

In our analysis, apart from considering the effects of the implementation of affirmative action policy given the order of moves, we also compared the games with different orders of moves, including the simultaneous-move one, given the policy option: the equal treatment or the affirmative action. This part of our analysis revealed that under the affirmative action policy option the order of moves is irrelevant. In this situation, independently of the order of moves, each individual exerts his effort at the same level, which implies that the total effort level is always the same.

As a part of our analysis we also studied preferences of the contest organizer for the two policy options, assuming that he is purely interested in maximizing the total effort level exerted by the contestants. We showed that if the order of moves is determined endogenously by the players, then the contest organizer always prefers the implementation of affirmative action policy to equal treatment policy. In such a situation, the equilibrium order of moves is the one in which the discriminated individual moves before the non-discriminated one and, as we noticed earlier, the implementation of affirmative action policy produces then the rise in the total equilibrium effort level of the contestants. These preferences are somehow changed if specified order of moves is exogenous for the players and the contest organizer is able to make them move according to this order. We distinguished here two cases: one in which this order is given to the contest organizer exogenously and the other in which the contest organizer can specify it, including the simultaneous-move one, on his own. When the order of moves is exogenous for the contest organizer, then he always prefers the implementation of affirmative action policy to equal treatment policy in all cases of this order but the one in which the non-discriminated individual moves before the discriminated one. In this particular situation he may sometimes prefer the implementation of equal treatment policy to affirmative action policy. This happens when the underlying heterogeneity of the individuals is not too severe. The same particular order of moves emerges as the one which maximizes the utility of the contest organizer in the second case – when the contest organizer is able to specify the order of moves in a game on his own. And here again, he prefers the implementation of equal treatment policy to affirmative action policy only if the individuals are not too heterogenous.

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Appendix

In the main part of our paper we concentrated on different results related to the comparison of the two policy options using as a standard of comparison the sum of equilibrium effort levels. However, apart from studying the total equilibrium effort level, it is interesting to look at its ingredients, that is individual effort decisions of the contestants. This allows us to understand deeper mechanisms that govern behavior of the players under different policy options, and lead to changes in the total equilibrium effort level. Therefore, in Appendix we provide complementary results on the individual effort levels of the contestants.

Case 1

Our analysis in this section we start with the case, in which contestant 1 is the leader. Let's consider first the effort levels of this contestant.

Proposition 7 *In a sequential-move game in which the stronger contestant is the leader, if $\beta < 4$ ($\beta > 4$) the equilibrium effort level of the leader is lower (higher) under the affirmative action policy than under the equal treatment policy option, and if $\beta = 4$ his equilibrium effort levels are the same. This can be summarized as*

$$e_1^*(ET) \gtreqless e_1^*(AA), \text{ if } \beta \lesseqgtr 4, \text{ respectively.}$$

Proof. Setting $i = 1$, $j = 2$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i ($i \in N$) in eq. (13) and (14), we obtain that

$$\begin{cases} e_1^*(ET) = \frac{\beta}{4}V, \text{if}\, \beta < 2, \\ e_1^*(ET) = \frac{1}{\beta}V, \text{if}\, \beta \geq 2, \end{cases}$$

and

$$e_1^*(AA) = \frac{1}{4}V.$$

To prove the proposition we need to show that for $\beta < 2$ the inequality

$$\frac{\beta}{4}V > \frac{1}{4}V, \tag{24}$$

and for $\beta \geq 2$ the inequality

$$\frac{1}{\beta}V \gtreqless \frac{1}{4}V, \tag{25}$$

if $\beta < 4$, $\beta = 4$ and $\beta > 4$, respectively, are satisfied.

Consider first the case of $\beta < 2$, given by the inequality (24). Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. It follows, that the inequality (24) is always true for $\beta \in (1, 2)$.

In the second case, if $\beta \geq 2$, within the domain of $V > 0$ and $\beta > 0$, the inequality given in (25) reduces to

$$\beta \lesseqgtr 4,$$

which is exactly the conditions on β in our claim. ■

It follows from Proposition 7 that in a sequential-move game in which the stronger player is the leader, the implementation of the affirmative action policy option may have detrimental effect on the leader's equilibrium effort level, as compared to the level under the equal treatment policy option. In particular if the leader and his opponent are rather homogenous in terms of their marginal cost of effort, then the implementation of the affirmative action policy option leads in equilibrium to the drop in the leader's effort level. The opposite happens if the players are very heterogenous - in such case his equilibrium effort level increases.

Let's concentrate now on equilibrium effort levels of contestant 2 – the follower.

Proposition 8 *In a sequential-move game in which the stronger contestant is the leader,* *(i) in equilibrium under affirmative action policy the follower is always active, that is, exerts effort at a positive level; and*

(ii) his equilibrium effort level is always higher under the affirmative action policy than under the equal treatment policy option, that is

$$e_2^*(ET) < e_2^*(AA).$$

Proof. Setting $i = 1$, $j = 2$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i $(i \in N)$ in eq. (13) and (14), we obtain that

$$\begin{cases} e_2^*(ET) = \frac{2-\beta}{4}V, \text{if}\, \beta < 2, \\ e_2^*(ET) = 0, \qquad \text{if}\, \beta \geq 2, \end{cases}$$

and

$$e_2^*(AA) = \frac{1}{4\beta}V. \tag{26}$$

The proof of the part (i) of the proposition is trivial. Given that by assumption $V > 0$ and $\beta > 0$, the equilibrium effort level of the follower under affirmative action policy in (26) is always positive, which proves the claim.

To prove the part (ii) of the proposition we need to show that for $\beta < 2$ the inequality

$$\frac{2-\beta}{4}V < \frac{1}{4\beta}V, \tag{27}$$

and for $\beta \geq 2$ the inequality

$$0 < \frac{1}{4\beta}V, \tag{28}$$

are satisfied.

Consider first the case of $\beta < 2$, given by the inequality (27). Using some algebra, within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$(\beta - 1)^2 > 0,$$

which is true for all $\beta \neq 1$. In our model this is always satisfied, given that by assumption $\beta > 1$. It follows, that the inequality (27) is always true for $\beta \in (1, 2)$.

Consider now the inequality (28), for $\beta \geq 2$. Given that by assumption $V > 0$ and $\beta > 0$, this inequality is always satisfied. It follows, that the inequality (28) is always true for $\beta \geq 2$. ■

It follows from Proposition 8 that in a sequential-move game in which the stronger player is the leader, the implementation of the affirmative action policy option always leads to increasing of the equilibrium effort level of the follower, as compared to the level under the equal treatment policy option. Moreover, under the affirmative action policy option the follower is always active, that is exerts effort at a positive level.

Case 2

Now, we study the case in which contestant 1 is the follower. Let's consider first the effort levels of this contestant.

Proposition 9 *In a sequential-move game in which the stronger contestant is the follower,*

(i) in equilibrium under both policy options the follower is always active, that is, exerts effort at a positive level; and

(ii) his equilibrium effort level is always higher under the affirmative action policy than under the equal treatment policy option, that is

$$e_1^*(ET) < e_1^*(AA).$$

Proof. Setting $i = 2$, $j = 1$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i $(i \in N)$ in eq. (13) and (14), we obtain that

$$\begin{cases} e_1^*(ET) = \frac{2\beta-1}{4\beta^2}V, \\ e_1^*(AA) = \frac{1}{4}V. \end{cases} \tag{29}$$

The proof of the part (i) of the proposition is trivial. Given that by assumption $V > 0$ and $\beta > 1$, the equilibrium effort levels of the follower under both policy options in (29) are always positive, which proves the claim.

To prove the part (ii) of the proposition we need to show that for all $\beta > 1$, the inequality

$$\frac{2\beta-1}{4\beta^2}V < \frac{1}{4}V \tag{30}$$

is satisfied.

Using some algebra, within the domain of $V > 0$ and $\beta > 0$, the inequality in (30) reduces to

$$(\beta-1)^2 > 0,$$

which is true for all $\beta \neq 1$. In our model this is always satisfied, given that by assumption $\beta > 1$. Hence, the inequality (30) is always true for $\beta > 1$. ■

It follows from Proposition 9 that in a sequential-move game in which the weaker player is the leader, the implementation of the affirmative action policy option has always positive effect on the equilibrium effort level of the follower – of the stronger player, as compared to the level under the equal treatment policy option.

Let's concentrate now on effort levels of contestant 2, who is now the leader.

Proposition 10 *In a sequential-move game in which the stronger contestant is the follower, the equilibrium effort level of the leader is always higher under the affirmative action policy than under the equal treatment policy option, that is*

$$e_2^*(ET) < e_2^*(AA).$$

Proof. Setting $i = 1$, $j = 2$ and using corresponding values of policy weights α_i^P and marginal cost parameter β_i $(i \in N)$ in eq. (13) and (14), we obtain that

$$\begin{cases} e_2^*(ET) = \frac{1}{4\beta^2}V, \\ e_2^*(AA) = \frac{1}{4\beta}V. \end{cases}$$

To prove the proposition we need to show that in our model, that is for all $\beta > 1$, the inequality

$$\frac{1}{4\beta^2}V < \frac{1}{4\beta}V, \tag{31}$$

is satisfied.

Using some algebra, within the domain of $V > 0$ and $\beta > 0$, the inequality (31) reduces to

$$\beta > 1,$$

which is satisfied in our model by assumption. It follows, that the inequality (31) is always true for $\beta > 1$. ■

The conclusion that flows from Proposition 10 is similar to one from Proposition 9: in a sequential-move game in which the weaker player is the leader, the implementation of the affirmative action policy option, as compared to the level under the equal treatment policy option, has positive effect on the equilibrium effort level – in this case on the effort level of the leader.

Case 1 vs Case 2

Our objective in this section is to compare Case 1 and Case 2 in terms of individual effort levels that they generate. Additionally, to highlight relations between sequential-move and simultaneous-move games, we consider also in our analysis Case 3, in which contestants make their effort level decisions simultaneously.

We begin our analysis by looking at equilibrium effort levels of contestants under the equal treatment policy option. Let's consider first effort levels of contestant 1.

Proposition 11 *Under equal treatment policy, the stronger contestant, in equilibrium exerts effort at the highest level in the case in which he is the leader (Case 1) and at the lowest level – when he is the follower (Case 2), in a sequential-move game. The level of his equilibrium effort when the contestants move simultaneously (Case 3) is (strictly) in-between those produced by Case 1 and 2. It may be summarized as*[16]

$$e_{1,2}^*(ET) < e_{1,3}^*(ET) < e_{1,1}^*(ET)$$

Proof. According to Table 1, under equal treatment policy, the level of the equilibrium effort for contestant 1 in each Case is

$$e_{1,1}^* = \begin{cases} \frac{\beta}{4}V, \text{if}\, \beta < 2, \\ \frac{1}{\beta}V, \text{if}\, \beta \geq 2, \end{cases}$$

$$e_{1,2}^* = \frac{2\beta - 1}{4\beta^2}V,$$

$$e_{1,3}^* = \frac{\beta}{(\beta + 1)^2}V.$$

To prove the proposition we need to show that for $\beta > 1$ the inequality

$$\frac{2\beta - 1}{4\beta^2}V < \frac{\beta}{(\beta + 1)^2}V, \tag{32}$$

and for $\beta < 2$ the inequality

$$\frac{\beta}{(\beta + 1)^2}V < \frac{\beta}{4}V, \tag{33}$$

and for $\beta \geq 2$ the inequality

$$\frac{\beta}{(\beta + 1)^2}V < \frac{1}{\beta}V \tag{34}$$

are satisfied.

Consider first the inequality (32). Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$\left(\beta + \frac{1}{2}\right)(\beta - 1)^2 > 0,$$

[16]Recall that $e_{i,k}^*(P)$ denotes an equilibrium effort level of a contestant i in Case k given a policy option P.

which is satisfied within the domain of $\beta > 0$ for all $\beta \neq 1$. In our model this is always true, given that by assumption $\beta > 1$. Hence, the inequality (32) is always satisfied for $\beta > 1$.

Consider now the inequality (33), for $\beta < 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$(\beta + 3)(\beta - 1) > 0,$$

which is satisfied within the domain of $\beta > 0$ for all $\beta > 1$. It follows, that the inequality (33) is always true for $\beta \in (1, 2)$.

Finally consider the inequality (34), for $\beta \geq 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$1 > 0,$$

which is always true. It follows, that the inequality (34) is always satisfied for $\beta \geq 2$. ■

Proposition 11 states that under equal treatment policy the stronger player exerts effort at the highest level when he is the leader, and at the lowest level when he is the follower in a sequential-move game. His effort level in a simultaneous-move game is in between the previous sequential-move game effort levels. It follows that there exist a first-mover advantage for the stronger player – as the leader he exerts effort at the highest level, higher than when he is the follower or when he moves simultaneously with his opponent.

Let's focus now on effort levels of contestant 2, still for the equal treatment policy option.

Proposition 12 *Under equal treatment policy, the weaker contestant,*

(i) in equilibrium exerts effort at the highest level in the case in which the contestants move simultaneously (Case 3), that is

$$\left\{e^*_{2,1}(ET), e^*_{2,2}(ET)\right\} < e^*_{2,3}(ET);$$

and

(ii) in a sequential-move case, if $\beta \lesseqqgtr \frac{1+\sqrt{5}}{2}$*, then in equilibrium he exerts effort at a higher (equal, lower, respectively) level when he is the follower (Case 1) than when he is the leader (Case 2). It may be summarized as*

$$e^*_{2,1}(ET) \gtreqqless e^*_{2,2}(ET), \ \textit{if}\ \beta \lesseqqgtr \frac{1+\sqrt{5}}{2}.$$

Proof. According to Table 1, under equal treatment policy, the level of the equilibrium effort for contestant 2 in each Case is

$$e^*_{2,1} = \begin{cases} \frac{2-\beta}{4}V, \text{if}\, \beta < 2, \\ 0, \quad\quad \text{if}\, \beta \geq 2, \end{cases}$$
$$e^*_{2,2} = \frac{1}{4\beta^2}V,$$
$$e^*_{2,3} = \frac{1}{(\beta+1)^2}V.$$

To prove the part (i) of the proposition we need to show that for $\beta < 2$ the inequality

$$\frac{2-\beta}{4}V < \frac{1}{(\beta+1)^2}V, \tag{35}$$

and for $\beta \geq 2$ the inequality

$$0 < \frac{1}{(\beta+1)^2}V, \tag{36}$$

and for $\beta > 1$ the inequality

$$\frac{1}{4\beta^2}V < \frac{1}{(\beta+1)^2}V, \tag{37}$$

are satisfied.

Consider first the inequality (35), for $\beta < 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$(\beta+2)(\beta-1)^2 > 0,$$

which is satisfied within the domain of $\beta > 0$ for all $\beta \neq 1$. In our model this is always true, given that by assumption $\beta > 1$. Hence, the inequality (35) is always satisfied for $\beta \in (1, 2)$.

Consider now the inequality (36), for $\beta \geq 2$. Given that by assumption $V > 0$ and $\beta > 0$, this inequality is always satisfied. It follows, that the inequality (36) is always true for $\beta \geq 2$.

Finally consider the inequality (37). Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$\left(\beta + \frac{1}{3}\right)(\beta - 1) > 0,$$

which is satisfied within the domain of $\beta > 0$ for all $\beta > 1$. In our model this is always true, given that by assumption $\beta > 1$. Hence, the inequality (37) is always satisfied in our model.

To prove the part (ii) of the proposition we need to show that for $\beta < 2$

$$\frac{2-\beta}{4}V \gtreqless \frac{1}{4\beta^2}V, \tag{38}$$

if $\beta < \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1+\sqrt{5}}{2}$ and $\beta > \frac{1+\sqrt{5}}{2}$, respectively; and that for $\beta \geq 2$

$$0 < \frac{1}{4\beta^2}V. \tag{39}$$

Consider first the inequality (38), for $\beta < 2$. Within the domain of $V > 0$ and $\beta > 0$, this inequality reduces to

$$(\beta - 1)\left(\beta - \frac{1-\sqrt{5}}{2}\right)\left(\beta - \frac{1+\sqrt{5}}{2}\right) \gtreqless 0,$$

whose LHS within the domain of $\beta > 0$ is positive for all $\beta \in \left(1, \frac{1+\sqrt{5}}{2}\right)$, equal to zero for $\beta = 1 \cup \beta = \frac{1+\sqrt{5}}{2}$ and negative for all $\beta \in (0,1) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$. As in our model by assumption $\beta > 1$, and now $\beta < 2$, this result implies that the LHS of the inequality (38) is higher than the RHS for all $\beta \in \left(1, \frac{1+\sqrt{5}}{2}\right)$, equal to for $\beta = \frac{1+\sqrt{5}}{2}$, and lower for all $\beta \in \left(\frac{1+\sqrt{5}}{2}, 2\right)$, which is exactly our claim.

Consider now the inequality (39), for $\beta \geq 2$. Given that by assumption $V > 0$ and $\beta > 0$, this inequality is always satisfied. It follows, that the inequality (39) is always true for $\beta \geq 2$. ■

It follows from Proposition 12, that under equal treatment policy the weaker player exerts effort at the highest level when a simultaneous-move game is played. However, for a sequential-move game the ranking of his effort levels depends on his marginal cost parameter β. If the players are rather homogenous in terms of their marginal cost of effort, then the weaker player exerts effort at higher level as the follower than as the leader. The opposite happens when the players are very heterogenous – in that case the weaker player exerts effort at higher level as the leader than as the follower.

Using our results we may compare equilibrium effort levels generated under the two policy options and in different Cases. However, before proceeding we need to state a supplementary result, related to the simultaneous-move case (Case 3).

Lemma 3 *In a simultaneous-move contest game (Case 3), both contestants exert effort at a higher level under the affirmative action policy* *option than under the equal treatment policy option. This result can be summarized as*

$$e^*_{i,3}(ET) < e^*_{i,3}(AA) \text{ for } i \in N,$$

Proof. See Franke (2007). ■

Lemma 3 means that in a simultaneous-move contest game (Case 3), the equilibrium effort levels of contestants are always higher under the affirmative action policy option than under the equal treatment policy option.

Now, using Propositions 4, 7, 9, 11 and Lemma 3 we may now rank all equilibrium effort levels exerted by the stronger contestant – contestant 1.

Conclusion 3 *The equilibrium effort level of the stronger contestant satisfies the following relations:*

$$e^*_{1,2}(ET) < e^*_{1,3}(ET) < \left\{e^*_{1,1}(ET), e^*_{1,1-3}(AA)\right\}$$

and

$$e^*_{1,1}\left(ET\right) \gtreqqless e^*_{1,1-3}\left(AA\right), \text{ if } \beta \lesseqqgtr 4, \text{ respectively.}$$

Similarly, using Propositions 4, 8, 10, 12 and Lemma 3 we can rank all equilibrium effort levels exerted by the weaker contestant – contestant 2.

Conclusion 4 *The equilibrium effort level of the weaker contestant satisfies the following relations:*

$$\left\{e^*_{2,1}(ET), e^*_{2,2}(ET)\right\} < e^*_{2,3}(ET) < e^*_{2,1-3}(AA)$$

and

$$e^*_{2,1}(ET) \gtreqqless e^*_{2,2}(ET), \text{ if } \beta \lesseqqgtr \frac{1+\sqrt{5}}{2}, \text{ respectively.}$$