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# Dundee Discussion Papers in Economics

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## A Foundation System and a State System - Private-School Implications on Welfare and Education Expenditure

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# A Foundation System and a State System - Private-School Implications on Welfare and Education Expenditure\*

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## Abstract

This paper examines the effects of two different education financing systems: a foundation system and a state system on the level and distribution of resources devoted to education in the presence of private schools. We use political economy approach where households differ in their level of income, and the central tax rate used to finance education is determined by a majority vote. Our analysis focuses on implications of allowing for a private-school option. To evaluate the importance of private schools we develop a computational model and calibrate it using USA data. The results reveal that the private school option is very important quantitatively in terms of welfare, total resources spent on education and equity.

*Keywords:* Education finance reform, Private schools

*JEL classification:* I22, I28, H42

## 1 Introduction

Many authors examine how different education finance systems affect the level of spending on education, its distribution and associated total welfare<sup>1</sup>. However, their research is concentrated mainly on public schools, with little attention paid to the existence of private schools. As

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<sup>1</sup>Among many we mention works of Fernández and Rogerson (1997, 1998, 1999, 2003), Barse, Glomm and Ravikumar (2001).

a result, there is relatively little analysis addressing how different education system outcomes are affected by the presence of a private education sector. The aim of this paper is to fill this gap in the case of selected education systems.

Central to the understanding how a school finance system affects the sum of education resources and the welfare is an analysis of the mechanisms through which it transfers resources across individuals. The objective of this paper is to study these mechanisms in a simple general equilibrium model and to perform a calibration exercise to assess their quantitative significance. In order to do so, we use a standard model of local public finance and extend it by allowing for the existence of a private alternative. In our model we have a large number of households that are heterogeneous with regard to income and perfectly sorted into homogenous communities (districts). The education finance system sets down the rules that govern how revenues are raised and distributed across communities for education spending. The key parameters of the education finance system are determined by a majority vote.

The framework we employ could be used to study diverse changes in education finance systems. Our analysis is motivated by the experience of California, since this case received considerable attention in the literature. Legislative regulations introduced in the 1970's changed California's education finance from a foundation system in which local expenditures supplement expenditure levels guaranteed by the state, to one in which effectively all financing is done at the state level. Subsequent to these changes, California's share of personal income going to public education fell by 10% and the enrolment in private schools increased by 3%.

The literature that studies transition from a foundation system to a state system abstracts from the existence of private schools and changes in their enrolment. Since in general private schools are attended by children of the wealthiest families, who spend on education more than poorer families, the impact of private schools on levels of total education spending and of total welfare may be quantitatively very substantial. In this paper, by allowing for the existence of private schools, we are trying to assess the importance of a private-school sector in an educational system. We do this by comparing our results with those obtained by other authors, who did not consider private schools in their models. We study the impact of private schools on changes in total education spending and welfare when we go from a foundation system to a state system of the education finance.

In our paper we show that compared to pure systems without private schools the private alternative introduces substantial distortion in

welfare and education spending distributions. In particular we argue that introducing the private-school alternative reverses the present-in-the-literature ranking in which a foundation system is welfare superior to a state system (for instance in the work of Fernández and Rogerson (2003)). In the light of our findings this ranking turns out to be true only if we do not allow in the model for the existence of private schools. We also show that the private-school alternative influences the equity under both finance systems, especially strongly in the case of a state system. A pure state system, which is considered to be the most equitable among different education finance systems, after introducing a private-school option represents an equity level only a little higher than a foundation system. Moreover, this new equity level is far from the perfect equity level attributed to the pure state system. Our findings confirm the present-in-the-literature result (for instance in the work of Fernández and Rogerson (1999)) that a shift from a foundation to a state system causes a fall in total resources spent on public education. They also reveal that this change of a finance system may result in decreasing per-student spending in both public and private sector. This happens, though private schools benefit from higher total spending under a state system than under a foundation one.

Our work is closely related to several papers. It builds directly on the work of Fernández and Rogerson (1999), who study transition from a foundation system to a state system in a static model. A key difference between their work and ours is that they do not allow in their model for the existence of private schools. Our comparisons of the finance systems follow the work of Fernández and Rogerson (2003), who compare several education finance systems, including a foundation one and a state one, in terms of total expenditure and welfare. They also do not consider private schools in their model, which is similar to Fernández and Rogerson (1997, 1998), who study the change of a pure local education finance system into a pure state system in a dynamic context. Barse, Glomm and Ravikumar (2001) allowing for the private-school alternative, examine change from a decentralized to a centralized finance system in two-school-district economy in both static and dynamic contexts. A large volume of the literature focuses on private schools as an alternative way of obtaining education. Epple and Romano (1996a) and Glomm and Ravikumar (1998) study properties of a voting equilibrium under mixed regime of public and private education. Epple and Romano (1998) and Epple, Figlio and Romano (2004) using complicated setting with peer effects study competition between private and public schools. Martínez Mora (2003) and Nechyba (1999) study households' choices between public and private education in models with mobility among communities

and housing issues. Cohen-Zada and Justman (2003) try to link a theoretical model of mixed private-public education with empirical evidence. Our work is also related to papers that focus on a foundation system. For instance Epple and Romano (1996b) study properties of a voting equilibrium under the system, in which a publicly provided good can be supplemented further with private purchase, like in a foundation system.

An outline of the paper follows. In Section 2 we provide an analytical characterization of a foundation system and of a state system of public education finance when the presence of private schools is allowed. Section 3 presents our calibration exercise and provides results of quantitative comparison of both education finance systems. Section 4 concludes.

## 2 Theoretical Analysis of a Foundation and a State System with Private Schools

### 2.1 The Model

The impact of a particular education-finance system on welfare and a level of total education expenditure depends generally upon many elements. The most important cover the details of the financing system itself, their interaction with the state and local tax systems, the distribution of income, the way in which tax rates, spending, and other key variables are chosen, the distribution of employment and housing locations, etc. In our model we choose to abstract away from many of these important factors in order to concentrate mainly on two variables: the distribution of income and the endogenous determination of the parameters of the financing system.

In our model the economy consists of a continuum of households, with their population normalized to one. Each household consists of a parent and a child. Households have identical preferences over a private consumption good  $c$  (the numeraire) and over the child's education quality  $q$ . These preferences are described by the utility function, whose general formula is given by  $U(c, q) = u(c) + v(q)$ , where  $U : R_+^2 \rightarrow R$  is twice differentiable, strictly increasing and strictly concave. We also assume that Inada conditions hold and that both  $c$  and  $q$  are normal goods. Throughout the paper we will in particular refer to a specific class of the utility functions that satisfy these conditions: the class of the constant-elasticity-of-substitution (CES) utility functions of the form

$$U(c, q) = \begin{cases} \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \delta \frac{q^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, & \sigma \neq 1 \\ \ln(c) + \delta \ln(q), & \sigma = 1 \end{cases}, \quad (1)$$

where  $\delta$  and  $\sigma$  are strictly positive constants and  $\sigma$  is the elasticity of

substitution between consumption and schooling. These CES utility functions are commonly used to model education demand.

By assumption households differ only in their endowments of income  $y$ . Its distribution is described by the cumulative distribution function  $F(y)$ , where  $F : R \rightarrow [0, 1]$ . The mean income  $m$  is assumed to be finite and greater than the median income  $me$ , i.e  $F(m) > 0.5$ .

In order to focus on distribution across communities we will use the assumption of perfect sorting, which will allow us to abstract from issues of distribution within the community. This assumption is commonly used in public finance literature. Hence, in our model we have equal-sized districts, indexed by  $i$ , consisting of households with the same level of income, which at the district level results in perfect agreement over the preferred levels of consumption and education quality.

By assumption, children go to school in the district where they live. In our model we have two types of schools: public and private ones. In a state system, public schools are financed only centrally from the state budget. In a foundation system, public schools are financed in a mixed way: centrally from the state budget by means of so called grants, and locally by means of additional taxation within districts. Private schools are modelled as clubs formed by parents under an equal cost sharing rule, which follows the Nechyba (1999) approach. These schools are financed by parents from their disposable income.

The quality of education that a child receives  $q$  is solely a function of spending per student  $s$ . In different types of schools we allow for the possibility that parents perceive a dollar spent on education in private schools as more effective than a dollar spent on public education, which affects schooling quality. We assume that the relationship between the spending and the quality is expressed by  $q = a \cdot s$ , where  $a$  is some positive constant (we will call it "the quality parameter"), which accounts for what is perceived by parents as the differences between different types of schools. These differences are reported by many authors. Cohen-Zada and Justman (2003) argue that private schools provide schooling more efficiently than public schools and that many parents of private-school children perceive it to be the case. In the United States, opting out of public education does not reduce household's school-tax liability. Thus it makes sense only if parents believe that the private school of their choice provides a better education than public schools. Average tuition levels in private schools are considerably lower than the spending per student in public schools. It means that private tuition dollars are perceived to be more effective than tax dollars (at least by those who opt for cheaper private education). However, a lower tuition level doesn't necessarily mean that total private-school expenditures are

lower. Cohen-Zada and Justman (2003) argue that a considerable part of private schools' costs is covered by subsidies and donations from different sources like local households, dioceses, etc. As an example, they provide a finding that about 50% of costs in Catholic elementary schools are covered by donations from local households and dioceses, and implicitly by teachers who are members of religious orders and accept less than the going wages. Another possible source of the heterogeneity of private and public schools, that also may be decisive in the choice of a school type, lies in the fact that often public schools are secular, and private ones religious. Some such private schools teach religious education, together with the usual academic subjects to impress their particular faith, beliefs and traditions in the students who attend. For instance, in the United States among all elementary and secondary schools, Catholic schools account for half of all private enrolment, and other religious schools for another third (Cohen-Zada and Justman (2003)), which means that the vast majority of private education is non-secular. Hence, apart from the factors mentioned earlier, choosing private education may be as a result of preferences for religious education instead of a secular one. In terms of our earlier discussion it just means that parents who have decided to send their child to a private (religious) school perceive every tuition dollar to be more productive than a dollar spent on public education because it provides them (their children) with the type of education that they value much and which cannot be obtained in a public school. All these arguments point out that the perceived advantage of private education is the result of the combined effect of a variety of possible factors: greater operating efficiency, subsidized tuition, an innate preference for a religious school environment, and other factors. We introduce the final combined effect of all these factors on parents' perception of private schooling by setting  $a = 1$  for public schools and  $a > 1$  for private ones and hold it constant for a given type of school. Hence we assume the same (perceived by parents) advantage level for a set of schools of the same type. However, to focus our analysis and simplify the model we abstract from any other factors that might affect the quality such as peer effects (i.e., the possibility that who you go to school with matters)<sup>2</sup>, child's abilities or parental characteristics other than income. By assumption they do not affect schooling quality.

Also to simplify matters we will restrict our attention to proportional taxes based on income both at the state and local levels. In reality however, in some countries, e.g. the USA, state taxes are often based on income, but local taxes are based on property value. Introducing

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<sup>2</sup>The literature doesn't provide clear conclusion about the role of peer effects in education. More details in Nechyba (1999).



this distinction to the model would require endogenizing the value of housing, which would complicate matters considerably.

Throughout the paper, we will assume that households have an alternative option of obtaining schooling services from a competitive private-school sector, where the state financial support is not available, but still having tax liability. The price of private schooling services is normalized to unity.

## 2.2 Foundation System with Private Schools

In this model all districts are required to collect income taxes at a uniform rate  $\tau$  from all households in exchange for some guaranteed base level of expenditures per student - the foundation grant  $g$ . However, households in a district may only spend on education this grant or decide to supplement it by further taxing their own income. Households have also an alternative option of obtaining schooling services from a competitive private-school sector, where the grant is not available, but still having tax liability. Let  $f \in [0, 1]$  denote the proportion of households choosing publicly provided services under this system.

### 2.2.1 Public schools under a foundation system

Let's assume for a while that households in a district utilize grants and that  $f$  is fixed. Given that the tax revenues are used to fund the foundation grant and that the central budget is balanced, the grant level is

$$g = \frac{\tau m}{f}.$$

However, households in a district may spend on education only this grant or supplement this grant by further taxing their own income. Letting  $t_i$  denote the district  $i$ 's tax level we have

$$\begin{aligned} c_i &= (1 - \tau - t_i)y_i \\ s_i &= \frac{\tau m}{f} + t_i y_i \end{aligned}.$$

The foundation tax rate  $\tau$  (and thus the foundation grant level) is chosen by a majority vote (i.e.  $\tau$  must be preferred to any other tax rate in a pair-wise comparison by at least 50% of the voters). To solve for preferred tax rates in districts that use grants, we will assume that tax rate decisions are made in two stages. In the first one, a majority vote at the state level determines the foundation tax, and in the second stage districts make their district tax choice.

Let's start with the district tax decision. Given a state-wide foundation tax rate outcome  $\tau$ , a district's preferred tax rate  $t_i$  is the solution to the representative household in district  $i$ 's maximisation problem (recall

that for public schools  $q = s$ )<sup>3</sup>

$$\begin{aligned} \max_{t_i} & \{u((1 - \tau - t_i)y_i) + v(\frac{\tau m}{f} + t_i y_i)\} \\ \text{s.t} & \quad t_i \geq 0 \end{aligned}, \quad (2)$$

yielding the first order condition

$$-u'((1 - \tau - t_i)y_i)y_i + v'(\frac{\tau m}{f} + t_i y_i)y_i \leq 0, \quad (3)$$

with equality for  $t_i > 0$ .

As the solution to the problem (2) we obtain a function  $t_i(\tau)$  which corresponds to the representative household's optimal choice of  $t_i$  given  $\tau$ .

Next we find the state-wide foundation tax rate by solving for a preferred foundation tax rate as a function of individual income, i.e., we solve

$$\begin{aligned} \max_{\tau} & \{u((1 - \tau - t_i(\tau))y_i) + v(\frac{\tau m}{f} + t_i(\tau)y_i)\} \\ \text{s.t} & \quad \tau \geq 0 \end{aligned}$$

and obtain the first order condition

$$-u'(\cdot)y_i + v'(\cdot)\frac{m}{f} + (-u'(\cdot) + v'(\cdot))y_i \frac{dt_i}{d\tau} \leq 0,$$

which using (3) and the Envelope Theorem gives

$$-u'((1 - \tau - t_i)y_i)y_i + v'(\frac{\tau m}{f} + t_i y_i)\frac{m}{f} \leq 0, \quad (4)$$

with equality for  $\tau > 0$ .

Suppose now that a preferred foundation tax rate  $\tau$  is positive, which implies that  $u'((1 - \tau - t_i)y_i)y_i = v'(\frac{\tau m}{f} + t_i y_i)\frac{m}{f}$ . Then (3) can be satisfied only if  $y_i \leq \frac{m}{f}$ . It follows that all households in districts with income above  $\frac{m}{f}$  must prefer a zero foundation tax rate. And now suppose that a preferred district tax rate  $t_i$  is positive, which implies that  $u'((1 - \tau - t_i)y_i)y_i = v'(\frac{\tau m}{f} + t_i y_i)y_i$ . Then (4) will be satisfied only if  $y_i \geq \frac{m}{f}$ . It follows that all households in districts with income below  $\frac{m}{f}$  must prefer a zero district tax rate (conditional upon obtaining their preferred foundation tax rate).

These results reveal that each district prefers to rely exclusively on either central or local funding<sup>4</sup>. As state and local spending are perfect

<sup>3</sup> Assumed strict concavity of the utility function guarantees that both district and state-wide preferred tax rates are unique. This also implies that first order conditions are necessary and sufficient for optimum.

<sup>4</sup> This statement is valid for all districts except for the district with income equal to  $\frac{m}{f}$ . This district is indifferent between the two types of funding. Since the measure of households in this district is zero, this fact is of no practical consequence in terms of our computational results in Section 3.

substitutes, each district chooses the type of funding which is cheaper in terms of the tax price<sup>5</sup>. For households in districts with income  $y_i > \frac{m}{f}$  local funding is cheaper than state funding and hence they prefer a zero foundation tax rate and would rather fund schooling from local taxes. The opposite applies to households in districts with  $y_i < \frac{m}{f}$ . They prefer the foundation grant to fund their entire demand for education.

In equilibrium households in some districts will supplement the grant with their own additional funds. As we stated earlier the foundation tax rate  $\tau$  (and thus the foundation grant level) is chosen by a majority vote of all districts, which by the perfect income sorting assumption is equivalent to a majority vote of all households. To determine in which districts the grant will be supplemented we have to refer to the income of the decisive district. As we will see later when the problem of a voting equilibrium will be discussed in a detailed way, the income level in the decisive district is never greater than the median income. Since the income distribution is skewed, the median income is below the mean income. Using this and previous results we see that the decisive voter will always prefer a zero district tax rate and a positive foundation tax rate. Let  $y_d$  denote the income of the decisive voter and  $\tau^*$  its most preferred foundation tax rate. Using the first order condition (3) we obtain that for the decisive voter

$$v' \left( \frac{\tau^* m}{f} \right) < u' \left( (1 - \tau^*) y_d \right).$$

Hence there exists the income level  $\tilde{y}$  satisfying

$$v' \left( \frac{\tau^* m}{f} \right) = u' \left( (1 - \tau^*) \tilde{y} \right).$$

It follows from properties of the utility function that  $\tilde{y} > y_d$ . Then we have that all households in districts with income lower than  $y_d$  will set  $t_i = 0$ . By continuity, households in the decisive district and households in some other districts with income  $y_i \geq y_d$  but smaller than  $\tilde{y}$  will also not supplement the grant with additional taxation since for all of them still  $v' \left( \frac{\tau^* m}{f} \right) < u' \left( (1 - \tau^*) y_i \right)$ . Finally all households in the districts with income  $y_i > \tilde{y}$  will set a positive district tax rate  $t_i$ .

For further reference the induced indirect utility function of a household in a district  $i$  that uses public services is

$$V(\tau, t_i, f, y_i) = u \left( (1 - \tau - t_i) y_i \right) + v \left( \frac{\tau m}{f} + t_i y_i \right).$$

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<sup>5</sup>Tax price is defined as the amount of extra local tax revenue that a district must generate in order to yield an extra unit of local spending on education. This is equal to marginal rate of substitution between schooling and consumption  $\frac{v'(\cdot)}{u'(\cdot)}$ . For local funding this is equal to one, and for state funding  $\frac{y_i f}{m}$ .

### 2.2.2 Private schools under a foundation system

Now we will concentrate on districts that utilize private schooling. In our model, all households in all districts pay taxes but each one is free to choose between publicly and privately provided school services. All private-sector users have full freedom to adjust schooling expenditures to their needs, whereas in a public sector this applies only to a part of its users. In private schooling the education spending level is specific to households in a district and may vary across districts. It is not completely the case for those districts which have chosen public-sector services. In a public sector, households in poor districts have the same level of education spending determined by the grant level. Only richer districts are able to supplement this grant and hence obtain a higher education spending level, specific to them.

Each household in a district that utilizes private-school services allocates its after-tax (disposable) income to consumption expenditures and private schooling services. Note that we do not allow for mixing public and private services. No household can choose publicly provided services (with supplementation or not) and top it up with some private services or the other way round. The representative household in a district  $i$  that uses private-school services faces the following budget constraint

$$(1 - \tau)y_i = c_i + s_i.$$

Taking a foundation tax rate  $\tau$  as given and assuming that technology of converting school expenditures into quality is  $q_i = a \cdot s_i$  ( $a > 1$ ) the utility maximization problem becomes

$$\begin{aligned} \max_{c_i, s_i} & \{u(c_i) + v(a \cdot s_i)\} \\ \text{s.t.} & \begin{cases} c_i + s_i \leq (1 - \tau)y_i \\ c_i \geq 0 \\ s_i \geq 0 \end{cases} \end{aligned}$$

The solution to this problem is unique and interior, given the assumptions made about the utility function<sup>6</sup>. Let  $W(\tau, y_i)$  denote the induced indirect utility function of the representative household in district  $i$  that chooses private services. The improved quality of public education doesn't affect the utility of such a household, hence the indirect utility  $W(\tau, y_i)$  varies inversely to changes in the tax rate  $\tau$ .

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<sup>6</sup>Assumed strict concavity of the utility function guarantees uniqueness. Inada conditions imply interior solution.

### 2.2.3 Equilibrium under a foundation system with private schools

Assume now that all districts in the economy anticipate the same public enrolment rate  $f^e$ . Once the foundation tax rate  $\tau$  has been determined by majority voting (recall that  $\tau^*$  denotes the decisive voter most preferred foundation tax rate) all households in districts compare their  $V(\tau^*, t_i, f^e, y_i)$  and  $W(\tau^*, y_i)$  and decide which type of schooling to choose. Households in a district  $i$  will choose to send their children to a public school if  $V(\tau^*, t_i, f^e, y_i) > W(\tau^*, y_i)$ .

Since each household in a district, while making this public-private choice, takes as given the proportion of households (districts) choosing publicly provided services  $f^e$ , we have to be sure that in an equilibrium, the individual decisions are consistent with the aggregate outcomes. That is the proportion of households for which  $V(\tau^*, t_i, f^e, y_i) > W(\tau^*, y_i)$  must be exactly equal to  $f^e$ .

Now given a foundation tax rate  $\tau$  we will compare utility of the representative household in a given district when it uses public and private education services. Notice that in both cases the representative household pays the tax at the rate  $\tau$ , hence its after-foundation-tax disposable income is the same. The grant that the representative household receives using public services increases its disposable income. If the quality parameter  $a$  is equal to one for both public and private services, which means that there are no quality gains from switching from public to private schools, the representative household can obtain a higher utility level using the grant and a public school (without supplementation of the grant for low-income districts and with supplementation for higher-income ones) than funding its education entirely from its disposable income and using a private school. It follows that if there is no quality advantage of private schooling, then for any level of income the representative household is better off using public rather than private services. However, if the quality parameter  $a$  is still equal to one for public schools but higher than one for private schools and also high enough, then gains in perceived quality in private schools may compensate the effect of the grant on disposable income and - as a consequence - on the utility level of the representative household and allow it to switch to a private school. From now on we will assume that the value of the quality parameter  $a$  guarantees that some districts will opt out of a public sector.

As opting out of public education does not reduce tax obligations of the representative household in a given district, and this household is aimed at obtaining a higher quality of education (and education quality is a normal good), other things being equal, households in districts that opt out of public schooling will be those with higher incomes. For a

given foundation tax level  $\tau \in (0, 1)$ , there exists a threshold income level  $\underline{y} = \underline{y}(\tau, f^e)$  such that all households in districts with income below  $\underline{y}$  send their children to public schools and all households in districts with income above  $\underline{y}$  send their children to private schools. This threshold income is implicitly defined by  $V(\tau, t_i, f^e, \underline{y}) = W(\tau, \underline{y})$ .

Using the threshold income  $\underline{y}$  the consistency condition for an equilibrium given a foundation tax rate  $\tau$  can be written as  $f^e = F(\underline{y}(\tau, f^e))$  - the anticipated public-school enrolment rate must equal an actual enrolment rate.

All districts in the economy vote on tax rates and the equilibrium foundation tax rate  $\tau$  is the one chosen by a majority of voters. Households in districts can choose between publicly and privately funded education services. Consequently, after the foundation tax rate has been set they consider the effect of this tax on both indirect utility functions  $V(\cdot)$  and  $W(\cdot)$ . If the tax rate is sufficiently close to zero, the quality of publicly provided services is low and households in a typical district will choose private services. If the tax rate increases marginally, private services are still preferred to over publicly provided services. But a small increase in the foundation tax rate decreases utility, because disposable income is then lower. If the tax rate increases further, the households in the district become indifferent between public and private services. Increasing the tax rate above this level induces the households to use public services and the utility increases until the most preferred foundation tax rate is reached. Any further increase in the tax rate lowers the household's utility<sup>7</sup>.

Districts with the richest households prefer a zero foundation tax rate, because they use private schooling and benefit more from private schooling than from any level of public schooling. In turn households in districts with income below  $\frac{m}{f}$  anticipating that they will utilize public services prefer a positive foundation tax rate  $\tau$  at which  $V(\cdot)$  reaches its maximum<sup>8</sup>. Households in districts with income above  $\frac{m}{f}$  anticipating that they will also utilize public services prefer however a zero foundation tax rate  $\tau$ , because they are better off funding their schooling with local taxes than with the central foundation tax<sup>9</sup>.

To identify the decisive voter we need to know how a preferred foun-

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<sup>7</sup>In this case preferences over tax rates are generally not single peaked and in general a global majority voting equilibrium may not exist. See Glomm and Ravikumar (1998), Epple and Romano (1996a) for details. However some special assumptions about the way people vote always guarantee the existence of the equilibrium. We will get back to this problem later.

<sup>8</sup>Recall, that these households prefer a zero district tax rate.

<sup>9</sup>As noted earlier, the district with income equal to  $\frac{m}{f}$  is indifferent between local and central funding.

dation tax rate changes with income. By skewness of the income distribution the decisive district's income level will be below the mean resulting that its preferred district tax rate  $t_i$  is zero. Using this we apply the implicit function theorem to (4) with equality and obtain

$$\frac{d\tau}{dy_i} \Big|_{y_i < \min\{\frac{m}{f}, \underline{y}\}} = \frac{u''((1-\tau)y_i)c_i + u'((1-\tau)y_i)}{u''((1-\tau)y_i)y_i^2 + v''(\frac{\tau m}{f})(\frac{m}{f})^2}, \quad (5)$$

whose sign is that of  $-\{u''((1-\tau)y_i)c_i + u'((1-\tau)y_i)\}$ .

If the sign of (5) is negative<sup>10</sup>, then a preferred foundation tax rate decreases with income among districts that prefer public to private schooling. Moreover, all districts that anticipate using public schooling with income satisfying  $y_i > \frac{m}{f}$  and high-income districts that anticipate using private services with income  $y_i > \underline{y}$  prefer a zero foundation tax rate. This implies that a preferred foundation tax rate decreases monotonically with income throughout the entire range of incomes. Hence, using the single-crossing property rule we obtain that a global political equilibrium exists, and that the equilibrium foundation tax rate is the tax rate preferred by the households in the median income district. The income of the decisive voter  $y_d$  satisfies in this case

$$F(y_d) = 0.5. \quad (6)$$

If (5) equals zero<sup>11</sup>, then a preferred foundation tax rate doesn't depend on income among districts that prefer public to private schooling. Moreover, like in the previous case, all districts that anticipate using public schooling with income satisfying  $y_i > \frac{m}{f}$  and high-income districts that anticipate using private services with income  $y_i > \underline{y}$  prefer a zero foundation tax rate. By skewness of the income distribution a coalition supporting a zero foundation tax rate consists of less than 50% of the districts<sup>12</sup>. It follows that in this case the median income voter is still decisive and its income satisfies (6).

If the sign of (5) is positive<sup>13</sup>, then a preferred foundation tax rate increases with income among districts that prefer public to private schooling. Moreover, as before, all districts with income satisfying  $y_i > \frac{m}{f}$  and high-income districts with income  $y_i > \underline{y}$  prefer a zero foundation tax rate. In this case an "ends against the middle" coalition is formed. Poor

<sup>10</sup>For the class of the CES utility functions it implies  $\sigma > 1$ .

<sup>11</sup>For the class of the CES utility functions it implies  $\sigma = 1$ .

<sup>12</sup>We assume here that in equilibrium  $me < \underline{y}$ . Since private school users are those with the highest income levels, this assumption is practically of no consequence. Note also that  $me < \frac{m}{f}$  for any  $f \in (0, 1]$ .

<sup>13</sup>For the class of the CES utility functions it implies  $\sigma < 1$ .

districts, which prefer less public spending on education, join forces with the districts that prefer a zero foundation tax rate. The opposition is the middle-income districts, which prefer higher spending levels. In this case in general single-crossing property doesn't hold and a global political equilibrium may not exist<sup>14</sup>. Since the problem of the existence of a voting equilibrium may hamper the research seriously, some authors advocate some additional conditions that would guarantee the existence. For instance Nechyba (1999) uses in his papers the assumption of "myopic voting". In our context this assumption means that districts decide which type of schooling to choose (public or private) before they vote on the foundation tax rate. Then the voting preferences are single-peaked and the existence holds in any case. To avoid problems with the existence of a voting equilibrium we follow this assumption and incorporate it to our work. Given this assumption the income of the decisive voter in an equilibrium of this type satisfies

$$F(y_d) + 1 - F(\hat{y}) = 0.5, \quad (7)$$

where  $\hat{y} = \min\{\frac{m}{f}, \underline{y}\}$ . Note that since  $f = F(\underline{y}) \leq 1$  and  $F(\frac{m}{f}) \leq 1$ , the income of the decisive voter  $y_d$  in this case is not greater than the median income.

## 2.3 State System with Private Schools

Under this system districts are required to collect income taxes at a uniform rate  $\tau$  from all households in exchange for some per capita revenue for education, equal for all districts<sup>15</sup>. Districts are however restricted to using only these funds for education and cannot supplement it in the way similar to this under a foundation system. Like in the previous model, households in all districts have an alternative option of obtaining schooling services from a competitive private-school sector, where the state financial support is not available, but still having tax liability. As before, let  $f \in [0, 1]$  denote the proportion of households choosing publicly provided services.

### 2.3.1 Public schools under a state system

Let's assume for a while that all households in a district utilize public schools and that  $f$  is fixed. Given that the tax revenues are used to fund the education and that the state budget is balanced the per-capita

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<sup>14</sup>See Glomm and Ravikumar (1998), Epple and Romano (1996a) for details. The authors study this problem in similar context to ours where public schooling is homogenous (no supplementation).

<sup>15</sup>The presentation of a state system with private schools follows Gradstein, Justman and Meier (2005).



schooling expenditure level is

$$s_i = \bar{s} = \frac{\tau m}{f}. \quad (8)$$

It follows that for the representative household in a district  $i$

$$\begin{aligned} c_i &= (1 - \tau)y_i \\ s_i &= \frac{\tau m}{f}. \end{aligned}$$

The state-system tax rate  $\tau$  is chosen by a majority vote. To find the majority vote outcome we have to determine first a preferred tax rate for every voter. This preferred tax rate is the solution to the representative household in a district  $i$ 's maximization problem (recall that for public schools  $q = s$ )

$$\begin{aligned} \max_{\tau} & \{u((1 - \tau)y_i) + v(\frac{\tau m}{f})\} \\ \text{s.t.} & \quad \tau \geq 0 \end{aligned},$$

yielding the first order condition (note that the assumptions about the utility function guarantee that the solution is unique and interior<sup>16</sup>)

$$-u'((1 - \tau)y_i)y_i + v'(\frac{\tau m}{f})\frac{m}{f} = 0. \quad (9)$$

The solution tax rate  $\tau$  optimally balances spending on consumption and spending on education.

For further reference the induced indirect utility function of a household in a district  $i$  that uses public services is

$$V(\tau, f, y_i) = u((1 - \tau)y_i) + v(\frac{\tau m}{f}).$$

### 2.3.2 Private schools under a state system

Under a state system private-school users face the same maximization problem as under a foundation system. Hence our previous discussion about private schooling in the case of a foundation system applies here also.

### 2.3.3 Equilibrium under a state system with private schools

The crucial difference between a foundation system and a state system (both with a private-school option) is that under a foundation system we allow districts to supplement the grant, whereas under a state one the supplementation of revenues obtained from the state budget for education is forbidden. Therefore under a state system no group of districts

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<sup>16</sup>Assumed strict concavity of the utility function guarantees that this preferred tax rate is unique. This also implies that the first order condition is necessary and sufficient for optimum. Inada conditions imply interior solution.

supplements these revenues. Formally, the grant (under a foundation system) and the revenues from the state budget (under a state system) have the same meaning. The difference between them is only in the fact that the grant can be further supplemented using local taxes and the revenues not. It follows that formally we can treat a state system as a special case of a foundation system with local tax rates  $t_i$  equal to zero for all districts. Hence we can present an equilibrium under a state system concentrating only on some important aspects that make this equilibrium slightly different from the one under a foundation system. All other issues remain the same.

Assume that all districts in the economy anticipate the same public enrolment rate  $f^e$ . Like under a foundation system, once the state-system tax rate  $\tau$  has been determined by majority voting (like earlier  $\tau^*$  denotes the decisive voter most preferred tax rate), all households in districts compare their  $V(\tau^*, f^e, y_i)$  and  $W(\tau^*, y_i)$  and decide which type of schooling to choose. The representative household in a district  $i$  will choose to send its child to a public school if  $V(\tau^*, f^e, y_i) > W(\tau^*, y_i)$ . As before, we have to be sure that in an equilibrium the individual decisions of households are consistent with the aggregate outcomes. That is the proportion of households for which  $V(\tau^*, f^e, y_i) > W(\tau^*, y_i)$  must be exactly equal to  $f^e$ . As opting out of public education does not reduce tax obligations of the representative household in a given district, and the household is aimed at obtaining a higher quality of education, other things being equal, households in districts that opt out of public schooling will be those with higher incomes. Glomm and Ravikumar (1998) proved that for a given tax level  $\tau \in (0, 1)$  there exists a threshold income level  $\underline{y} = \underline{y}(\tau, f^e)$  such that all households in districts with income below  $\underline{y}$  send their children to public schools and all households in districts with income above  $\underline{y}$  send their children to private schools<sup>17</sup>. This threshold income is implicitly defined by  $V(\tau, f^e, \underline{y}) = W(\tau, \underline{y})$ . Hence, using the threshold income  $\underline{y}$  the consistency condition for an equilibrium given a state-system tax rate  $\tau$  can be written as  $f^e = F(\underline{y}(\tau, f^e))$  - the anticipated public-school enrolment rate must equal an actual enrolment rate.

Like under a foundation system all districts in the economy vote on tax rates and the equilibrium state-system tax rate  $\tau$  is the one chosen by a majority of voters. The households in the richest districts prefer

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<sup>17</sup>They proved it assuming that  $a = 1$  for public and private schools. However their result is valid also in our case. The quality parameter  $a > 1$  for private schools shifts only  $\underline{y}$  downwards, since higher quality in private schools induces more households (districts) to choose private schooling. This effect was shown by Cohen-Zada and Justman (2003).

a zero state-system tax rate, because they benefit more from private schooling than from any level of public schooling. In turn households in poor districts anticipating that they will utilize public services prefer a positive state-system tax rate at which it  $V(\cdot)$  reaches its maximum. To identify the decisive voter we need to see how a preferred state-system tax rate changes with income. Using the implicit function theorem on (9) we obtain

$$\frac{d\tau}{dy_i} \Big|_{y_i < \underline{y}} = \frac{u''((1-\tau)y_i)c_i + u'((1-\tau)y_i)}{u''((1-\tau)y_i)y_i^2 + v''(\frac{\tau m}{f})(\frac{m}{f})^2}, \quad (10)$$

which is formally the same as under a foundation system (eq. (5)). The sign of (10) is that of  $-\{u''((1-\tau)y_i)c_i + u'((1-\tau)y_i)\}$ .

As before, if the sign of (10) is negative<sup>18</sup>, then a preferred tax rate decreases with income among districts that prefer public to private schooling. Since high-income districts that anticipate using private services prefer a zero state-system tax rate, this implies that a preferred tax rate decreases monotonically with income throughout the entire range of incomes. Hence, using the single-crossing property rule we obtain that a global political equilibrium exists and that the equilibrium state-system tax rate is the tax rate preferred by the households in the median income district. The income of the decisive voter  $y_d$  satisfies in this case

$$F(y_d) = 0.5. \quad (11)$$

If (10) equals zero<sup>19</sup>, then a preferred foundation tax rate doesn't depend on income among districts that prefer public to private schooling. Moreover, all high-income districts that anticipate using private-school services prefer a zero state-system tax rate. Hence, by skewness of the income distribution a coalition supporting a zero state-system tax rate consists of less than 50% of the districts<sup>20</sup>. It follows that in this case the median income voter is still decisive and its income satisfies (11).

If the sign of (10) is positive<sup>21</sup>, then a preferred tax rate increases with income among districts that prefer public to private schooling. Like under a foundation case, an "ends against the middle" coalition is formed. Poor districts, which prefer less public spending on education, join forces with the rich districts, which would rather send their children to private schools, and therefore prefer a zero state-system tax rate. The opposition

<sup>18</sup>For the class of the CES utility functions it implies  $\sigma > 1$ .

<sup>19</sup>For the class of the CES utility functions it implies  $\sigma = 1$ .

<sup>20</sup>We assume here that in equilibrium  $me < \underline{y}$ . Since private school users are those with the highest income levels, this assumption is practically of no consequence.

<sup>21</sup>For the class of the CES utility functions it implies  $\sigma < 1$ .

is the middle-income districts, which prefer higher spending levels. As Epple and Romano (1996a) argue in this case in general single-crossing property doesn't hold and a global political equilibrium may not exist. To avoid this problem we will follow the assumption of "myopic voting" used by Nechyba (1999), which we discussed in the case of a foundation system equilibrium. Given this assumption the income of the decisive voter in an equilibrium of this type satisfies

$$F(y_d) + 1 - f = 0.5. \tag{12}$$

Notice that - like under a foundation system - since  $f = F(\underline{y}) \leq 1$ , the income of the decisive voter  $y_d$  in this case is not greater than the median income.

## 2.4 Some Analytical Comparisons of the Systems for the CES Utility Functions

In our computational exercise we will use the class of the CES utility functions given by (1). We will discuss now some analytical properties of a voting equilibrium under this class of utility functions. Throughout this section we will concentrate only on districts that use public education services.

Note that the first order condition (9) for a preferred central tax rate under a state system is identical to the one obtained under a foundation system in (4) (with equality for  $t_i = 0$ ). Using these first order conditions we obtain that for the CES utility functions the formula for a central tax rate preferred by the representative household in a district  $i$  under both systems can be written as

$$\tau = \frac{\delta}{\left(\frac{fy_i}{m}\right)^{\sigma-1} \delta^{1-\sigma} + \delta}. \tag{13}$$

It is straightforward to show that a preferred tax rate level is increasing with  $f$  for  $\sigma < 1$  and decreasing with  $f$  for  $\sigma > 1$ . If  $\sigma = 1$ , then changes in  $f$  do not affect a preferred tax rate.

The same formula (13) for both systems doesn't necessarily mean that under both systems the representative household in a given district will also prefer the same central tax rate. As can be noticed, a preferred central tax rate depends on a public enrolment rate. This rate may be different in general under both systems, as we will argue later. As a result, preferred levels of a central tax rate may also differ between both systems. Moreover, under a foundation system some groups of districts prefer a zero central tax rate, independently of the value given by (13). This fact may also affect the relation between central-tax-rate

levels preferred by the representative household in a given district under both systems.

It follows from our previous discussions and from (13) that if  $f = f_f = f_s$ <sup>22</sup>, then a preferred central tax rate among districts with income  $y_i < \frac{m}{f}$  is the same under both systems. However, districts with income  $y_i > \frac{m}{f}$  do not prefer the same central tax rate under both systems<sup>23</sup>. Under a foundation system they prefer a zero tax rate, whereas under a state system a positive tax rate. It is the result of the fact that under a foundation system districts have an option of funding their education locally and all districts with income  $y_i > \frac{m}{f}$  prefer this type of funding. Under a state system it is not possible to fund education locally, and consequently districts with income  $y_i > \frac{m}{f}$  prefer a positive tax rate.

However, in general we expect that in equilibrium  $f_f > f_s$ . Districts that supplement the grant under a foundation system will lose such possibility under a state system. Hence the quality of education in districts that supplement the grant and whose income is just below  $y_f$  will likely fall. This will induce some of the districts to opt out of a public sector and to use private-school services. As a result, using (13) we obtain that in an equilibrium the central tax rate preferred by households in a given district with income  $y_i < \frac{m}{f_f}$  under a state system will be lower (if  $\sigma < 1$ ), the same (if  $\sigma = 1$ ) or higher (if  $\sigma > 1$ ) than under a foundation system. In turn, households in districts with income  $y_i > \frac{m}{f_f}$  will always prefer a higher central tax rate under a state system (a positive tax rate) than under a foundation system (a zero tax rate), independently of the value of  $\sigma$ .

These results affect also the equilibrium tax rates. Notice first that by eq. (6), (7), (11) and (12) the income level of the decisive voter under both systems is never greater than the median income level. By our assumption about the income distribution we have that the median income  $me$  and the mean income  $m$  satisfy  $me < \frac{m}{f_f}$  for any  $f_f \in (0, 1]$ . Hence the decisive voter under both systems has always the income level lower than  $\frac{m}{f_f}$ . Consequently, in the remaining part of this section while discussing the relationship between the equilibrium tax rates we restrict our attention only to districts in this income interval.

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<sup>22</sup>We will use index  $f$  to denote the values of variables related to a foundation system and  $s$  for the values related to a state system.

<sup>23</sup>It is very hard to draw similar conclusion for the district with income equal to  $\frac{m}{f}$ . As we noted earlier, under a foundation system this district is indifferent between central and local funding of education. It follows that determination of the relation between preferred central tax rates under a foundation and a state system is very difficult. Consequently, in this section we concentrate our discussion on districts with income different than  $\frac{m}{f}$ .

If a preferred central tax rate is decreasing with income or constant in income (that is when  $\sigma \geq 1$ ), then under both systems the decisive voter is the median-income district defined in (6) and (11). Since in all districts with income  $y_i < \frac{m}{f_f}$  (in particular in the median-income district) under a state system the preferred central tax rate is the same (if  $\sigma = 1$ ) or higher (if  $\sigma > 1$ ) than under a foundation system, the equilibrium central tax rate under a state system will also be respectively the same or higher than under a foundation system. Notice also that when  $\sigma \geq 1$  it is not possible that in equilibrium  $f_f = f_s$ . If a given district opts out of a public sector as a consequence of the reform, then the only way to make it go back to this sector under a state system is to raise the quality level of public education. In our model this can be achieved only by increasing the per-capita spending in the public education, given by (8) (recall that for all public schools the relationship between quality of schooling  $q$  and the per-capita spending in education  $s$  is  $q = s$ ). Since we would like to have under a state system the same public enrolment rate as under a foundation system ( $f = f_f = f_s$ ), raising this per-capita education spending requires to increase appropriately the equilibrium state-system tax rate compared to the equilibrium foundation tax rate. However, using (13) (with the fixed value of  $f = f_f = f_s$ ) and the fact that under a foundation system and under a state system the decisive voter has the same level of income (the median income), we obtain that under both systems the equilibrium central tax rate necessarily has to be the same. It follows that the desired increase in the equilibrium state-system tax rate and - consequently - the equality of the public enrolment rates is in this case not possible.

If a preferred central tax rate is increasing with income and an "ends against the middle" coalitions are formed (that is when  $\sigma < 1$ ), then the relationship between the equilibrium tax rates is now not so obvious. Now the decisive voter under both systems has in general different income, defined in (7) and (12). Depending upon the equilibrium levels of public enrolment rates  $f_f$  and  $f_s$ , which determine the relation between values of  $\hat{y} = \min\{\frac{m}{f_f}, \underline{y}_f\}$  and  $\underline{y}_s$ , the size of a coalition supporting a zero central tax rate under a state system may be smaller, the same or larger than under a foundation system. If the equilibrium values of  $f_f$  and  $f_s$  are such that  $\hat{y} < \underline{y}_s$ , then the size of a coalition supporting a zero central tax rate under a foundation system is larger than the size of a similar coalition under a state system. It follows that in this case the decisive voter under a state system will have higher income than under a foundation system. Now two opposite factors determine the relation between the equilibrium central tax rates under both systems. On the one hand, higher income of the decisive voter under a state system in-

creases its preferred central tax rate compared to the central tax rate preferred by the decisive voter under a foundation system. On the other hand, using previous arguments we have that for  $\sigma < 1$  in all districts with income  $y_i < \frac{m}{f_f}$  (in particular in the decisive district, whose income belongs to this interval) a preferred central tax rate under a state system is in general lower than under a foundation system (it may be the same if in equilibrium  $f_f = f_s$ , which is a very unlikely outcome). As a result, the equilibrium central tax rate under a state system may be lower, the same or higher than under a foundation system. The outcome in a given case depends on the relationship between the two opposite factors.

However, it may happen that the equilibrium values of  $f_f$  and  $f_s$  are such that  $\hat{y} \geq \underline{y}_s$ . Then the size of a coalition supporting a zero central tax rate under a foundation system is the same or smaller than the size of a similar coalition under a state system. As a result, the decisive voter under a state system will have the same or lower income than under a foundation system. The same or lower income of the decisive voter under a state system respectively keeps constant or decreases its preferred central tax rate compared to the central tax rate preferred by the decisive voter under a foundation system. Moreover, we have here that for  $\sigma < 1$  in all districts with income  $y_i < \frac{m}{f_f}$  (in particular in the decisive district) a preferred central tax rate under a state system is lower than under a foundation system (as we will show in a while it cannot be the same since here always  $f_f > f_s$ ). In total these two effects result in the equilibrium state-system tax rate lower than the foundation tax rate. Notice that in this case it is not possible that in equilibrium  $f_f = f_s$ . It might happen here only if  $\underline{y}_s = \underline{y}_f \leq \frac{m}{f_f}$  were satisfied (recall that  $f_f = F(\underline{y}_f)$  and  $f_s = F(\underline{y}_s)$ ). Using similar arguments as before, in order to have in equilibrium under a state system the same public enrolment rate as under a foundation system we need to raise appropriately the equilibrium state-system tax rate compared to the equilibrium foundation tax rate. However, using (13) (with the fixed value of  $f = f_f = f_s$ ) and the fact that under a foundation system and under a state system the decisive voter has the same level of income (if  $\underline{y}_s = \underline{y}_f \leq \frac{m}{f_f}$ , then the size of a coalition supporting a zero central tax rate is the same under both systems, determined by values of  $\underline{y}_s$  and  $\underline{y}_f$ ), we obtain that under both systems the equilibrium central tax rate necessarily has to be the same. It follows that the desired increase in the equilibrium state-system tax rate and - consequently - the equality of the public enrolment rates is in this case not possible.

The change in the equilibrium central tax rate is one of the factors that affect changes in total education expenditure. This factor leads to

the increase or decrease in the expenditure level under a state system relative to that one under a foundation system, depending on whether the equilibrium state tax rate is respectively higher or lower than the foundation tax rate. In particular the expenditure may remain constant if the equilibrium tax rates are the same. However, there is another factor that decreases education spending under a state system: under a state system, districts cannot supplement the revenues obtained from the state for education. Total education spending is also affected by changes in a private enrolment rate. As we showed before, under a state system this enrolment rate likely increases. Hence districts that switch to use private-school services decide about their new level of education spending in order to adjust to the new environment.

### 3 Quantitative Results

In this section we report the results of quantitative comparison of a foundation and a state system when we allow for the presence of private schooling. A foundation system will be treated as the benchmark system for our comparisons, since it is considered to be the best approximation of the existing "average" system in the USA. Both education finance systems affect total schooling expenditure and welfare. Our objective is to obtain some sense for how large differences might be if we introduce private schools to the model.

#### 3.1 Functional Forms and Parameter Values

We begin by specifying functional forms and assigning parameter values. Our model's structure is very parsimonious - we need to choose only the distribution of income and define in a detailed way preferences over consumption and quality of education.

A possible choice for the income distribution is to use households' income distribution for a typical year. But if we interpret the model as referring to the entire schooling period of a child, then presumably it is the distribution of income over a longer time span (like life-time income) that is more relevant. However, Fernández and Rogerson (2003), who do comparison similar to ours, report that the choice between these two types of income distribution is not important quantitatively. Following their remark we concentrate in our calibration exercise on annual income distribution. Specifically, we use data on the United States income distribution in 1989. In our computational analysis we utilize approximation to the income distribution by the log-normal distribution of the form  $\ln(y) \sim N(\mu, \rho)^{24}$ , where  $\mu$  is the expected value and  $\rho$  is the standard

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<sup>24</sup>If  $\ln(y) \sim N(\mu, \rho)$ , then the mean  $m$  of  $y$  is  $m = E(y) = e^{(\mu + \frac{\rho^2}{2})}$ , and the median



deviation of  $\ln(y)$ . As Epple and Romano (1996a) report, in the United States in 1989 mean and median household income were 36,250 \$ and 28,906 \$, respectively. These pieces of data imply that  $\mu = 3.36$  and  $\rho = 0.68$  (for income measured in thousands).

To describe preferences over consumption and quality of education we use the family of the CES utility functions defined in (1). This choice of a functional form is dictated by the empirical evidence that over the long run educational expenditures and personal income grow at (approximately) the same rate. Over the last thirty years public spending on primary and secondary education as a fraction of personal income has remained roughly constant, despite roughly a doubling of real personal income. This finding holds after considering other potential factors (like observed changes in the fertility rate or standard deviation of earnings) that could influence the education spending. In our model, for both education finance systems we consider, the requirement that a proportional shift in the income distribution (holding fixed the education finance system) keeps constant the share of income spent on education (implying income elasticity of demand equal to one) requires that preferences be homothetic. Given this the natural class of utility functions to consider is that of constant elasticity of substitution (CES) (and its monotone transformations).

Given this specification, there are two parameters to determine,  $\sigma$  - the elasticity of substitution between consumption and schooling and  $\delta$ . However, quality  $q$  depends on the parameter  $a$ , which has to be specified for private schools (for public schools it is equal to one). Hence full specification of the utility function for our problem requires to find values of three parameters.

How to determine the value of  $\sigma$  is not completely obvious. A natural restriction on values of parameter  $\sigma$  seems to be  $\sigma < 1$ . Under this restriction a preferred tax rate is increasing with income. Fernández and Rogerson (2003) argue that such restriction in richer models with endogenous stratification of income types into communities is required to ensure that richer communities have higher quality education. The authors also study the empirical literature and find strong support for this restriction on values of parameter  $\sigma$ . They also report that vast majority of empirical results related to a foundation system suggests  $\sigma$  between 0.33 and 1. However, to start the computational exercise we need to restrict further values of  $\sigma$ . Epple and Romano (1996a) obtained that  $\sigma = 0.65$  does the best job in their model of education with private schools. We incorporate their finding in our model but also

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*me* of  $y$  is  $me = e^\mu$ . Given the mean and the median of  $y$ , these can be solved for  $\mu$  and  $\rho$ .

do some sensitivity analysis of our results to changes of this parameter, examining selected values of  $\sigma$  between 0.33 and 1.

Parameters  $\delta$  and  $a$  are chosen simultaneously. We choose  $\delta$  to generate in an equilibrium the average expenditure per student in public schools under a foundation system equal to 2,111 \$, which is the observed level of per-household public education expenditure in the USA in 1988 (Epple and Romano (1996a))<sup>25</sup>. Since we expected that the quality parameter  $a$  would strongly influence the equilibrium public enrolment rate, our goal was to find such value of  $a$  that would guarantee the public-school enrolment level equal to 88%, reported by the empirical data for the USA (Epple and Romano (1996a)). There is little literature which could help us to find exact value of  $a$ . Cohen-Zada and Justman (2003) in their calibration for the case of a state system with private schools received a value of  $a$  for private schools of the order of 1.5. We tried to incorporate this value of  $a$  for private schools in our computational exercise, but unfortunately it turned out that  $a = 1.5$  is too small to guarantee the desired public-school enrolment rate of 88%. However, the process of calibration showed that  $\delta$  had very little impact on the equilibrium public enrolment rate but substantial on the equilibrium per-student public expenditure level and conversely,  $a$  strongly influenced the equilibrium public enrolment rate but very little the equilibrium per-student public expenditure level. Using this fact we found for different values of  $\sigma$  corresponding values of  $\delta$  and  $a$ , which guaranteed in an equilibrium the desired levels of a public-school enrolment rate and per-student expenditure.

## 3.2 General Calibration Results

Table 1 presents the results of our parameter calibration with corresponding values of the equilibrium central tax rates and the equilibrium public-school enrolment rates.

As we expected, when we go from a foundation system to a state system of education financing, the equilibrium public-school enrolment rate decreases. This is indicated by two last columns of Table 1. As can also be seen, in general the equilibrium central tax rate is higher under a state system than under a foundation system. These results correspond to the ambiguous case when the decisive voter under a state system has higher income than under a foundation system ( $\hat{y} < y_s$ ), which we discussed in Section 2.4. In our exercise we have that the positive effect (on a preferred central tax rate) of shifting the decisive voter income

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<sup>25</sup>The observed per-student public expenditure level was 4,222 \$, but there were 0.5 students per household. This gives the per-household public expenditure level equal to 2,111 \$.

$\sigma$	$\delta$	$a$	$\tau_f$	$\tau_s$	$f_f$	$f_s$
0.40	0.0012861	1.5600	0.0387	0.0338	0.8800	0.7000
0.50	0.0045490	1.6165	0.0409	0.0419	0.8800	0.7805
0.60	0.0104510	1.6770	0.0429	0.0459	0.8800	0.8181
0.65	0.0143362	1.7083	0.0439	0.0471	0.8800	0.8302
0.70	0.0187540	1.7410	0.0448	0.0480	0.8800	0.8393
0.80	0.0288330	1.8085	0.0465	0.0490	0.8800	0.8517
0.90	0.0399850	1.8789	0.0479	0.0493	0.8800	0.8586
0.99	0.0504330	1.9441	0.0490	0.0491	0.8800	0.8615

Table 1: Parameter calibration results

upwards is stronger than the negative effect of decrease in the public enrolment rate. As a result, we have that the equilibrium central tax rate is higher under a state system than under a foundation system. However, for  $\sigma = 0.4$  this is not the case<sup>26</sup>. Here, the equilibrium central tax rate is lower under a state system than under a foundation system. This result corresponds to the case when the income of the decisive voter under a state system is below the income of the decisive voter under a foundation system ( $\hat{y} > \underline{y}_s$ ). We have here that the negative effect (on a preferred central tax rate) of shifting the decisive voter income downwards adds up to the negative effect of decrease in the public enrolment rate. As a result, we observe a drop in the equilibrium central tax rate. We discussed this likely effect in Section 2.4.

Note also that as  $\sigma$  approaches one, we observe that the equilibrium central tax rates under both systems converge to each other. This is consistent with our previous results. Using eq. (13) we obtain that for  $\sigma$  equal to one a preferred central tax rate is the same under both systems and independent of a public enrolment rate. Moreover, for this value of  $\sigma$  under both systems the median income district is decisive. As a result, the equilibrium tax rates will be the same. Note also that similarly - as  $\sigma$  approaches one - the equilibrium public enrolment rates converge to each other. However, we do not expect them to be equal in the limit, when  $\sigma$  equals one. As we argued in Section 2.4, the equilibrium public enrolment rates cannot be equal for this value of  $\sigma$ .

### 3.3 Education Spending Analysis

In this section we report our results concerning the education expenditures under both finance systems. Tables 2-5 present the predicted expenditure effects of the change from a foundation system to a state

<sup>26</sup>Of course by continuity this applies also to other values in the close neighbourhood of  $\sigma = 0.4$ .

$\sigma$	$\frac{Q_s - Q_f}{Q_f}$	$\frac{Q_s^{PUB} - Q_f^{PUB}}{Q_f^{PUB}}$	$\frac{Q_s^{PRIV} - Q_f^{PRIV}}{Q_f^{PRIV}}$	$\frac{Q_f^{PUB} - Q_g^{PUB}}{Q_f^{PUB}}$	$\frac{\bar{s} - g}{g}$
0.40	-0.0760	-0.3400	0.8254	0.2458	0.1002
0.50	-0.0303	-0.1820	0.4924	0.2027	0.1568
0.60	-0.0103	-0.1048	0.3186	0.1622	0.1494
0.65	-0.0050	-0.0809	0.2603	0.1434	0.1374
0.70	-0.0019	-0.0636	0.2144	0.1254	0.1226
0.80	-0.0007	-0.0438	0.1516	0.0928	0.0890
0.90	-0.0043	-0.0383	0.1166	0.0650	0.0543
0.99	-0.0102	-0.0418	0.1024	0.0445	0.0244

Table 2: Comparison of total expenditures on education under both systems

$\sigma$	$CV_f$	$CV_s$	$\frac{Q_f}{m}$	$\frac{Q_s}{m}$
0.40	0.4839	0.4863	0.0663	0.0612
0.50	0.4653	0.4315	0.0661	0.0641
0.60	0.4469	0.4070	0.0660	0.0653
0.65	0.4381	0.3997	0.0659	0.0656
0.70	0.4295	0.3945	0.0659	0.0657
0.80	0.4138	0.3890	0.0658	0.0657
0.90	0.4007	0.3883	0.0657	0.0654
0.99	0.3915	0.3906	0.0656	0.0649

Table 3: Total expenditures on education under both systems - the coefficient of variation and the fraction of total income spent on education

system.

Table 2 shows the impact of finance system change on a total education expenditure level. We utilize here the following notation:  $Q$  denotes total education expenditure; superscripts  $PUB$  and  $PRIV$  denote expenditures in a public and private sector, respectively;  $Q_g^{PUB}$  denotes total central funds spent on foundation grants under a foundation system;  $\bar{s}$  denotes per-student expenditure in a public sector under a state system;  $g$  denotes a foundation grant level under a foundation system. The second column of this table gives information about the change of total resources spent on education. The third and the fourth columns provide information about the change of total resources spent on education within sectors - public and private, respectively. The penultimate column shows the fraction of total expenditures within a public sector under a foundation system that are accounted for by spending above a grant level  $g$ . The last column shows the fraction of a foundation system grant level  $g$  by which it is exceeded by per-student spending in a public

sector under a state system.

As can be seen, the model predicts that a shift from a foundation to a state system will result in a drop in total education spending. This drop is accompanied by a substantial fall in resources spent on public education. However, total spending in a private sector increases with the change of a system. A clear pattern emerges as  $\sigma$  increases: the effect of the reform on total expenditures, also within sectors, becomes smaller. These results confirm findings of Fernández and Rogerson (1999). The authors studied effects of the same finance reform on total expenditure in a public education sector but without considering the existence of private schools. Similarly to their findings our model predicts that switching from a foundation system to a state one causes a fall in the total expenditures for education in a public sector. However, detailed comparison of theirs and our findings sheds new light on the effects of the reform. First, for similar range of parameter  $\sigma$  Fernández and Rogerson (1999) obtained that total public expenditures decreased by between 8% and 13%. Our model predicts much greater range of this drop by between 4.18% and 34%. This difference is caused by the private-school alternative. As our model predicts, switching of a finance system induces many households to start to use private education as the response to centralization of a finance system and equalization of per-student spending in a public sector. These changes in private-school enrolment additionally affect total funds spent on public schools in the economy. Second, a relatively large drop in total expenditures in a public sector doesn't necessarily mean a large fall in total resources spent on education in the whole economy. As can be seen in Table 2 these are affected very little by the change of a finance system compared to a public sector, with a drop ranging between 1.02% and 7.6%, depending upon the exact value of  $\sigma$ . This smaller drop is again due to the existence of private schools. In consequence of the reform substantial amount of total resources is just shifted from a public sector to a private one. A private-school sector benefits from the system change in terms of total expenditure on education. This expenditure increases within the range between 10.24% and 82.54%, depending upon the exact value of  $\sigma$ .

Table 3 reports for both systems a value of the coefficient of variation of education spending across students and the fraction of total income devoted to education, denoted by  $CV$  and  $\frac{Q}{m}$ , respectively. For any value of  $\sigma$ , the fraction of total income spent on education remains roughly constant: between 6.56% and 6.63% under a foundation system, and between 6.12% and 6.49% under a state system. The coefficient of variation can be interpreted as a measure of inequality of educational resources across the income distribution. The lower it is, the more equi-

$\sigma$	$\frac{AQ_s^{PUB} - AQ_f^{PUB}}{AQ_f^{PUB}}$	$\frac{AQ_s^{PRIV} - AQ_f^{PRIV}}{AQ_f^{PRIV}}$
0.40	-0.1702	-0.2699
0.50	-0.0777	-0.1843
0.60	-0.0370	-0.1305
0.65	-0.0256	-0.1103
0.70	-0.0181	-0.0934
0.80	-0.0121	-0.0682
0.90	-0.0143	-0.0525
0.99	-0.0212	-0.0449

Table 4: Comparison of per-student expenditures on education under both systems

table is the finance system. Some authors provide results showing that the dispersion of school expenditures falls with centralization (Fernández and Rogerson (2003)). They also show that a pure state system without a private-school option has the value of the coefficient of variation equal to zero. Our results confirm the former finding, but not the latter one. In our model with private schools a state system is more equitable than a foundation system, but the difference in inequality measured by the coefficient of variation is very small. What we observe is only a small fall in the coefficient of variation under a state system compared to a foundation system. Our results also show that if we introduce private schools to a pure state system, then the coefficient of variation can no longer be equal to zero. This great difference between a pure state system and our case comes from the fact that at the top of the income distribution there is more households opting out to private schools, which raises inequality.

Since the number of public and private-sector users changes with the change of a finance system, the natural question to ask is how it affects per-student expenditures on education. Table 4 presents our findings about per-student education expenditures in a public and private sector for different values of  $\sigma$ . In the table  $AQ$  denotes per-student education spending. As can be seen, the model predicts a drop in per-student education spending in both public and private sector. This fall is even greater in private schools. The drop in a public sector is caused by the fact that districts no longer can supplement their education spending. The supplementation under a foundation system accounts for between 4.45% and 24.58% of total education spending in a public sector (as shown in Table 2). Under a state system these additional sources of funds are not available and increased central per-student revenues (compared to a foundation grant level, as shown in the last column of Table

<b>Income percentile range</b>	<b><math>AQ_f</math></b>	<b><math>AQ_s</math></b>
0.00-0.02	1.8084	2.0569
0.02-0.10	1.8084	2.0569
0.10-0.20	1.8084	2.0569
0.20-0.30	1.8084	2.0569
0.30-0.40	1.8084	2.0569
0.40-0.50	1.8084	2.0569
0.50-0.60	1.9071	2.0569
0.60-0.70	2.2459	2.0569
0.70-0.80	2.7094	2.0569
0.80-0.90	3.3016	2.6478
0.90-0.98	4.0576	4.0440
0.98-1.00	7.2335	7.2093
<i>CV</i>	0.4381	0.3997

Table 5: Distributions of per-student education spending under both systems for  $\sigma = 0.65$

2) are not able to compensate this loss. The drop in spending per student in private schools has two main sources. The first one is that under a state system the central tax rate (as presented in Table 1) is higher than under a foundation one (except for  $\sigma = 0.4$ , when it is lower). This decreases disposable income of households under a state system and - in consequence - their education spending. The second one is that under a state system the private-school enrolment rate (as suggested by data in Table 1) is higher than under a foundation one. Under a state system new districts start to use private-school services. These new districts entering a private sector are populated by households with lower income levels than districts that have already used private schools. The new private-school users necessarily spend on education less than richer districts, which negatively affects per-student spending in a private-school sector.

Table 5 presents an example of distributions of per-student education spending under both systems for  $\sigma = 0.65$ . It allows us to see which households lose and which benefit from the finance system change in terms of per-student education spending. Clearly, approximately 60% of all households (the group of the poorest households) benefit from the change into the state system. Per-student expenditure increases for them. However, households above the 60th income percentile lose in consequence of this change. Note also that the reform results in a more equitable distribution of per-student education spending under a state system than under a foundation system, as indicated by corresponding

values of the coefficient of variation.

### 3.4 Welfare Analysis

In this section we examine the welfare implications of our two education finance systems. To compare the overall welfare we compute for each system the expected utility that a household would obtain, if under that system its income were a random draw from the income distribution. Specifically, for a foundation system and for a state system we compute the expected utilities denoted  $EU_f$  and  $EU_s$ , respectively. Although comparisons of these two welfare indices allow us to conclude whether welfare has increased or not, it does not provide any information as to how significant the change in welfare is. Hence, we are interested in using a measure of welfare change that is not affected by monotone transformations of a utility function. Using a foundation system as the benchmark for our utility comparisons, we compute the fraction  $\Delta$  by which the entire income distribution would have to be changed under a foundation system so that a foundation system and a state system provide the same aggregate welfare (Fernández and Rogerson (1997)). For the class of CES utility functions that we use, it means to find the value  $\Delta$  such that

$$(1 + \Delta)^{1 - \frac{1}{\sigma}} EU_f = EU_s.$$

This expression follows from the fact that for this class of utility functions indirect utility functions are homogenous of degree  $(1 - \frac{1}{\sigma})$  in income.

Table 6 presents the results of our welfare comparisons for different values of parameter  $\sigma$ . An immediate result is that in the presence of the private-school alternative a state system is welfare superior to a foundation system (note that a positive values of  $\Delta$  indicates that households obtain higher expected utility under a state system than under a foundation system). For  $\sigma = 0.65$  all households in the economy would have to have income higher by 0.14% under a foundation system to obtain the same expected utility as under a state system. This welfare difference may seem to be not very big. However, we have to keep in mind that total education expenditures comprise only a small fraction of the total income (under a foundation system it is between 6.56% and 6.63%) and while interpreting this welfare difference we should scale it in proportion to the size of the education sector.

The advantage of a state alternative decreases when values of  $\sigma$  increase, but even for  $\sigma$  close to one welfare is higher under a state system than under a foundation one. These gains reflect the fact that resources are being reallocated from wealthier to less wealthy individuals, or equivalently - that the variance of the consumption and schooling expenditures distributions are being diminished, as indicated by values of the



$\sigma$	$\Delta$
0.40	0.0066
0.50	0.0033
0.60	0.0018
0.65	0.0014
0.70	0.0011
0.80	0.0007
0.90	0.0006
0.99	0.0007

Table 6: Welfare comparison of the education finance systems

coefficient of variation in Table 3.

These results are more striking, when we compare them with ones obtained by other authors. Fernández and Rogerson (2003) in very similar static comparison, but not allowing for the presence of private schools, obtained a converse result that a foundation system was welfare superior to a state system. Our result shows that a private-school option has great importance in terms of total welfare. It strongly affects total welfare induced by a finance system. The observed shift upwards of a welfare level can be explained by the combined effect of quality gains in private schools, increased private-school enrolment under a state system and also higher per-student expenditure in public schools under a state system in a group of the poorest districts. Under a state system a larger group of households benefits from increased schooling quality in a private sector. Also a group of the poorest public-sector users benefits from increased per-student expenditure. These effects in total positively affect the expected utility under a state system.

## 4 Conclusions

In this paper we used a political economy approach to analyse the outcomes generated by a foundation system and a state system of education finance in the presence of private schools. In addition to analytically characterizing both systems we carried out a calibration exercise to assess quantitative significance of the differences between these systems. We found that compared to pure systems without private schools the private alternative introduces substantial distortion in welfare and education spending distributions. In particular, we showed that considering private schools in the model reverses the present-in-the-literature ranking in which a foundation system is welfare superior to a state system. In the light of our findings this ranking turns out to be true only if we do not allow in the model for the existence of private schools. The private-

school alternative also influences the equity under both finance systems, especially strongly in the case of a state system. A pure state system is considered to be the most equitable among different education finance systems. Our results show, however, that introducing private schools to the model results in equity under a state system only a little higher than under a foundation system. Moreover, this equity, as measured by the coefficient of variation, is far from perfect equity attributed to a pure state system. Our findings confirm the result, present in the literature, that a shift from a foundation to a state system causes a fall in total resources spent on public education. They also reveal that this change of a finance system may result in decreasing per-student spending in both public and private sector. This happens, though private schools benefit from higher total spending under a state system than under a foundation one.

Our model was purposefully simple in order to explore the relations between total resources to education, welfare and also equity. This approach allowed us to obtain analytical and quantitative results regarding both finance systems and compare them. We close with some cautions about interpreting our results. First, due to calibration issues, in our quantitative exercise we used data for 1989, which are now a little outdated. So probably the results obtained for current data would be slightly different in details. Second, our analysis is static and doesn't consider any dynamic long-term implications of the change of an education finance system on the income distribution, which could possibly change our findings.

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