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Munns, Andrew

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Andrew Munns

School of Science and Engineering, University of Dundee, Perth Road, Dundee DD1 4HN

Email: akmunns@dundee.ac.uk

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Abstract

Professional bodies expect engineers to show competence in both mathematics and engineering topics such as mechanics, using their abilities in both of these to solve problems. Yet within engineering programmes there is a phenomena known as ‘The Mathematics Problem’, with students not demonstrating understanding of the subject. This paper will suggest that students are constructing different concept images in engineering and mathematics, based on their perception of either the use or exchange value for the topics. Using a mixed methods approach, the paper compares ten different types of concept image constructed by students, which suggests that familiar procedural images are preferred in mathematics. In contrast strategic and conceptual images develop for mechanics throughout the years of the programme, implying that different forms of competence are being constructed by students between the two subjects. The paper argues that this difference is attributed to the perceived use-value of mechanics in the career of the engineer, compared to the exchange-value associated with mathematics. Questions are raised about the relevance of current definitions of competence given that some routine mathematical operations previously performed by engineers are now being replaced by technology, in the new world of work.

Keywords

Competence, Engineering, Mathematics, Concept Image, Knowledge Types

Introduction

One definition of engineering is that it is 'the application of science and mathematics to projects for the benefit of society' (Kirschenman and Brenner, 2010, 54). To benefit society in this way, it would be expected that the engineer is competent in both science and mathematics, to enable them to use their knowledge of both disciplines to solve problems. The notion that a professional is competent has been central to the development of the professions. As suggested by Eraut(1994) the assessment of the ability of members became important for professional bodies as they tried to distinguish themselves to a public who were not able to distinguish between someone who was 'capable' and someone who was not.

Society would therefore expect a professional engineer to display competence in their use of mathematics to solve problems in conjunction with their other subjects. A problem that is faced on engineering degree programmes is that they often have high attrition rates in mathematics (Chen 2013). This would suggest that students are not achieving a level of competence that is expected from them.

This paper will highlight how students, in an engineering programme, develop competence in two contrasting subjects, mathematics and mechanics. It will explore the concept of knowledge to examine how students create concept images in different subjects. A concept image is defined by Vinner(1983) as a mental image of a mathematical concept that is formed by a student over time by personal experience. Using a mixed methods approach this paper will consider whether different concept images are constructed in mathematics compared to other core subjects in engineering, and if this could affect their competence as engineers.

Competence in Mathematics

The profession of engineering is governed by the Engineering Council in the United Kingdom, which was formed in 1982 to create a society for the public benefit. One important role of the Council was to 'advance education ..... for the public benefit'(Engineering Council, 2011). This resulted in two developments, the publication of standards for registration, and the formation of professional engineering institutions to maintain standards in particular domains such as civil, mechanical and chemical engineering. The latest edition of the standards for registration is the UK Standard for Professional Engineering Competence (UK-SPEC) (Engineering Council, 2013), which defines the assessment of competence that is required for a professional engineer. The forward to UK-SPEC identifies how engineers solve problems using scientific knowledge and mathematics, which would imply that competence in mathematics is essential to the role of a professional engineer, and could be expected to be an outcome of their education.

The engineering subject benchmark statement for the United Kingdom identifies mathematics as one of three core subjects in engineering, noting that it is a 'language used to communicate parameters, model and optimise solutions' (Quality Assurance Agency (QAA) 2015, 8). The idea of it as a language may cause some students to consider it differently from other subjects in their programme, for example in a comparison of students taking degrees in mathematics and mechanical engineering, Bingolbali et al(2007) found that engineering students saw mathematics as a tool. They were seeking applications of concepts, which is different from mathematics students, who see the scientific understanding as more important. Treating the subject as a tool, to help them to reach a solution, without understanding the underlying concepts of how the tool works, may cause students to struggle with the topic, leading to what has been termed 'The Mathematics Problem' in engineering.

It is suggested by Firouzian et al(2012) that mathematics is a difficult subject, hence ‘The Mathematics Problem’, and that students struggle because of their perceptions of the terminology and concepts. This may be because there is a hierarchy structure in the learning of the topic, which Loch and McLoughlin(2012) suggest is built on throughout a programme, and a failure to understand a concept early on would limit the ability to grasp concepts later in their course.

Getting the right early understanding of key mathematical concepts is therefore important, and one of the problems in the early years in an engineering curricula is that mathematics is taught as a separate subject by mathematicians rather than engineers. As Sazhin(1998) identifies, this results in students not seeing its relevance. Flegg et al(2012) found this is especially true when it is not applied until later in the course, therefore as Harris et al(2014) argue, the material becomes decontextualised and students fail to make the connection between the topic and its application. This might suggest that students are not bridging between the subjects, which Cardella(2008) suggests can be addressed by innovative approaches to teaching which integrates the subjects. Without this integration, students fail to gain understanding of basic mathematical concepts at an early stage in their engineering education, and as Hennig et al(2014) note various attempts have been made to address these issues with bridging courses, mathematics support centres and curriculum changes to integrate the technical with the mathematical.

Another aspect of ‘The Mathematics Problem’ is that students tend to learn procedures rather than understanding concepts. It is pointed out by Engelbrecht et al(2009) that the method of teaching symbol manipulation tends to encourage learning of procedures. This is then reinforced through examinations which focus on procedures, so students tend to learn by rote to solve standard problems, but have no deeper understanding. They can then be caught out by unusual problems, or questions in a different format. Appleby et al(2000) identify the tendency to 'Drill and Kill' students, where they are required to repeatedly solve problems and take tests in areas where they have a weakness, until they reach the required level. This would encourage rote learning to pass assessment, rather than fostering the broader competence expected of the professional.

These problems will cause issues for the ability of students to become competent in their use and application of mathematics. To begin to understand the extent to which competence is affected it is important to understand how students acquire knowledge within the subject.

Types of Knowledge in Mathematics

It is argued in this paper that the type of knowledge is important in understanding what is meant by competence in mathematics. A common distinction is made between procedural and conceptual knowledge, attributed to Hiebert and Lefevre(1986) and developed by the likes of Star(2005), Baroody et al(2007) and Star and Stylianides(2013). Conceptual knowledge relates to the building of relationships in knowledge, where bridges can be built between facts in a network of knowledge. Whereas procedural knowledge is concerned with two aspects of knowledge, being familiar with symbols and their conventions, and the procedures for solving problems. Star(2005) suggested an extension to these two dimensions, by also identifying the quality of knowledge, producing a two by two matrix with four possible conditions. The quality variable was seen to provide a continuum between shallow or rote learning at one extreme to deep or flexible learning at the other.

When using mathematics, the competent engineer should understand not just the process of solving a problem, but when to use different techniques. This is the nature of competence that is suggested in the requirement for the engineer to be able to 'integrate their knowledge and understanding of mathematics, science .... to solve a substantial range of engineering problems, some of a complex nature' IMechE(2013). This implies the networked type of conceptual knowledge linking mathematics with science and other knowledge, together with a deeper procedural knowledge to be able to solve a range of complex problems, which would go beyond a mere superficial use of standard techniques.

There are, however, other forms of competence that might be derived from the requirements for the engineer. One example is the requirement to have 'the ability to apply learned knowledge and skills to perform operations intuitively'(IMechE2013, 27). The intuitive knowledge that is required here is comparable to the professional artistry that Schon refers to when considering the question of professional knowledge, what he describes as 'knowing-in-action'(1992, 9). This form of knowledge would lie outside of the earlier definitions of quality and type of knowledge, suggesting that there are broader aspects of knowledge that need to be included in any assessment of professional competence in mathematics.

An alternative theoretical model is provided by De Jong and Fergussion-Hessler(1996) who identify a classification of knowledge from a general learning perspective. They introduce the two variables of type and quality, but also provide a finer granularity along both scales. Their model has four types of knowledge, namely; situational, conceptual, procedural and strategic. Conceptual and procedural map onto the definitions of Hiebert and Lefevre(1986), the two new types relate to the domain and the approach to problem solving. These four elements are considered by De Jong and Fergussion-Hessler(1996) in terms of the quality of learning between deep and surface to identify eight types of knowledge that an engineer could be competent in.

These eight categories do not include the intuitive component of professional knowledge. This is what has been described by Savelsbergh et al(2002) as the experience that comes from considerable practice in problem-solving, which is an impediment to novices because of their inexperience.

Combining the definitions of De Jong and Fergussion-Hessler(1996) with the intuitive component of knowledge, creates ten forms of knowledge. This new combined model is proposed for the purposes of understanding competence in this paper, and has the following categories:

- Familiar situational knowledge - being able to use knowledge in a domain that is typical and routine.
- Unfamiliar situational knowledge - the ability to apply knowledge in a new context, to apply concepts in a different domain.
- Shallow conceptual knowledge - to know facts and principles as rote knowledge, not developing connections between related ideas.
- Deep conceptual knowledge - an understanding of the relationships and links between concepts in a domain, and to build links into other related areas of knowledge.
- Shallow procedural knowledge - to be able to perform standard routines to solve a problem.
- Deep procedural knowledge - understanding the use of procedures and the reasons for the selection of the appropriate methods.
- Basic strategic knowledge - to be able to identify values and variables required in problem solving when these are clearly defined.
- Advanced strategic knowledge - to plan a method to resolve complex problems, defining information required and stages in the process

- Pictorial modality - a preference for using graphical knowledge compared to textual, when defining and resolving problems
- Heuristic knowledge - intuitive knowledge to solve a problem, search for solutions, and recognise the correctness of a solution.

These ten types of knowledge will be used to assess how students build their understanding of a concept, which is defined by Meyer and Land(2005) as a ‘building block’ within a subject, that has to be understood but does not lead to a different view of the subject. In contrast a threshold concept is a pivotal element of a curriculum which presents a significant challenge to students, but once they are understood, this provides a significant difference to understanding in a subject (Rountree, Robins et al. 2013).

It is necessary to distinguish between the meaning of a concept, and how someone develops an understanding of it. Vinner(1983) distinguishes between a concept definition, and a concept image. A concept definition is the accurate and formal meaning associated with a function, symbol or process. The concept image is the mental interpretation of the definition by the individual. The definition will be a constant that is true for all, the image will be unique, that is derived from experience, examples and interpretations by the individual.

In a study of how students understand functions in mathematics, Tall and Bakar(1992) suggest that complex mathematical concepts such as functions can take years to develop, as the concept matures in their thinking. They suggest that an individual develops a prototype example of a concept. When resolving a problem, the student will try to match against this prototype. The construction of the correct prototype is therefore one element of being able to competently resolve a problem. It is the way the student develops these concepts in the ten types of knowledge that this paper will attempt to establish.

This paper argues that students approach the development of concept images in mathematics in a different manner to other engineering topics. This is because students consider mathematics as a ‘tool’ to be applied, which could make it different from other knowledge that they will need when practicing as an engineer. This would imply that the value attached to it would be different than an engineering subject, which a student would see as important in their future career. Harris et al(2014) identify that students fail to see the relevance of the mathematics they are studying in early years, arguing that this causes an exchange-value attitude to the subject, where it becomes only of benefit to acquiring grades and qualifications, rather than having a use-value, to be able to perform competently to solve a problem. Gibbins(1976) suggests that the use-value and exchange-value are independent of each other, therefore a student who fails to see the relevance of a subject, might approach the learning of it purely to gain the grade and progress on their programme, encouraging a rote type procedural learning. In contrast, a subject that has an apparent direct relevance to their future career would be approached in a different manner, building a conceptual understanding that would be employed in complex problem-solving.

There is some evidence that students may view the study of mathematics differently. In interviews with students Solomon(2007) found that it was seen as a set of rules that had to be followed, there was no construction of knowledge. This might suggest that a student who fails to see the need to construct an understanding, will just learn the subject in a shallow procedural manner in order to obtain the exchange-value of passing a module. They will miss the conceptual understanding that would come from a use-value interpretation of the subject. The motive to study a subject will be determined by the student's perception of the value of the material to their future aspirations. The long-term value of an engineering topic may be considered greater than the exchange-value of the less relevant subject.

The objective of this study is to examine the different conceptual images developed by students in the two areas of mathematics and mechanics. To engineering students, mechanics would be considered an essential subject which would possess a high use-value, which is taught with many examples that are drawn from the discipline. If a student is approaching the study of mathematics purely for the exchange-value, then the types of knowledge associated with mathematical concepts are expected to be different from a topic like mechanics where the use-value would be more obvious.

This is assuming a constructivist viewpoint from the student and their learning process, suggesting that an individual student is developing a point of view towards their subject as they progress through their studies, implying an active approach to learning and the sense-making process. To make sense of the material they are taught, they will construct their own concept image of a topic, it is the different forms of constructed image that this research is seeking to uncover.

Research Design

Purpose of Study

This study investigates how students construct concept images of different topics in an engineering curriculum, by comparing cross-sections of data from cohorts of students at various stages in a programme. This is a cross-section rather than a longitudinal study, because it will not focus on how understanding changes over time within the individual, rather it will consider the general trends in understanding between the stages of a programme. It is recognised that in a cross-sectional study it will not be possible to draw inferences about how concept images change in individuals over time.

The study will seek to answer the following questions:

- Is the type of concept image created by students different between mathematics and mechanics?
- Does the nature of the concept image change within a topic during their education?
- Does the quality of understanding that occurs in mathematics alter during the degree programme compared to mechanics?

Theoretical Framework

This is an exploratory study to discover the concept images that students develop for threshold concepts in engineering. An empirical approach is employed which adopts a constructivist view of how students develop their individual concept images of mathematical and mechanics knowledge. As Vinner(1983) identifies, although the concept definition in a subject is constant, the image that is created by the individual will not necessarily match with this theory. As understanding grows, then the image will be developed or changed to come into line with the theory. To measure the concept images of students and how these develop and change, a quantitative approach is adopted, to allow the assessment of population-wide characteristics in the development of knowledge by students. It is recognised that this would not allow causal relationships to be established, but the nature of the research question is to measure the nature of the understanding and not to establish any links to other variables. This is supported in a mixed methods approach with interviews to explore the lived experience of individuals when studying the two subjects.

Research Methodology

A mixed methods approach was adopted in this research, using a quantitative multiple-choice questionnaire and dialogic interviews. This approach was adopted because it was considered that the initial phase of quantitative data would allow the relative emphasis on each type of learning to be measured, but not to explore the processes that individual students were adopting. It is suggested by Creswell and Clarke(2007) that a mixed methods approach is one way of trying to gain a further explanation of a research problem.

The use of multiple-choice tests for measuring knowledge acquisition and cognitive skills has been identified by Royer et al(1993) as one method to assess if a learner has the knowledge to function in a domain, noting the benefit when trying to determine if specific knowledge has been accumulated. In this study specific forms of knowledge related to identified concepts are being assessed, which would lend itself to a multiple-choice approach. The use of this type of instrument has been identified by Hofer(2004) as a common tool for assessing conceptions of knowledge and learning when working with broader issues of epistemic beliefs. Royer et al(1993) do identify that this type of analysis would not determine mastery, as it would be understood in the context of competency. It is however assumed that the design of questions to measure different qualities of knowledge, rather than just factual recall, could allow the broader application of a concept to be assessed.

The instrument used in this study has been designed around eight threshold concepts, four for each of mechanics and mathematics. The threshold concepts have been identified from the study of engineering concepts that was conducted by Male(2012), and were compared to the first year curriculum for engineering and mathematics students at a Scottish university. Staff responsible for the delivery of the relevant modules were then engaged in the study and asked to determine how they would assess the ten knowledge types for each concept. This resulted in eight summary concept statements, and ten applications of each concept in the form of statements that relate to that concept.

The statements against each of the concepts were all possible correct answers. It was considered important not to try and confuse participants with incorrect answers, because they may see the survey as more of a test than a study of their understanding. Consequently each of the ten statements were possible answers, seeking to assess the concept image constructed by the participant. As the concept could be viewed in many different ways, as a process, a concept, image or strategy, the survey did not ask them to choose between different statements. Instead they were asked to indicate how strongly they related to each statement, allowing the instrument to compare the understanding between each type of knowledge. It is assumed a learner would commence their understanding with relatively superficial understanding of a concept or procedure, but as the image developed this could move to a deeper appreciation.

An example of two statements from the survey are shown in Figure 1, which shows two different *Pictorial Modality* statements, one from each of mathematics and mechanics. The concept, such as a bending moment, is outlined at the start of each statement, followed by ten possible concept images for each of the types of knowledge being assessed. To establish the extent of association with a statement students were asked to rate their understanding using a seven point Likert-type scale. A seven point scale was recommended by Schwarz et al(1991) who suggest that this produces the most reliable results, improving the ability of respondents to discriminate between scale values.[1]

In a mixed methods approach five engineering students were also invited to participate in an interview to explore their experiences of learning during their degree. They engaged in a dialogic interview which, as suggested by Knight and Murray(1999), could be appropriate when trying to elicit constructed meanings from individuals. Excerpts from these interviews will be used in the analysis of the quantitative data to explore how the participants developed meaning during their programmes. Where excerpts from interviews are used in the text, the identities of students are denoted as ‘Student-A’.

The Research Context

The research was completed in a university in Scotland, where undergraduate degree courses in engineering and mathematics were taught. The participants in the survey were students in their early and final years of their degrees in civil and mechanical engineering. Participants were enrolled to the survey at the end of lectures, where a brief introduction to the project was given, and volunteers asked to complete the survey instrument. The participants were provided with three documents, including an explanation of the study, which outlined the purpose of the questionnaire and how data would be used. This informed students that the survey was anonymous and would not be associated with their performance on the course. It was also stressed verbally that the work was anonymous, because it was seeking to measure their understanding, and there was a concern that students might be worried about the effect of giving a wrong answer. Participants were also provided with the questionnaire as a booklet to complete, and a consent form, to comply with the ethical requirements of the institution. After completing the survey participants were invited to engage in interviews to explore further their responses to the survey, and five individuals responded to this request.

Results and Discussion

A total of 58 valid questionnaires were completed, 26 from students in their final year and 32 from early years. The average Likert scores were calculated for the four concepts in both mathematics and mechanics, with a single average determined for each subject for all ten of the types of knowledge being tested. This average was tested for normality using the Shapiro-Wilk test which showed that the null hypothesis that the data is normally distributed could not be rejected. It was therefore decided to compare the populations using a student's t-test to determine if they were significantly different. To test against the research questions the data was compared for the differences in scores between mathematics and mechanics in each of the ten types of knowledge, and between the years on the programme. The results are summarised in Table 1.

When considering the question '*is the type of concept image created by students different between mathematics and mechanics?*' the statistical differences in final years would indicate that students have created very different images for the two subjects, with strong statistical differences evident in nine of the ten knowledge types, the exception being *Familiar Situation*. It is interesting to note that this is the only category where students are showing the least difference in their concept image in mathematics when compared to mechanics. A *Familiar Situation* would be closest to a rote style of learning, which would be expected if students were adopting a more exchange-value approach to a subject. This style of learning is also supported by the higher scores in mathematics attributed to the *Shallow Procedural* knowledge, which is the highest types that students could relate to in mathematics for both student cohorts, demonstrating that they are learning the *how* of solving a problem which might be associated with seeing a test as something to be passed without any deeper understanding of the *why* in the process. In interviews this was supported by the Student-D, who when asked about their studying of mathematics said '*You need to keep practicing it. If you leave it too long you lose the skill.*' This shows that this particular student was more interested in rehearsing the process involved in solving a known problem, a *shallow procedural* concept image of the knowledge which is reinforced by practising with *familiar situation* problems. The mathematics is being used as a tool which has to be exercised regularly, otherwise the ability will be lost. This is supporting an exchange-value attitude to the topic, gaining the knowledge just to progress through the assessment, rather than constructing deeper concept images. This is reinforced by Student-E who said that you learnt the mathematics to '*prepare for the examination but once the examination is over it is forgotten.*' This student could see no use-value in the material, to need to learn it for any reason other than the assessment.

A strong association with the concept images for using procedures in *familiar situations* in mathematics for both student cohorts can be contrasted with higher scores in mechanics for *conceptual* and *strategic* knowledge for the final year students. These types of knowledge would be associated with a use-value perception, which is supported by Student-B who stated that studying mechanics was different because *'you think it is what you are going to be involved with.'* This can be contrasted with Student-D who when studying mathematics stated that '*I never felt that I needed it, so maybe I did not try as much as I would have done*' which shows that students are making strategic choices about how they study subjects based on the value they perceive.

The analysis of the ten knowledge types and dialogic comments from students would tend to support the idea that within a programme students do create different types of concept images for subjects based on how they perceive their relevance. Where there is a perceived exchange-value, learning is concentrated on the process to solve known problems likely to be encountered in assessment. In contrast, there is a conceptual and strategic conceptual image formed when it is thought the subject has a use-value.

For the professional engineer, the expectation that an engineer is competent in not only the science in subjects like mechanics, but also the language of mathematics, would suggest a similar type of concept image should be constructed in both areas. When solving 'a substantial range of engineering problems, some of a complex nature' IMechE(2013) an engineer would need competence that is going beyond the *shallow procedural* which are currently constructed by students in mathematics. Complex problems such as this would require ability in *unfamiliar situations* which the results suggest that the graduating student is significantly more able to relate to in mechanics than in mathematics.

The second research question asked '*Does the nature of the concept image change within a topic during their education?*' and is concerned with changes in the concept images between cohorts. Table 2 compares the mean scores of students ability to relate to each of the concept images within both subjects. This table indicates that there are significant differences between the two subjects as students progress through their course. In mathematics there is only one significant difference in their average scores between the two groups. It is interesting to observe that although only one was significant, in nine of the ten knowledge types the average score drops from the early to the final years. This shows that students in the early years are relating better to the majority of the concepts in mathematics than in the final years. When asked about this, one of the final year students suggested that '*You remember it more if you are constantly doing something. It does drop off if you are not using it*'(student E). He is suggesting that learning in mathematics decreased because their practice had stopped in these subjects two years earlier in their course. The difference to him was that the mathematics was not being refreshed through regular use and assessment.

This contrasts with the significant differences in the averages for mechanics, where all the scores are higher indicating they related more to these concept images in all the knowledge types. Seven of these scores are significant at a 95% level or more. Mechanics is showing a completely different level of development through the programme, with stronger concept images being established as the course progresses. This result might be expected, because a topic like mechanics would be one that is built on throughout all the years of the programme. Student A commented that the difference was that *'you are doing it all the time rather than just for the maths'* which for him was reinforcing the knowledge and understanding of the subject.

The contrast in development in the two subjects could indicate that the level of competence in the graduate engineer is significantly different between the two subjects. This would raise the issue of whether the weaker appreciation of mathematics would affect their ability to practice. In the interviews all the participants were asked about how they thought they would make use of their knowledge of mathematics once they graduated. One of the participants, who had experience in industry during summer placements, noted that

> ‘You would still need some maths but a lot of the programmes do it for you now. You still need that understanding is what the computer saying right. You have to have some sort of responsibility.’ (Student-E)

Here he is suggesting that the procedural mathematics taught in the programme was not a skill that he envisaged the practising engineer using much, with this being replaced by computer software. Instead, he recognises two important qualities that a competent engineer should display, one is the notion of responsibility, which returns us to the opening quote from Kirschenman and Brenner(2010) which raised the issue of engineers working to the benefit of society. The engineer has a duty to ensure that their solutions are technically correct, which is a part of the broader concept of competence. The second is the ability to recognise the correctness of the solution that is generated by the software, which relates to the intuitive knowledge that is an element of the concept image in the last of the ten knowledge types. In Table 2 the *heuristic* knowledge in mechanics has shown notable development during the programme, suggesting that although the procedural mathematics required to arrive at a solution may not be evolving, the experience to recognise the solution has. With an increasing reliance on software to perform the routine processes, this competence in recognising and understanding the correctness of the output from software could be more significant for the engineer in the new world of work.

The ability to interpret and recognise potential errors in the solution provided by software might be a new competence that is becoming more important to professionals as they increasingly rely on computers to model complex problems. It might indicate that the current definitions of competence for engineers need to change to reflect developments in technology, with an increasing emphasis on the competence needed to use software and the ability to interpret the results that are produced.

The third research question focuses on the specific development of the quality of learning, using the second dimension of knowledge that was suggested by Star(2005). '*Does the quality of understanding that occurs in mathematics alter during the degree programme?'* The quality of learning is assessed against four types of learning between the two extremes of shallow and deep, and the average scores shown in Table 3.

In mathematics, the relationship between the two aspects of quality does not change during the course, with shallow and deep learning both displaying a significant difference comparing the mean differences in three of the four knowledge types. In three of these it is the shallow quality that students relate most to. This reinforces the arguments made earlier that the enhancement in mathematical skills are not being seen as students' progress, with students concentrating on routine processes and not building on the knowledge they acquire.

It is interesting to compare these results with those in mechanics, because Table 2 had shown that students were exhibiting a progression in their concept images in this subject as they proceed through their programme. Table 3 would indicate that this development in mechanics still favours the shallow quality of learning, with three of the four categories being significantly different for final year students. A difference might be expected in the earlier years when they are familiarising themselves with the new subject domain. Student-A noted this difficulty in learning elements of mechanics at the start of the course because '*I did not know how to draw a bending moment or shear force diagram*', so the processes of solving basic elements of mechanics were alien at this stage, and learning the basic processes were important.

Comparing the average scores for mechanics in Tables 2 & 3, they show that while the ability to relate to the concept images has improved over time, the difference is not significant when comparing the shallow qualities to the deeper appreciation. This might be explained by Student-A who said that they were '*continually using that throughout any subsequent structural modules*' so the processes that he found difficult at first were being reinforced throughout the programme. This returns to the aspect of continual practice element of learning, when considering the learning of processes by repeating exercises. The basic processes of calculating values for bending moments and shear forces in mechanics were a drill that became second nature as they became well practiced in this, which could be equated to the process of learning to change gear in a car, the more times it is done the less the driver has to consciously think about the process of depressing the clutch and moving the gear stick. In the context of competence, this could be considered a strength, because they have mastered a procedure and are confident in their use. It is this familiarity and confidence that could be enhancing the heuristic knowledge that has matured during their time on the programme.

The results indicate that within this one institution, students are developing different types of concept images in the two subjects, which will have implications for their competence as engineers. The need to construct the same type of images in the two subjects is questioned because the need for competence in mathematics is changing with developments in computing removing the need for engineers to manually complete complex calculations. This would suggest that the teaching of mathematics may need to change for engineering students, away from the mechanical process of performing the calculation, towards an understanding of the results and the ability to appreciate the output generated by computer software.

Conclusions

Society and professional bodies would expect an engineer to be competent in the areas of knowledge that are essential to their practice. Two areas of competence that would be included in this are science and mathematics, but students are displaying different types of knowledge in the two subjects, which will have an influence on their levels of competence in solving problems. Their approach to mechanics displays a development in knowledge and concept images that are associated with deeper qualities of knowledge, derived from a perception of use-value of this topic. In contrast mathematics is seen as having more of an exchange-value, resulting in a rote learning style that is suitable for solving familiar problems. Competence might be compromised when faced with more complex problems which require mathematical techniques that are not understood by the graduate. Their attitudes to mathematics might be partly attributed to changes in the use of information technology which has shifted the importance of some skills in mathematics away from manual calculations, and towards computers performing routine tasks. In this context the heuristic knowledge of the engineer becomes more important, as the engineer is required to interpret the results, a type of knowledge that is not currently articulated in the standards expected from engineers. This heuristic knowledge is being developed during the mechanics syllabus of the degree programme studied, through a familiarity with problem solving as the course progresses. This would suggest that the notions of competence for the engineer are shifting with changes in the workplace, with professional practice for the engineer moving away from the ability to perform extensive manual calculations towards the manipulation of computers and interpretation of results.

There are implications for the teaching of mathematics, and the bridging between subject areas within the curriculum. Cardella(2008) has identified a number of approaches to teaching of mathematics in the engineering curriculum to try and integrate the subject. This integration might help to remove the exchange-value perception of mathematics which appears to be contributing to the type of concept images that are formed by students.

This work was conceived as an exploratory study and has been completed in a single institution, which could be developed in further studies. With only a single establishment included in the data collection, the results are conditioned by the culture and practices of the individual programmes studied. The integration of the mathematics curriculum, and the emphasis given to the application and end use of procedures could influence student perceptions of the topic. A broader study across a number of institutions would allow for an assessment of the generality of the findings. The work has also been on a cross-section of students which has not allowed for individuals to be tracked through a programme to determine how they develop in their understanding of a topic. A longitudinal study tracking individuals and how their personal concept images change could allow more insight into individual development. An important question of the use of mathematics by professional engineers is raised, and whether this is being replaced by technology. The use of technology to replace some the routine tasks completed by the engineer, and the competence to use software and interpret results, would be another area for future study.

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Notes

[1] Full copies of the questionnaire can be obtained from the author on request.

	Final Years (26)			Early Years (32)		
	Mechanics	**Mathematics**	**Sig. of Mean Diff.[a]**	**Mechanics**	**Mathematics**	**Sig. of Mean Diff.[a]**
Familiar Situation	5.56	5.21	*	4.90	5.39	***
Unfamiliar Situation	5.42	4.58	***	5.05	4.78	
Shallow Conceptual	6.14	5.22	***	5.33	5.39	
Deep Conceptual	5.16	3.86	***	4.59	4.31	
Shallow Procedural	6.14	5.72	**	4.83	5.73	***
Deep Procedural	4.82	3.95	***	4.45	4.59	
Basic Strategic	6.13	5.29	***	5.55	5.11	**
Deep Strategic	5.44	5.00	**	4.81	5.30	**
Pictorial	6.63	4.57	***	5.79	4.59	***
Heuristic	6.08	4.79	***	4.88	4.89	

[a]The mean for each group was compared using a t-test and the significance of that test is reported. Where there is no significant difference cells are left blank.

Key to levels of significance: * $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table 1 Mean scores and t-test results between final year and early year cohorts

	Mathematics (58)			Mechanics (58)		
	Final Years	**Early Years**	**Sig. of Mean Diff.**[a]	**Final Years**	**Early Years**	**Sig. of Mean Diff.**[a]
Familiar Situation	5.21	5.39		5.56	4.90	***
Unfamiliar Situation	4.58	4.78		5.42	5.05	
Shallow Conceptual	5.22	5.39		6.14	5.33	***
Deep Conceptual	3.86	4.31	*	5.16	4.59	*
Shallow Procedural	5.72	5.73		6.14	4.83	***
Deep Procedural	3.95	4.59		4.82	4.45	
Basic Strategic	5.29	5.11		6.13	5.55	***
Deep Strategic	5.00	5.30		5.44	4.81	**
Pictorial	4.57	4.59		6.63	5.79	***
Heuristic	4.79	4.89		6.08	4.88	***

[a]The mean for each group was compared using a t-test and the significance of that test is reported. Where there is no significant difference cells are left blank.

Key to levels of significance: * $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table 2 Mean scores and t-test results between mathematics and mechanics

Final Years (26)	Mathematics			Mechanics		
	Shallow	**Deep**	**Sig. of Mean Diff.[a]**	**Shallow**	**Deep**	**Sig. of Mean Diff.[a]**
Situation	5.21	4.58	***	5.56	5.42	
Conceptual	5.22	3.86	***	6.14	5.16	***
Procedural	5.72	3.95	***	6.14	4.82	***
Strategic	5.29	5.00	*	6.13	5.44	**
Early Years (32)						
Situation	5.39	4.78	***	4.90	5.05	
Conceptual	5.39	4.31	***	5.33	4.59	***
Procedural	5.73	4.59	***	4.83	4.45	***
Strategic	5.11	5.30		5.55	4.81	***

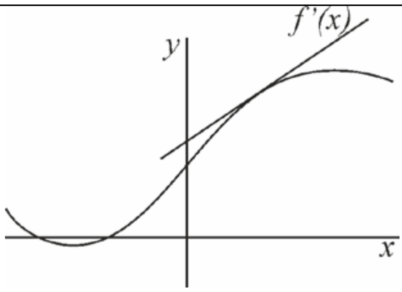
[a]The mean for each group was compared using a t-test and the significance of that test is reported. Where there is no significant difference cells are left blank.

Key to levels of significance: * $p < 0.10$
** $p < 0.05$
*** $p < 0.01$

Table 3 Mean scores and t-test results between deep and shallow knowledge types

$$\frac{dy}{dx} = f'(x)$$

Each of the following options can all be associated with the above concept. Please rate each of them according to how closely, in your own understanding, they are associated with it.

	Nothing like it						Exactly right
	1	2	3	4	5	6	7

A bending moment is a behaviour that causes an object to curve on its longitudinal axis.

Each of the following options can all be associated with this concept. Please score each of them according to how closely you associate with it in your own understanding.

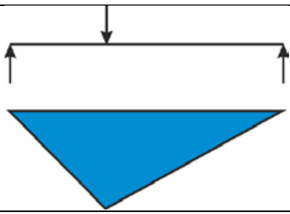
	Nothing like it						Exactly right
	1	2	3	4	5	6	7

Figure 1 Example of two Pictorial Modality statements from the questionnaire