Multiproduct firms and market structure: an explorative application to the product life cycle

Allanson, Paul; Montagna, Catia

Publication date:
1999

University of Dundee

Dundee Discussion Papers in Economics

Multiproduct Firms and Market Structure: An Explorative Application to the Product Life Cycle

Paul Allanson

and

Catia Montagna

Department of Economic Studies,
University of Dundee,
Dundee.
DD1 4HN

Working Paper
No. 101
July 1999
ISSN:1473-236X

Multiproduct Firms and Market Structure:

An Explorative Application to the Product Life Cycle

Paul Allanson **and** **Catia Montagna***

Department of Economic Studies

University of Dundee

Abstract

We extend the standard Dixit-Stiglitz model of imperfect competition to allow for multiproduct firms. We fully endogenise market structure by determining both the number of varieties per firm and the number of firms in the industry. A crucial feature of the model is that firms internalise the effects of both intra- and inter-firm competition in making their product range and pricing decisions. The model is used to explore the proposition that shakeout in some industries may result from a shift from a fragmented market structure with many single-product firms to a concentrated equilibrium with a few large firms each offering many products.

Keywords Imperfect competition, multiproduct firms, shakeout

JEL L11, L13

We would like to thank Martin Chalkley, Hassan Molana, and three anonymous referees for helpful comments and suggestions. The usual disclaimer applies.

* **Corresponding author:** Department of Economic Studies, University of Dundee, Dundee DD1 4HN, UK. Tel. +44-1382-344845, e-mail: c.montagna@dundee.ac.uk

1. INTRODUCTION

Markets for many manufactured goods are in differentiated products rather than in homogeneous commodities, and firms producing in these industries typically offer not one but a range of varieties of the same product to cater to the diversity of consumer tastes. Yet, product differentiation and multiproduct firms have mainly been considered separately in theoretical studies. Most of the work on product differentiation has dealt with single product firms while research on multiproduct firms has largely followed Baumol et al. (1982) in focusing on the role of cost factors in generating outcomes in which firms produce several products. As a result, the literature on multiproduct firms is particularly poorly developed in relation to the role of demand factors in the determination of equilibrium in differentiated goods markets.

In this paper, we develop a model that extends the standard Dixit-Stiglitz framework of product differentiation (Dixit and Stiglitz, 1977) to allow for multiproduct firms. On the demand side, we represent preferences for the differentiated product by a nested CES utility function, with the degree of substitutability between the varieties produced by any one firm being higher than that between varieties produced by different firms. On the supply side, we allow for both economies of scope and scale through the introduction of both firm-level and variety-level fixed costs. Firms choose the size of their product range and compete in price, with the number of firms determined by the free entry equilibrium.

At a theoretical level, the main implication of the extension of the standard Dixit-Stiglitz framework to multiproduct firms is to break the identity between the mass of varieties and the mass of firms. Two main (partially related) issues then arise. The first concerns the determination of market structure and the second concerns the optimal behaviour of firms. Both of these issues are dealt with in this paper.

Market structures with multiproduct firms may range from *fragmented equilibria* (in which a large number of firms offer either one or a small range of products) to *concentrated equilibria* (in which either one or a small number of firms each offer many products). Shaked and Sutton (1990) argue that the key issue that the literature on multiproduct firms in differentiated markets ought to address is what basic features of consumer preferences lead to the appearance of the various possible equilibria. Yet, with the notable exceptions of Anderson and de Palma (1992), and Ottaviano and Thisse (1999) this literature has not fully endogenised market structure by considering the joint determination of the equilibrium numbers of firms, total product varieties and varieties per firm on offer. Some studies focus on the choice of the size of product range by individual firms in differentiated markets while taking the number of firms as given (see, for example, Schmalansee, 1978, Raubitschek, 1987, and Champsaur and Rochet, 1989)[1]. Other work examines the emergence of multiproduct firms in the context of inter-related markets while holding the total number of distinguishable product varieties fixed (see, for instance, Wolinsky, 1986, Shaked and Sutton, 1990, and Hanly and Cheung, 1998). Finally, there are a number of papers which deal with product line selection by multiproduct firms while treating both the number of firms and the number of products per firm as exogenous (see, especially, Brander and Eaton, 1984). Clearly, any analysis which takes either the number of firms, total product varieties or varieties per firm as fixed can at best provide only a partial explanation of the emergence of the equilibrium market structure. A first crucial feature of our analysis is that it endogenously determines both the equilibrium number of firms and varieties.

With respect to the optimal behaviour of firms, two aspects deserve attention. The first is that of intra-firm competition (known as '*cannibalisation*'): when a firm reduces its price on one of its varieties this will decrease demand for all the other substitute varieties the firm produces. Each firm therefore needs to internalise competition within its product line. Second, there is an issue of inter-firm competition: even assuming that the number of firms in the market is large, the multidimensionality of the firm's product range implies that each firm is likely to be a 'large actor'. This clearly casts doubt on the plausibility of the atomistic assumption of the standard monopolistic competition model whereby firms behave in a non-strategic manner. A second crucial feature of our model is to allow both for (i) the full internalisation of the cannibalisation effect by firms, and (ii) the strategic interaction between firms which therefore behave like oligopolists and not as monopolistic competitors as in the standard Dixit-Stiglitz framework (Beath and Katsoulacos, 1991). To our knowledge, this is the first paper to do so within a CES framework of multiproduct firm competition. Raubitschek (1987), who specifies a single-level CES utility function on the assumption that all products compete as equal substitutes, makes no allowance for price co-ordination within firms or strategic interaction between firms. Anderson and de Palma (1992) and Ottaviano and Thisse (1999) allow for both intra-firm decision coordination and for inter-firm competition, but within a nested multinomial logit and a quadratic utility model respectively.

Our model is similar in spirit to the nested multinomial logit model of Anderson and de Palma (1992) but the adoption of the representative consumer framework on the demand side facilitates the interpretation of the analytical results. In particular, paralleling Shaked and Sutton (1990), we identify a *competition* effect, which is given by the Lerner index of market power, and an *expansion* effect, which measures the responsiveness of the variable profits of a firm to a change in the size of its product range, as determinants of the market equilibria. The resultant interpretation of the equilibrium outcomes is particularly simple and intuitive. Moreover, the competition and expansion effects have the further appeal that they are potentially observable market characteristics that could be employed to generate testable predictions about the relationship between market size and market structure.

The development of a multiproduct firm version of the standard representative consumer framework of product differentiation so as to capture an important feature of real world industries, is particularly relevant given the widespread popularity of the Dixit-Stiglitz model in many areas of economics. Thus, the model has a number of potentially interesting applications both within and outside industrial organisation, in areas such as trade theory and macroeconomics. We use it here to explore the proposition that the shakeout phenomenon, in which the number of firms in an industry approaching maturity falls despite continuing growth in the market, may result in some cases from a change in the *scope* as well as the *scale* of firms over the course of the product life cycle.

This analysis complements the standard explanation of shakeout, which describes it as the outcome of a period of scale-intensive process innovations initiated by the emergence of a dominant design (Utterback and Suarez, 1993), by suggesting a richer menu of possible mechanisms. On the supply side, process innovations may generate economies of scope, as well as of scale, promoting the emergence of large, multiproduct firms (Chandler, 1990). On the demand side, product standardisation would of itself be expected to compress price-cost margins leading to a more competitive market capable of supporting fewer firms, and may also favour multiproduct firms to the extent that changes in the constellation of product varieties on offer increase the relative contribution of an additional variety to the total variable profits of a firm.

The structure of the paper is as follows. In the following section we set up our model of imperfect competition with multiproduct firms and in section 3 we derive the market outcome in the symmetric, free entry equilibrium. In Section 4 we then use the model to consider the theoretical possibility that the emergence of multiproduct firms is a contributory factor in the shakeout phenomenon. Specifically, we conjecture that shakeout may in part reflect a shift in industry structure from a larger number of firms supplying only one or a few products, to a smaller number of firms each offering many products. Section 5 concludes the paper.

2. THE MODEL

We consider an imperfectly competitive industry with multiproduct firms producing a horizontally differentiated good to cater to the diversity of consumer tastes. The varieties of the good are grouped into nests with the degree of substitutability between varieties within nests being higher than that between nests. The analysis is developed within a partial equilibrium framework.

2.1. Consumers

On the demand side of the model, we adopt the representative consumer framework as in Dixit and Stiglitz (1977)[2]. The behaviour of the representative consumer may be regarded as the outcome of a three-stage utility maximisation procedure[3]. In the first stage, the consumer optimally allocates expenditure between the differentiated good and a single outside good. In the second and third stage consumption decisions are made respectively over nests and over the varieties within each nest. In both of these stages, preferences – which reward product diversity – are described by CES utility functions. This is in essence a *nested* version of the CES model[4]. In common with the nested logit model (Ben-Akiva, 1973; Anderson and de Palma, 1992; Anderson *et al.*, 1992), it allows for the possibility of localised competition since the elasticity of substitution between varieties belonging to the same nest may differ from that between nests[5]. The nested CES therefore provides a richer characterisation of the nature of competition than the standard Dixit-Stiglitz formulation.

We assume for the sake of simplicity that preferences in the first stage are described by a Cobb-Douglas utility function $U = z^{\eta} x^{1-\eta}$, $0 < \eta < 1$, where z is the quantity of an outside composite good used as the numeraire and x is the aggregate or industry quantity index of the differentiated good (to be defined below). The total expenditure on the differentiated good $y = (1-\eta)I$ is simply given in this first stage as a constant share of the exogenously determined consumer income I.

The second stage utility function is given by the industry-level quantity index of the differentiated good and is defined as follows:

(1) $$x = \left(\int_{i=0}^{n} x_i^{\frac{\alpha-1}{\alpha}} di \right)^{\frac{\alpha}{\alpha-1}}$$

where $\alpha > 1$ is the elasticity of substitution between nests and n is the mass of available nests [6]; x_i, the quantity index of the varieties in a typical nest $i \in [0, n]$, corresponds to the third stage sub-utility function and is given by:

(2) $$x_i = \left(\int_{k=0}^{m_i} x_{ik}^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution between varieties within a nest and x_{ik} is consumption of a typical variety $k \in [0, m_i]$ from nest i. We further make the natural assumption that $\sigma > \alpha$: that is, substitutability between varieties is higher within nests than between nests. When $\sigma = \alpha$, all varieties are equally substitutable and nests cannot therefore be distinguished: it can easily be verified that (1)-(2) reduces to a single-stage CES function with a total mass of varieties $v=nm$.

The price indexes dual to the utility functions in (1) and (2) are respectively given by:

(3) $$p = \left(\int_{i=0}^{n} p_i^{1-\alpha} di \right)^{\frac{1}{1-\alpha}}$$

and

(4) $$p_i = \left(\int_{k=0}^{m_i} p_{ik}^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}},$$

where p is the industry price index, p_i is the price index of the varieties within the typical nest i, and p_{ik} is the price of a typical variety k in nest i. As a reflection of love of variety, the price indexes in (3) and (4) are decreasing in n and m_i respectively.

Thus, in the second stage of utility maximisation, the representative consumer maximises (1) subject to the budget constraint $y = px = \int_{i=0}^{n} p_i x_i di$ to obtain the following system of demand functions:

(5) $$x_i = \frac{y}{p}\left(\frac{p_i}{p}\right)^{-\alpha}$$

which give the 'aggregate' demand for each nest.

In the third stage, the representative consumer will maximise the sub-utility function in (2) subject to the budget constraint $p_i x_i = \int_{k=0}^{m_i} p_{ik} x_{ik} dk$. The resulting demand function for each variety *k* in nest *i* will be given by

(6) $$x_{ik} = \frac{y}{p}\left(\frac{p_i}{p}\right)^{-\alpha}\left(\frac{p_{ik}}{p_i}\right)^{-\sigma}$$

Thus, the demand for the individual variety will depend negatively on its price and positively on both the nest-level and industry-level price indexes.

2.2. Producers

In the literature, two main cases of multiproduct firms with horizontal product differentiation have been considered (see Brander and Eaton, 1984; Anderson and de Palma, 1992). In the first, each nest $i \in [0, n]$ is taken to correspond to a firm, with the typical firm *i* producing a mass m_i of varieties of the good. This is normally referred to as the '*market segmentation*' case. In the alternative case, referred to as '*market interlacing*', each nest $i \in [0, n]$ is assumed to consist of varieties produced by different firms with the typical nest *i* occupied by a mass m_i of firms.

Clearly, the choice between these two alternative industry configurations will be dictated by the definition and characteristics of the product market under consideration[7]. In this paper we formally consider only market segmentation[8] in which the products of a firm are perceived by consumers to be closer substitutes to each other than to those of other firms. Market segmentation is characteristic of those industries in which the brand name or 'label' is the primary locus of differentiation with other specific product attributes being of secondary importance[9]. For example, Levi Strauss & Co. and other leading suppliers of jeans are able to command significant price premia across their entire ranges in spite of independent test evidence that the quality of their products is not superior to that of other manufacturers (Consumers' Association, 1993; Asda, 2002). Brander and Eaton (1984) cite the restaurant trade as another plausible example of a segmented market in that customers will often first choose which restaurant to patronise and only then select specific items from the menu. In general, firms can seek to create and sustain segmented market structures by pursuing differentiation strategies based on advertising, brand image, product design, styling, distribution channels, delivery terms, credit facilities, service arrangements and other dimensions of the total offering to customers (see Levitt, 1980).

We therefore specify that each nest corresponds to a firm. All the varieties in a nest are produced by the same firm and there is a one-to-one correspondence between the mass of nests and the mass of firms in the industry. Firms employ a globally increasing returns to scale and scope technology. In addition to a fixed production cost per variety, which is standard in the monopolistic competition literature, we assume the existence of a firm-level fixed cost, which must be paid regardless of the size of the firm's product range. Technology is assumed to be identical across firms. The total cost function of a typical firm i is therefore given by

(7) $$C_i = \gamma + \phi m_i + \beta\left(\int_{k=0}^{m_i} x_{ik}\, dk\right)$$

where γ is the firm-level fixed cost, ϕ is the fixed cost per variety and β is the firm's marginal cost. Given the assumed symmetry, γ, ϕ and β are the same for all firms.

The firm-level fixed cost γ implies that the cost structures of the different varieties produced by a firm are not independent: γ can be thought of as being related to firm-specific activities, as for example marketing, distribution and management services. The existence of γ generates economies of scope and thereby provides an incentive for the firm to produce a mass of varieties of the good. Clearly, the size of each firm's product range will be limited by the existence of the variety-level fixed cost ϕ – which will typically include the cost of launching a new variety – and implies that there are scale economies at the variety level.

3. MARKET EQUILIBRIUM

Following Chamberlin (1933), we think of the *long-run* equilibrium as being determined by free-entry. In the short-run, market structure can be characterised by a given mass of firms *n*. Firms' decisions are modelled as a two-stage game. In the first stage firms choose the size of their product range (i.e. the mass of varieties to produce) and in the second stage they compete in price. The model is solved by backward induction using the sub-game perfect Nash equilibrium concept to determine the symmetric market solution. The symmetric long-run equilibrium mass of firms is determined by the zero-profit condition.

3.1. Pricing behaviour and product range choice

Firms choose the mass of varieties to produce and the price of each variety in a two-stage game. In the second stage of the game, the mass of firms and the mass of varieties produced by each firm are predetermined and firms compete in price. As in Anderson and de Palma (1992), we assume that each firm co-ordinates its pricing decisions across all the varieties that it produces so as to maximise overall profits. In other words, each firm will internalise the fact that when reducing the price of one of the varieties that it produces, demand for all its other varieties will fall (the cannibalisation effect). We shall further assume that, since firms may produce a non-negligible set of varieties, they will take into account the effects of their pricing decisions on the industry's price index, while taking the prices of all other firms as given. A Nash equilibrium in prices will therefore emerge, with each firm choosing a pricing rule for each variety within its nest. Given the demand functions in (5), the profit function of a typical firm *i* will be:

$$\textbf{(8)} \qquad \pi_i = \left(\int_{k=0}^{m_i} y p^{\alpha-1} p_i^{\sigma-\alpha} (p_{ik} - \beta) p_{ik}^{-\sigma} dk \right) - \phi m_i - \gamma .$$

Differentiating (8) with respect to p_{ij} yields the set of first order conditions:

$$\textbf{(9)} \qquad \frac{\partial \pi_i}{\partial p_{ij}} = x_{ij} - \sigma \frac{x_{ij}}{p_{ij}} (p_{ik} - \beta) + (\sigma - \alpha) \left(\frac{p_{ij}}{p_i} \right)^{1-\sigma} \left(\int_{k=0}^{m_i} \frac{x_{ik}}{p_{ij}} (p_{ik} - \beta) dk \right)$$

$$+ \left((\alpha - 1) \left(\frac{p_i}{p} \right)^{1-\alpha} \left(\frac{p_{ij}}{p_i} \right)^{1-\sigma} \int_{k=0}^{m_i} \frac{x_{ik}}{p_{ij}} (p_{ik} - \beta) dk \right) = 0; \qquad j \in [0, m_i].$$

Each first-order condition is an implicit function of the profit maximising price for one variety given the prices of all other varieties (both those produced by the firm and those produced by its competitors). Given symmetric preferences and the assumed cost structure, equation (9) implies that a typical firm i will charge the same price for all the varieties within its nest, i.e. $p_{ij} = p_{ik} \; \forall \; j,k \in [0, m_i]$[10]. Thus, for a typical variety k produced by firm i, (9) may be re-written as:

$$\textbf{(10)} \qquad \frac{\partial \pi_i}{\partial p_{ik}} = x_{ik}\left\{1-\sigma\frac{p_{ik}-\beta}{p_{ik}}\right\} + x_{ik}\left\{(\sigma-\alpha)\frac{p_{ik}-\beta}{p_{ik}} + (\alpha-1)\left(\frac{p_i}{p}\right)^{1-\alpha}\left(\frac{p_{ik}-\beta}{p_{ik}}\right)\right\} = 0\,.$$

where the first term gives the standard 'Chamberlinian' first-order condition and the second term reflects the behavioural assumptions that the firm internalises the effects of competition both within its product line and between firms within the industry[11]. From equation (10) we obtain

$$\textbf{(11)} \qquad p_{ik} = \beta\left[\frac{\alpha-(\alpha-1)(p_i/p)^{1-\alpha}}{\alpha-(\alpha-1)(p_i/p)^{1-\alpha}-1}\right] = \beta\frac{1}{1-L_i}; \qquad \forall \; k \in [0, m_i], \forall \; i \in [0, n]$$

where $L_i = (p_{ik}-\beta)/p_{ik} = 1/(\alpha-(\alpha-1)s_i)$ and $s_i = (p_i/p)^{1-\alpha}$. L_i is the Lerner index of market power, which determines the magnitude of the mark-up over marginal cost, and s_i is the market share of firm i defined as the proportion of the total expenditure on the differentiated good y that it holds. Thus, L_i – by determining the gap between the imperfectly competitive prices and those characterising a competitive outcome – can be seen as providing a measure of what Shaked and Sutton (1990) call the '*competition*' effect. The market power of the firm is lower the smaller is the market share of an individual firm.

In the first stage of the game, anticipating the subsequent price competition, firms decide on the mass of varieties they will produce in their respective nests taking as given the mass of competitors determined by the free-entry equilibrium. We assume that firms play a Nash game with each other, so that when a typical firm i chooses its product range m_i it takes as given the product ranges of all other firms.

Given equation (11), the definition of the firm-level price index in (4) and the result that a typical firm i will choose to charge the same price for all of its varieties, the profit function in (8) can be re-written as:

$$\textbf{(12)} \qquad \pi_i = yL_i s_i - \phi\, m_i - \gamma; \qquad i \in [0, n]$$

since $p_i = m_i^{\frac{1}{1-\sigma}} p_{ij}$, given that $p_{ij}=p_{ik}$ $\forall k \in [0, m_i]$. Differentiation of (12) with respect to m_i yields the first order condition $\frac{\partial \pi_i}{\partial m_i} = y \frac{\partial L_i}{\partial s_i} \frac{\partial s_i}{\partial m_i} s_i + y L_i \frac{\partial s_i}{\partial m_i} - \phi = 0$. In the closed loop solution, the firm is assumed to take into account the effect of its product range choice on both its own and all other firms' pricing decisions in the second stage of the game and the above first order condition will be equal to (see Appendix A.1):

(13) $$\frac{\partial \pi_i}{\partial m_i} = \frac{L_i s_i y}{m_i} \varepsilon_i - \phi = 0; \quad i \in [0, n]$$

where

(14) $$\varepsilon_i = \left[\left(\frac{\alpha - 1}{\sigma - 1} \right) (1 - s_i) \right] \left[\left(\frac{\alpha L_i}{\alpha L_i - s_i} \right) \left(1 - \frac{s_i}{\left(\frac{\alpha L_i - s_i}{1 - s_i} \right) \left(\int_{h=0}^{n} s_h \left(\frac{1 - s_h}{\alpha L_h - s_h} \right) dh \right)} \right) \right]$$

is the elasticity of variable profits (i.e. the total margin over variable costs) for a typical firm i with respect to the size of its product range m_i. In the spirit of Shaked and Sutton (1990), ε_i provides a measure of the expansion effect. Note that ε_i is bounded to lie within the open interval (0,1) for $\sigma > \alpha$ and $s_i < 1$,[12] which implies the existence of decreasing returns to the introduction of a new variety by a firm such that the optimal product range m_i will be finite. The term in the first set of square brackets corresponds to the open-loop solution, where the firm does not take into account the effects of a change in its mass of varieties on the other firms' pricing decisions. This term is less than unity, reflecting the cannibalisation effect whereby the introduction of a new variety will depress the sales of the firm's existing varieties and thereby lead to a less than proportionate increase in variable profits. The term in the second set of square brackets captures the strategic effect on the pricing decisions of the other firms. This term is also less than one, implying that the firm in the first stage will choose to under-expand its product range in order to limit price competition in the second stage.

In the next section we shall derive the long-run symmetric equilibrium that occurs when all firms have the same product range size (i.e. $m_i = m$ $\forall i \in [0,n]$). Before doing so, it is useful to consider the implications of such symmetry for the pricing and product range choice decisions of the firm.

Starting with the second stage of the game, when all firms have the same product range, they will all choose the same price for all their varieties (i.e. $p_{hk} = p_{il}$ $\forall h,i \in [0,n]$, $\forall k,l \in [0,m]$). In this case for all firms s_i is equal to $1/n$ and $L_i = L$. The latter varies inversely with both the mass of firms, tending to $1/\alpha$ as the mass of firms n tends to infinity, and with the degree of substitutability between the products of competing firms, tending to zero as $\alpha \rightarrow \infty$ and the products in different nests become perfect substitutes. In the short-run when n is fixed, the optimal price rule does not depend on the degree of substitutability between the varieties produced by the firm nor on the size of the firm's product range. This result rests on the assumption that the firm co-ordinates its own pricing decisions in order to maximise overall profits as a monopolist in the supply of its own product range, which implies in (10) that the negative own-price demand effect and the positive intra-firm competition effect of a change in the price of an individual variety exactly cancel out. The same result is obtained by Anderson *et al.* (1992) but they conjecture that it stems from the lack of an outside alternative in their nested multinomial logit demand model.

Noting that symmetry between firms also implies $\varepsilon_i = \varepsilon \ \forall i \in [0,n]$, in the first stage of the game, the first-order condition in (13) yields:

$$\textbf{(15)} \qquad m_i = m = \frac{y}{\phi n} L\varepsilon = \frac{y}{\phi n}\left\{\frac{1}{\alpha - \dfrac{\alpha-1}{n}}\right\}\left\{\left(\frac{\alpha-1}{\sigma-1}\right)\left(1-\frac{1}{n}\right)\left(\frac{1}{1+\dfrac{\alpha-1}{\alpha n(n-1)}}\right)\right\} \quad \forall i \in [0,n]$$

which is positive for $n>1$[13]. Hence, in the symmetric short-run equilibrium, where the mass of firms is given, the typical firm's product range is inversely related to the fixed cost associated with the launching of a new variety. An increase in the intra-firm elasticity of substitution between varieties σ will have a negative effect on m by reducing the product range elasticity of variable profits ε. Conversely, an increase in the inter-firm elasticity of substitution α will increase the size of the product range as the positive effect of the resultant increase in ε more than offsets the negative effect of a fall in L, the degree of market power enjoyed by the firm. Finally, m tends to zero when n tends to infinity, that is each firm wants to sell a single product (formally, a zero measure set of varieties) when the population of firms is arbitrarily large.

3.2. Free-entry equilibrium

The zero-profit equilibrium, which determines the endogenous mass of firms, can be thought of as the industry long-run. We solve for this equilibrium in the symmetric case where $m_i = m$, $\forall\, i \in [0,n]$ and therefore $p_{hk} = p_{il}$ $\forall\, h,i \in [0,n]$, $\forall\, k,l \in [0,m]$. In this case, substituting equation (15) back into the profit function in (12) and setting the latter equal to zero yields the implicit solution for the free-entry equilibrium mass of firms as:

$$\textbf{(16)} \qquad \pi(n) = \gamma\left(\frac{\psi}{n}L(1-\varepsilon)-1\right) = 0$$

where $\psi = y/\gamma$. As noted earlier, $n>1$ must hold for each firm's market share to be less than unity. Given that $1/\alpha \leq L<1$ and $0<\varepsilon \leq (\alpha-1)/(\sigma-1)<1$, then it is apparent from (16) that $n>1$ requires ceteris paribus that the size of the market relative to firm level fixed cost, i.e. $\psi = y/\gamma$, is sufficiently large. Substitution for L and ε yields

$$\textbf{(17)} \qquad \pi(n) = \gamma\left(\frac{An^3 + Bn^2 + Cn + D}{(\sigma-1)(\alpha n - \alpha + 1)(\alpha n^2 - \alpha n + \alpha - 1)}\right)$$

where A, B, C and D are defined in the appendix. Given that the denominator of the ratio in brackets is positive for $n \geq 1$, then $\pi(n)=0$ requires the polynomial in the numerator to be zero. As discussed in Appendix A.2, this polynomial has only one real root for all relevant values of parameters and this root corresponds to a stable equilibrium.

The main comparative static properties of the symmetric long-run equilibrium are summarised in Table 1 below. A ceteris paribus increase in the size of the market for the differentiated good (y) leads to an increase in the long-run equilibrium mass of firms. This in turn directly reduces firms' market power L leading to a fall in the price of each variety. The increase in the mass of firms also increases the firms' product range elasticity of variable profits ε. Coupled with the increase in market size, this results in an expansion of product ranges in spite of the higher price competition. Firm-level and industry price indexes both fall, with the effects of the reduction in individual variety prices reinforced by the increase in choice due to the expansion of product ranges and the mass of firms.

Table 1: Long-run equilibrium comparative statics

Changes in:	Resultant changes in:							
	n	L	ε	m	v=mn	p_{ik}	p_i	p
y	+	−	+	+	+	−	−	−
γ	−	+	−	+(a)	−	+	−(b)	+
ϕ	0	0	0	−	−	0	+	+
α	−	−	+(c)	+(c)	+(d)	−	−	−
σ	+	−	−	−	−	−	+	+

Sufficient conditions for the annotated signs to hold are: (a) $n>L+2$, (b) $n>\sigma L+2$, (c) $n>2$, (d) $n(n-1) > 3\alpha/(1-\varepsilon)$.

The fixed cost per firm γ determines the potential for the exploitation of scope economies and directly affects the mass of firms surviving in the free-entry equilibrium. Hence, a ceteris paribus increase in γ will reduce the mass of firms in the industry, increasing the market power of the remaining firms but reducing the product range elasticity of variable profits. In response, firms will raise the prices of individual varieties and, for $n>L+2$, expand product ranges to counter the fall in the number of competitors. For sufficiently large n, product range expansion may lead to a fall in the firm-level price index in spite of the increase in individual variety prices. But, at the industry level, the effects of the fall in the mass of firms dominate, leading to an unambiguous fall in the total number of varieties on offer in the market and an increase of the industry price index.

Given that the total cost function (7) is additive in the fixed cost terms, changes in the fixed cost per variety ϕ do not affect the equilibrium mass of firms n. As a result, both the degree of market power and the product range elasticity of variable profits are independent of ϕ and changes in the fixed cost per variety will not affect the price of individual varieties. Clearly though, the size of the product range is inversely related to ϕ in (15) so firms respond to increases in fixed costs per variety by cutting back on the number of varieties that each of them offers. This contraction of product ranges leads to increases in both the firm-level and industry-level price indexes.

The inter-firm elasticity of substitution α determines the distinctiveness of firms' product ranges. An increase in α implies that product ranges become more homogeneous and therefore results in an immediate decline in firms' market power L and a fall in the prices of individual varieties. This reduction in price-cost margins leads to exit from the industry and to a long-run decline in the mass of firms. For n>2, an increase in α will also give rise to an increase in the product range elasticity of variable profits ε. Coupled with the fall in the mass of firms, this results in a long-run expansion in product ranges in spite of the reduced price-cost margins. Firm-level and industry price indexes both fall, with the effects of the reduction in individual variety prices reinforced, for sufficiently large n, by the increase in choice both within product ranges and in the market as a whole.

The intra-firm elasticity of substitution σ determines the distinctiveness of individual varieties within each firm's product range. An increase in σ implies that the varieties offered by any single firm become more homogeneous and will therefore result in an immediate decline in the product range elasticity of variable profits ε and a contraction of product ranges. This contraction of existing firms' product ranges induces entry leading to a long-run rise in the mass of firms with negative consequences for firms' market power and the prices charged for individual varieties. Nevertheless, the reduction of choice, both within product ranges and in the market as a whole, leads to increases in the firm-level and industry price indexes.

In summary, depending on the value of its structural parameters, the model may give rise both to fragmented equilibria with a large population of firms each offering a small range of products, and to concentrated equilibria with a small mass of firms each offering many products. Specifically, the free-entry equilibrium will be more likely to consist of a large (small) population of firms with small (large) product ranges when (i) fixed costs per firm are small (large) relative both to the size of the market and to fixed costs per variety, (ii) the inter-firm elasticity of substitution is low (high), and (iii) the intra-firm elasticity of substitution is high (low). There will exist values of y, γ, ϕ, α and σ such that each firm in the free-entry equilibrium will choose a single variant.

3.4. Social optimum

To compare the free-entry equilibrium with the social optimum subject to a zero profit constraint, first note that the second-best welfare optimum corresponds to the market equilibrium outcome in the "large number" monopolistic competition case[14], and can therefore be found by setting L and ε equal to their respective limiting values of $1/\alpha$ and $(\alpha$-1)/(σ-1). Hence, given that $L > 1/\alpha$ and $\varepsilon<(\alpha-1)/(\sigma-1)$ for finite n, it is easy to show that prices will be too high in equilibrium with too many firms offering too few varieties, both individually and in total. These findings are also obtained for the nested multinomial logit demand model by Anderson and de Palma (1992) (see also Anderson *et al*., 1992, p.257) who argue that it arises as the net outcome of three distinct effects: a business stealing externality (an entrant does not internalise the detrimental effects on existing firms' profits) which is a tendency for excessive entry, a consumer surplus externality (an entrant can not extract the whole consumer surplus associated with producing its product line) which is a tendency to under-entry, and a multiproduct externality (an entrant does not account for the contraction of the product ranges of existing firms) which is again a tendency for over-entry. As in Anderson and de Palma (1992), we find that the net effect is too much inter-firm diversity but too little intra-firm diversity and, as a result, too little total diversity.

4. AN EXPLORATIVE APPLICATION TO INDUSTRY SHAKEOUT

In this section we highlight some of the implications of the model developed above for the evolution of market structure during the life cycle of an industry. It is a commonly accepted stylised fact of industry evolution that the number of producers in many new industries first increases to a peak and then, despite continuing growth in the size of the market, falls sharply until it finally reaches a stable level (see Sutton, 1997, for references). Here, we want to explore the proposition that industry shakeout may result in some industries from a change in the scope as well as the scale of firms over the course of the product life cycle.

The majority of models proposed in the literature to account for the shakeout in the number of firms during the product life cycle of an industry rest on the basic proposition that shakeout is the outcome of changes in the scale of production[15]. Changes in the scope of firm output have not been central to the analysis of shakeout despite empirical evidence that product proliferation may be associated with the shakeout phase. For example, Raff and Trajtenberg (1997) document how the early development of the United States automobile industry was characterised by a rise in the number of models (varieties) on offer, more through entry than through model proliferation, but that after 1910 the number of firms in the industry fell (from an average of 153 between 1910 and 1920 to an average of 30 in the 1930's) while the number of models offered by the surviving firms rose substantially (from an average of 5.1 body models per firm in the 1910's to 18.4 in the 1930's). Brander and Eaton

(1984) observe that a fairly common historical pattern is for firms to expand the scope of their product offerings and compete more directly with each other as the market grows.

In our opinion therefore, a full analysis of the evolution of market structure over the product life cycle should allow for the possible role played by the strategic product range choices made by individual firms[16]. Given the exploratory nature of our analysis, we shall not develop a dynamic model of industry evolution. Instead, we shall employ the partial equilibrium model developed above to assess whether changes in the scope of production are likely to be a contributory factor in the shakeout of firms. By means of comparative static analysis, we shall consider whether stylised changes in both process technology and product characteristics observed over the course of the product life cycle may serve in some industries to induce a shift from a fragmented equilibrium, in the early stages of the industry development, to a concentrated equilibrium in the mature industry.

4.1. Process technology: changes in scale and changes in scope

Industry shakeout is generally presumed to be the direct outcome of changes in the scale of firms' output which lead to a reduction in the number of firms that can survive in the industry, despite continuing growth in the market. In the product life cycle model (see Utterback and Suarez, 1993), firms invest in new capital-intensive technologies that yield lower unit costs of production but at higher volumes of output. Alternatively, Klepper (1996) assumes that firms must invest in research and development in order to maintain competitive levels of unit costs. Expenditure on advertising and other selling activities might also play a similar role in some industries given the existence of promotional scale economies (see Sutton, 1991).

In the typical shakeout model, firms produce a single product such that fixed investments give rise to simple scale economies in production. The fact that most firms are multiproduct, however, implies that they may seek to exploit economies of both scope and scale, where the former can arise from the cost advantages of making a number of products in the same production unit, of distributing a number of products though the same channels, and of managing a number of products within the same organisation. For example, Chandler (1990) cites the case of the German chemical dye industry as a prime example in which leading companies sought to exploit fully economies of scope by building plants that produced 'literally hundreds of dyes' (p.25).

In our model, the balance between scope and scale is determined by γ and ϕ, the firm-level and variety-level fixed costs respectively. Increases in either type of fixed cost will lead to a shakeout in the total number of varieties on offer in the market, but only increases in the firm-level fixed cost γ lead to exit from the industry. This suggests that the shakeout process may be more pronounced in industries where increases in firms' overhead costs dominate increases in variety-level costs. Moreover, the long run-equilibrium in these cases will be characterised by a smaller number of firms each producing a wider product range.

Thus, our analysis identifies the nature of the changes in the cost structures of firms that may tend to shift the industry from a fragmented equilibrium to a concentrated equilibrium over the course of the product life cycle.

4.2. Product characteristics: changes in heterogeneity

Changes in product characteristics are a central feature of most descriptions of the product life cycle: indeed, products without rich opportunities for both product and process innovations may not follow the prototypical product life cycle (see Klepper, 1996). Most accounts of the shakeout process note a reduction in the degree of heterogeneity in the products offered by different firms over time. In the product life cycle model, this is generally ascribed to the emergence of a 'dominant design' that, by creating the conditions for the investment in capital-intensive technologies, triggers the shakeout process. Conversely, in Klepper (1996) the diversity of competing versions of the product declines as a consequence rather than as a cause of the shakeout process.

We would add that, within a multiproduct firm framework, standardisation of the product offerings of different firms may also be accompanied by changes in the heterogeneity of the products offered by each individual firm. In the early stages of the product life cycle in which rival firms commonly offer products based on radically different technologies, variants offered by the same firm are likely to be relatively homogeneous compared to those offered by their rivals. The subsequent emergence of a dominant design may then lead to the complete 'commoditisation' of the product with firms competing on price alone. But the more likely scenario is for a number of distinct variants of the generic design of the product to continue to be offered on the market to cater for what may be increasingly discerning consumer tastes, with the locus of differentiation shifting from fundamental or primary characteristics to adaptive or secondary characteristics of the production technology (Saviotti, 1996). Given an established group of competing firms in the market, each firm has an incentive to develop products that are closer substitutes to each other than to those of other firms, since joint production of these products will lead to less intense price and output competition (Brander and Eaton, 1984)[17]. However, the scope for secondary differentiation is likely to be limited by the number of characteristics over which the varieties of the product may be differentiated from each other, and by crowding of the firm's characteristic space if the product range is extended for any reason.

The individual varieties offered by a single firm might therefore be expected to become somewhat more distinct as the product ranges of competing firms become increasingly uniform. In our model, a reduction in the degree of heterogeneity of the products offered by different firms may be characterised as an increase in α, the inter-firm elasticity of substitution. Conversely, any increase in the heterogeneity of the varieties offered by a single firm are captured by reduction in σ, the intra-firm elasticity of substitution. Thus the increasing standardisation of products offered by different firms and the increasing differentiation of varieties offered by individual firms over the product life cycle both serve to reinforce the shakeout of firms and the expansion of product ranges.

In conclusion, taking into account the main stylised facts about industry shakeout, our theoretical analysis of the product life cycle suggests the possibility that changes in product heterogeneity may complement and reinforce those arising from the exploitation of economies of scope and scale in shifting the industry from a fragmented equilibrium to a concentrated equilibrium characterised by a smaller number of firms each producing a larger number of varieties. Clearly, empirical research will be required in order to elucidate the relative importance of changes in process technology and product characteristics in the shakeout of individual industries. We would suggest that it may be of particular interest to examine the evolution of advertising-intensive industries, such as those producing packaged, branded goods, in which fundamental product and process innovation is limited but heavy advertising expenditures tend both to increase fixed costs on the supply side and to influence the relative size of the competition and expansion effects on the demand side.

5. CONCLUSION

In this paper we have extended the standard Dixit-Stiglitz model of imperfect competition (Dixit and Stiglitz, 1977) to allow for multiproduct firms. We endogenously determine both the number of varieties per firm and the number of firms in the industry. A crucial feature of the model is that firms internalise the effects of both intra- and inter-firm competition in making their pricing and product range decisions. Thus firms behave as oligopolists not as monopolistic competitors and each firm co-ordinates its pricing decisions across all the varieties that it produces so as to maximise overall profits.

Depending on the values of the structural parameters, the model may give rise to both fragmented equilibria in which a large number of firms offer either one or a small range of products, and concentrated equilibria in which either one or a small number of firms each offer many products. We identify a *competition* effect and an *expansion* effect as determinants of the market equilibria. This characterisation serves to clarify the way in which the nature of consumer preference influences market structure and may also facilitate model estimation from standard industrial cost data and market intelligence reports.

The model is used to explore the proposition that shakeout may result in some cases from a change in the scope as well as the scale of firms over the course of the product life cycle. We do not seek to endogenise fixed costs due to (sunk) investments in either new process technologies (Jovanovic and Macdonald, 1994), process R&D (Klepper, 1996) or advertising (Sutton, 1991). Nor do we consider vertical product differentiation either as a transient phenomenon due to the diffusion of product improvements or as a permanent feature of the market. Moreover, we abstract from possible asymmetries either in costs (Jovanovic and Macdonald, 1994; Klepper, 1996; Montagna, 1995) or in strategic behaviour (Sutton, 1991) between firms. Nevertheless, our comparative static analysis does serve to demonstrate that the forces leading to the emergence of multiproduct firms may complement the standard explanation of industry shakeout. In so doing, we hope to have shown that by using a nested-CES oligopolistic model of multiproduct competition it is possible to offer a richer characterisation of the shakeout process.

Appendix

A.1 Derivation of the closed-loop solution

The profit function of the typical firm is

(12) $\pi_i = yL_i s_i - \phi\, m_i - \gamma$

Partially differentiating (12) with respect to the firm's product range m_i we obtain:

$$\frac{\partial \pi_i}{\partial m_i} = y\frac{\partial L_i}{\partial m_i}s_i + yL_i\frac{\partial s_i}{\partial m_i} - \phi = yL_i^2\alpha(1-\alpha)r_i^{-\alpha}\frac{\partial r_i}{\partial m_i} - \phi$$

where

$$\frac{\partial r_i}{\partial m_i} = \frac{\partial\left\{\frac{p_i}{p}\right\}}{\partial m_i} = \left[\frac{1}{p}\right]\frac{\partial p_i}{\partial m_i} - \left[\frac{p_i}{p^2}\right]\int_{h=0}^{n} r_h^{-\alpha}\frac{\partial p_h}{\partial m_i} \quad \text{since } p = \left(\int_{h=0}^{n} p_h^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$$

with for $h \neq i$:

$$\frac{\partial p_h}{\partial m_i} = \frac{\partial\left\{m_h^{\frac{1}{1-\sigma}}p_{hj}\right\}}{\partial m_i} = m_h^{\frac{1}{1-\sigma}}\frac{\partial\{p_{hj}\}}{\partial m_i} = m_h^{\frac{1}{1-\sigma}}\beta\frac{\partial\left\{(1-L_h)^{-1}\right\}}{\partial m_i},$$

$$= -\frac{r_h\theta_h}{(1-\theta_h)}\frac{\partial p}{\partial m_i} \qquad \text{where } \theta_h = \frac{(1-\alpha)s_h L_h}{(1-s_h)}$$

since the firm assumes $\partial m_h / \partial m_i = 0$, but takes account of the effects of the change in m_i on its competitors' second-stage optimal prices $p_{hj} = \beta(1-L_h)^{-1}$;

and for $h = i$:

$$\frac{\partial p_i}{\partial m_i} = \frac{\partial\left\{m_i^{\frac{1}{1-\sigma}}p_{ij}\right\}}{\partial m_i} = \frac{\partial\left\{m_i^{\frac{1}{1-\sigma}}\right\}}{\partial m_i}\beta(1-L_i)^{-1} + m_i^{\frac{1}{1-\sigma}}\beta\frac{\partial\left\{(1-L_i)^{-1}\right\}}{\partial m_i}$$

$$= \frac{p_i}{(1-\theta_i)(1-\sigma)m_i} - \frac{r_i\theta_i}{(1-\theta_i)}\frac{\partial p}{\partial m_i} \qquad \text{where } \theta_i = \frac{(1-\alpha)s_i L_i}{(1-s_i)}.$$

It follows that:

$$\frac{\partial p}{\partial m_i} = \int_{h=0}^{n} r_h^{-\alpha}\frac{\partial p_h}{\partial m_i} = \frac{r_h^{-\alpha}p_i}{(1-\theta_i)(1-\sigma)m_i} - \int_{h=0}^{n}\frac{s_h\theta_h}{(1-\theta_h)}\frac{\partial p}{\partial m_i}$$

$$= \frac{r_h^{-\alpha}p_i}{\left(\int_{h=0}^{n}\frac{s_h}{(1-\theta_h)}\right)(1-\theta_i)(1-\sigma)m_i} \qquad \text{since } \int_{h=0}^{n} s_h = 1$$

which implies:

$$\frac{\partial r_i}{\partial m_i} = \frac{r_i}{(1-\theta_i)(1-\sigma)m_i}\left(1-\frac{s_i}{\left((1-\theta_i)\int_{h=0}^{n}\frac{s_h}{(1-\theta_h)}\right)}\right).$$

Hence:

$$\frac{\partial \pi_i}{\partial m_i} = \left(\frac{yL_i s_i}{m_i}\right)\left\{\left(\frac{\alpha-1}{\sigma-1}\right)\left(\frac{\alpha L_i}{(1-\theta_i)}\right)\left(1-\frac{s_i}{\left((1-\theta_i)\int_{h=0}^{n}\frac{s_h}{(1-\theta_h)}\right)}\right)\right\}-\phi$$

which gives (13) since $\frac{\partial\{yL_i s_i\}}{\partial m_i}=\frac{yL_i s_i}{m_i}\left(\frac{\partial\{yL_i s_i\}}{\partial m_i}\frac{m_i}{yL_i s_i}\right)=\frac{yL_i s_i}{m_i}\varepsilon_i$ where $yL_i s_i$ is variable profits and ε_i maybe written as in (14) since $1-\theta_i=\left(\frac{\alpha L_i - s_i}{1-s_i}\right)$.

A.2 Free-entry equilibrium

In the symmetric equilibrium the profit function is $\pi(n)=\gamma\left(\frac{\psi}{n}L(1-\varepsilon)-1\right)$ where ψ=y/γ.

Substituting in $L=\frac{1}{\alpha-\frac{\alpha-1}{n}}$ and $\varepsilon=\left(\frac{\alpha-1}{\sigma-1}\right)\left(1-\frac{1}{n}\right)\left(\frac{1}{1+\frac{\alpha-1}{\alpha n(n-1)}}\right)$, this becomes:

$$\pi(n)=\gamma\left(\frac{\left[1-\left(\frac{\alpha-1}{\sigma-1}\right)\frac{(1-1/n)^2\alpha}{n(n\alpha-\alpha+[(\alpha-1)/n])}\right]\psi}{\left(\alpha-\frac{\alpha-1}{n}\right)n}-1\right)$$

which can be rewritten as:

$$\pi(n)=\gamma\left(\frac{An^3+Bn^2+Cn+D}{(\sigma-1)(\alpha n-\alpha+1)(\alpha n^2-\alpha n+\alpha-1)}\right)$$

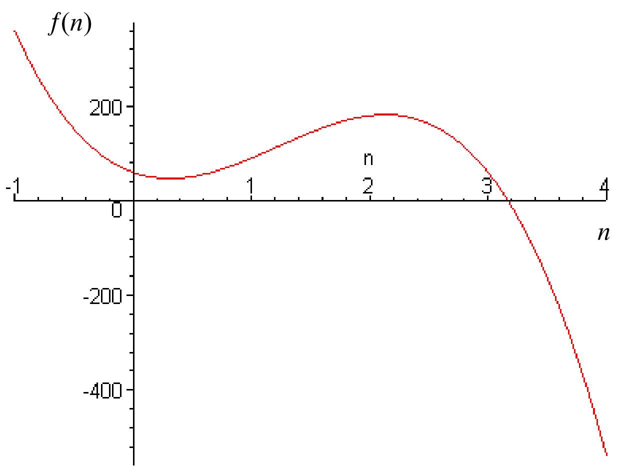
where:

$$A = -\alpha^2(\sigma - 1);$$
$$B = (\psi - 2(\sigma - 1))\alpha^2 + (\sigma - 1) - \sigma\alpha;$$
$$C = 2(\sigma - 1 - \psi)\alpha^2 - (2(\sigma - 1) + \psi(\sigma + 1))\alpha \qquad ;$$
$$D = (\psi + \sigma - 1)\alpha^2 + (2(\sigma - 1) - \sigma\psi)\alpha - (\sigma - 1) + \psi(\sigma - 1))$$

To determine the zero-profit free-entry equilibrium we need to solve $\pi(n)=0$ for n. The profit function in (A.2) is a ratio of polynomials in n: the denominator is positive for all $n \geq 1$. Hence, for $\pi(n)=0$ the numerator must be zero, i.e.

(A.3) $\quad f(n) = An^3 + Bn^2 + Cn + D = 0$

Note that $A<0$. Hence: $n \to \infty \Rightarrow f(n) \to -\infty$ and $n \to -\infty \Rightarrow f(n) \to +\infty$. Furthermore it is tedious but straightforward to show that $f(n)$ has only one real root for all plausible values of parameters. Hence, $f(n)$ is of the form[1]:



where the real root clearly corresponds to a stable equilibrium, since $f(n)$ cuts the horizontal axis from above at $n>1$: other things equal, firms' profit falls if n increases and vice-versa.

[1] This particular graph is obtained for: $\sigma=6$, $\alpha=3$, $\theta=10$.

REFERENCES

Abernathy, W.J. and Utterback, J.M. (1978). "Patterns of industrial innovation", *Technology Review*, 80, 41-47.

Anderson, S.P. and de Palma A. (1992). "Multiproduct firms: a nested logit approach", *Journal of Industrial Economics*, 40, 261-276.

Anderson, S.P. and de Palma A. (2000). "From local to global competition", *European Economic Review*, 44, 423-448.

Anderson, S.P., de Palma A. and Thisse, J-F (1992). *Discrete choice theory of product differentiation*, The MIT Press: Cambridge, Massachusetts.

ASDA (2002) *ASDA announces the end of the label love affair*, News release, February 18.

Baldwin, R.E. and Ottaviano, G.I.P. (1998). "Multiproduct multinationals and reciprocal FDI dumping", Working Paper 6483, National Bureau of Economic Research: Cambridge, MA.

Baumol, W., Panzar, J. and Willig, R. (1982). *Contestable markets and the theory of industry structure*, Harcourt Brace Jovanovic: New York.

Beath, J. and Katsoulacos Y. (1991) *The economic theory of product differentiation*, Cambridge University Press: Cambridge.

Ben-Akiva, M. (1973). *Structure of passenger travel demand models*, MIT Ph.D. Dissertation.

Brander, J.A. and Eaton, J. (1984). "Product line rivalry", *American Economic Review*, 74, 323-334.

Chamberlin, E.(1933) *The Theory of Monopolistic Competition*, Cambridge: Harvard University Press.

Champsaur, P. and Rochet, J-C. (1989). "Multiproduct duopolists", *Econometrica*, 57, 533-557.

Chandler, A.D. (1990). *Scale and scope: the dynamics of industrial capitalism*, Harvard Belknap: Cambridge, MA.

Consumers' Association (1993) Jeans, *Which?*, March, 26-29.

Deaton A. and Muellbauer J. (1990). *Economics and Consumer Behaviour*, Cambridge University Press.

Dixit, A.K. and Stiglitz, J.E. (1977). "Monopolistic competition and optimum product diversity", *American Economic Review*, 67, 22-33.

Hanly, D. and Cheung, K.C.K. (1998). "Market structure of multiproduct firms under free entry", *Economics Letters*, 6, 159-163.

Jovanovic, B. and MacDonald, G.H. (1994). "The life cycle of a competitive industry", *Journal of Political Economy*, 102, 322-347.

Katz, M.L. (1984). "Firm-specific differentiation and competition among multiproduct firms", *Journal of Business*, 57, S149-S178.

Klepper, S. (1996). "Entry, exit, growth and innovation over the product life cycle", *American Economic Review*, 86, 560-583.

Levitt, T. (1980). "Marketing success through differentiation of anything", *Harvard Business Review*, 1, 83-91.

Montagna, C. (1995). "Monopolistic competition with firm-specific costs", *Oxford Economic Papers*, 47, 318-28.

Ottaviano, G.I.P. and Thisse, J.-F. (1999). "Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity", CEPR Discussion Paper N. 2151.

Raff, D.M.G. and Trajtenberg, M. (1997) "Quality Adjusted Prices for the American Automobile Industry: 1906-1940", in T.F. Bresnahan and R.J. Gordon (eds.) The Economics of New Goods, Chicago: University of Chicago Press, 73-101

Raubitschek, R.S. (1987). "A model of product proliferation with multiproduct firms", *Journal of Industrial Economics*, 35, 269-279.

Saviotti, P.P. (1996). *Technological evolution, variety and the economy*, Edward Elgar: Cheltenham.

Schmalansee, R.L. (1978). "Entry deterrence in the ready-to-eat cereal industry", *Bell Journal of Economics*, 9, 305-327.

Shaked, A. and Sutton, J. (1990). "Multiproduct firms and market structure", *Rand Journal of Economics*, 21, 45-62.

Sutton, J. (1991). *Sunk costs and market structure*, The MIT Press: Cambridge, Massachusetts.

Sutton, J. (1997). "Gibrat's legacy", *Journal of Economic Literature*, 35, 40-59.

Utterback, J.M. and Suarez, F.F. (1993). "Innovation, competition and industry structure", *Research Policy*, 22, 1-21.

Utterback J.M. and Abernathy, W.J. (1975). "A dynamic model of product and process innovation", *Omega*, 3, 639-656.

Verboven F. (1996). "The nested logit model and representative consumer theory", *Economics Letters*, 50, 57-63.

Wolinsky, A. (1986). "The nature of competition and the scope of firms", *Journal of Industrial Economics*, 34, 247-259.

ENDNOTES

[1] Baldwin and Ottaviano (1998) have also employed the representative consumer framework to model FDI and trade patterns, but only between two established multiproduct multinationals.

[2] This is one of the three major approaches used in the literature to model the demand for differentiated products, the others being the address and the discrete choice (random utility) approaches. Anderson *et al.* (1992) show that there exists a class of discrete choice models that are consistent with both the representative consumer and the address approaches. Hence, the representative consumer and the address models of product differentiation can be linked "via the intermediary of the discrete choice approach" (Anderson *et al.*, 1992, page 7).

[3] Given homothetic preferences and linear budget constraints, this is equivalent to a one-shot utility maximisation (see Deaton and Muellbauer, 1990).

[4] Verboven (1996) generalises the nested logit model developed by Anderson and de Palma (1992) and Anderson *et al.* (1992), and shows that the demand functions derived from the nested version of the representative consumer CES model are equivalent to those generated from a version of the nested logit discrete choice model. This equivalence explains why the general character of the results obtained in this paper is similar to those obtained by Anderson and de Palma (1992).

[5] Anderson and de Palma (2000) introduce a framework that incorporates both localised and global competition and that has different oligopolistic models of product differentiation (the circle, the logit and the CES) as limit cases.

[6] For expositional convenience we may sometimes refer to 'number' rather than 'mass'.

[7] One may also conceive of complex market structures involving both interlacing and segmentation. In general, the type of market structure may be the endogenous outcome of the competitive process.

[8] The market interlacing case raises the substantive issue of the determination of the number of nests. One cannot simply assume an unlimited number of potential nests in which the firm might choose to operate with identical costs, because this provides no incentive for a firm to enter a nest occupied by another firm.

[9] As Katz (1984) observes, the pattern of advertising gives an indication of the importance of brand-wide effects, with firms in many industries using advertising to promote entire product lines rather than any single variant.

[10] Equation (9) can be re-written using (6) and (8) as:

$$x_{ij}\left\{\left(1-\sigma\left(\frac{p_{ij}-\beta}{p_{ij}}\right)\right)+\left(\frac{\pi_i+\phi m_i+\gamma}{y}\right)\left((\sigma-\alpha)\left(\frac{p_i}{p}\right)^{\alpha-1}+(\alpha-1)\right)\right\}=0$$

from which the symmetry of the firm's prices is clear.

[11] It is tedious but straightforward to show that the Hessian is negative definite for integer values of $m \geq 1$.

[12] In the limit, the expansion effect will tend to zero as $\sigma \to \infty$ and the products offered by the firm become perfect substitutes for each other, and to $(\alpha-1)/(\sigma-1)$ as $s_i \to 0$.

[13] It is straightforward to show that the second order condition is unambiguously negative if $n>1$.

[14] This result can readily be shown. See also Dixit and Stiglitz (1977).

[15] See Utterback and Abernathy (1975), Abernathy and Utterback (1978), Utterback and Suarez (1993), Jovanovic and MacDonald (1994) and Klepper (1996).

[16] Product proliferation is recognised in the literature (e.g. Schmalansee, 1978) as a possible form of strategic entry deterrence in mature differentiated-goods oligopolies. But what is not clear from this literature is the extent to which changes in the scope of firms might also be instrumental in the process leading to the mature industry stage (though see Sutton, 1991, on the specific case of the ready-to-eat breakfast cereals industry).

[17] Alternatively, firms may choose to develop products that more closely resemble those of their competitors, transforming an initially segmented market structure into an interlaced one through the fragmentation of the market into a number of separate sub-markets or nests within each of which all firms compete (this possibility cannot be formally explored in this paper).