An I(2) analysis of inflation and the markup
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An I(2) Analysis of Inflation and the Markup

Anindya Banerjee† Lynne Cockerell# Bill Russell##

An I(2) analysis of Australian inflation and the markup is undertaken within an imperfect competition model. It is found that the levels of prices and costs are best characterised as integrated of order 2 and that a linear combination of the levels (which may be defined as the markup) cointegrates with price inflation. From the empirical analysis we obtain a long-run relationship where higher inflation is associated with a lower markup and vice versa. The impact in the long-run of inflation on the markup is interpreted as the cost to firms of overcoming missing information when adjusting prices in an inflationary environment.

Keywords: Inflation, Wages, Prices, Markup, I(2), Polynomial Cointegration.
JEL Classification: C22, C32, C52, E24, E31
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1 INTRODUCTION

The pricing models of Bénabou (1992), Athey, Bagwell and Sanichiro (1998), Simon (1999), Russell, Evans and Preston (1997), Chen and Russell (1998), and Russell (1998), predict that higher inflation is associated with a lower markup. Furthermore, the latter three papers argue that the markup and the steady-state rate of inflation are negatively related. This paper empirically investigates the proposition that inflation and the markup are related in the long-run in the sense proposed by Engle and Granger (1987) and that higher inflation is associated with a lower markup and *vice versa*. We interpret this long-run relationship as being between steady-state rates of inflation and the markup.\(^1\)

The investigation of a long-run relationship under this interpretation implies a definite modelling strategy. First, the inflation data must follow an integrated statistical process so that there are persistent deviations from any given mean value in the data. This enables us to investigate the proposition that inflation and the markup are related during periods of high, low and medium rates of inflation.\(^2\) The second aspect of the modelling strategy follows

\(^{1}\) The steady state is defined by all nominal variables growing at the same constant rate.

\(^{2}\) It is possible that stationary processes with shifting means could generate a relationship between steady-state rates of inflation and the markup. This issue is explored further in section 2.1. Markup models treating inflation as stationary have been studied for Australia by Richards and Stevens (1987), Cockerell and Russell (1995), and de Brouwer and Ericsson (1998), and for Germany and the US by Franz and Gordon (1993). Bénabou (1992) assumes that inflation is a stationary process and finds that expected and unexpected inflation significantly reduces the markup. Given the assumption of stationarity this cannot be regarded as a long-run relationship in the sense of Engle and Granger (1987).
from the first. If inflation is integrated of order 1 then the empirical investigation must accommodate the possibility that the price level is I(2).

Graph 1: Wage and Price Inflation
Four Quarter Ended Percentage Change

Notes: Prices are defined as the private consumption implicit price deflator and wages as non-farm unit labour costs and are measured on a national accounts basis.

These two aspects of the modelling strategy are followed in the paper to model Australian inflation and the markup. Graph 1 shows that Australian inflation has varied widely and persistently over the past 25 years displaying a number of distinct inflationary periods. Following low inflation in the early 1970s, inflation rose substantially with the first OPEC oil price shock and successive wage shocks. Inflation rose again in the late 1970s and early 1980s with a wage boom associated with a buoyant mining industry and the second OPEC oil

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3 The notation I(d) represents the phrase 'integrated of order d'. For a comprehensive discussion on the statistical properties of data and the order of integration see Banerjee, Dolado, Galbraith and Hendry (1993) or Johansen (1995a).
price shock before moderating through the 1980s. During the recession beginning in 1989, inflation declined to rates not seen since the beginning of the period. The evidence of distinctly different periods of inflation is consistent with standard macroeconomic models.

Graph 2: The Markup of Price on Unit Labour Costs

Note: The markup is calculated as prices divided by unit labour costs and 100 = period average.

Following from the above, a necessary but not sufficient condition for the proposition to be correct is that the markup is also integrated. As a general measure of the markup, Graph 2 shows the markup of price on unit labour costs. The wage shocks in the early 1970s led to a large fall in the markup that persisted until the mid to late 1980s. The extended period of a low markup may simply represent slow price adjustment in response to the wage shocks in the early 1970s. However, this representation should be questioned since the adjustment appears so slow and a case could be established for the markup to be a genuinely integrated process.
It is proposed that the non-stationary characteristics of inflation and the wide variations in the markup are closely related and that high inflation is associated with a low markup as argued by Bénabou (1992), Athey et al. (1998), Simon (1999), Russell et al. (1997), Chen and Russell (1998) and Russell (1998). Comparing inflation and the markup in Graph 3 reveals the veracity of this proposition.

We argue that the variables of interest, namely the levels of prices and costs are best described as I(2) statistical processes. From this starting point we proceed to estimate an I(2) system using techniques developed by Johansen (1995a, b). We find that a linear combination of the levels of prices and costs (which may be defined as the markup)
cointegrate with a linear combination of the differences of the three core variables. In addition, a reduction of the I(2) system enables us to identify the relationship between the markup and price inflation alone. We do so by estimating the trivariate I(1) system given by the markup of prices on unit labour costs, the markup of prices on unit import costs and price inflation. This constitutes the reduction from I(1) to I(0) space and corroborates the original I(2) analysis. From the analysis we obtain a long-run relationship where higher inflation is associated with a lower markup.

The proposition that inflation and the markup are negatively related in the long-run has a number of important economic implications. First, with inflation negatively related with the markup, inflation is, therefore, positively related with the real wage for a given level of productivity. If unemployment is, in part, dependent on the real wage relative to productivity, it follows that it is unlikely that the long-run Phillips curve is vertical. Second, the relationship provides an explanation for the widely reported international evidence that stock returns and inflation are negatively correlated. The lower stock returns simply reflect the impact of inflation on the profitability of firms. Third, the relationship provides an explanation for why firms may desire a low rather than a high rate of steady-state inflation as lower inflation increases the markup of firms.

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4 The logarithm of the markup, $\mu$, is defined as $\mu = p - \sum_{i=1}^{k} \psi_i c_i$ where $p$ and the $c_i$’s are the logarithms of prices and the costs of production respectively, $\psi_i = 1$ where $k$ is the number of inputs. If the latter condition is not satisfied then the relationship between prices and costs cannot be termed the markup.

In order to motivate the empirical analysis, an imperfect competition model of prices is set out in the next section where inflation imposes costs on firms in the long-run. We then briefly consider the statistical properties of inflation and address the question of why the alternative modelling strategy of assuming the inflation data is I(0) with structural breaks is avoided. In Section 3 we estimate the long-run relationship between inflation and the markup using quarterly Australian data for the period 1972 to 1995.

2 AN INFLATION COST MARKUP MODEL OF PRICES

A markup model of prices for a closed economy may be derived using an imperfect competition model of inflation in the Layard-Nickell tradition. We can write the firm’s desired markup as:

\[ p - w = \omega_0 - \omega_1 U - \omega_2 \Delta U + \omega_3 z_p - \omega_4 \left( p - p^e \right) - \omega_5 \phi - \omega_6 \Delta p \]  

(1)

and labour’s desired real wage as:

\[ w - p = \gamma_0 - \gamma_1 U - \gamma_2 \Delta U + \gamma_3 z_w - \gamma_4 \left( p - p^e \right) + \gamma_5 \phi \]  

(2)

where \( p, \; p^e, \; w, \; U \) and \( \phi \) are prices, expected prices, wages, the unemployment rate and productivity respectively. The lower case variables are in logs, \( \Delta \) is the change in the variable and all coefficients are positive. The variables \( z_w \) and \( z_p \) capture shifts in the bargaining

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position of labour and firms respectively. For labour, $z_w$ includes unemployment benefits, tax rates, and measures of labour market skill mismatch. Similarly for firms, $z_p$ includes measures of the firm’s competitive environment or market power, indirect taxes, and non-labour input costs including oil prices. The unemployment term in the firm’s desired markup equation is simply a measure of output using Okun’s law.

The cost to firms of inflation in this model is represented by $\omega_6 \Delta p$. In the standard model $\omega_6 = 0$ and inflation imposes no costs on the firm. In the more general model where $\omega_6 > 0$, the desired markup of firms is lower with higher inflation.

These two equations represent the desired claims of firms and labour on the real output of the economy. We can eliminate the level of unemployment from (1) and (2) and assuming that $\Delta U = 0$ and $p = p^e$ in the long-run, the long-run relationship between the markup and inflation can be written:

$$\bar{p} - \bar{w} = (\gamma_1 + \omega_1)^{-1} \left\{ \omega_0 \gamma_1 - \omega_1 \gamma_0 - \omega_1 \gamma_3 z_w + \omega_3 \gamma_1 z_p - (\omega_5 \gamma_1 + \omega_6 \gamma_4) \phi - \omega_6 \gamma_1 \Delta p \right\}$$ (3)

where the bar over the variable $p - w$ indicates its long-run value conditional upon the long-run value of inflation $\Delta p$. Finally, if we assume that firms price independently of demand.

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7 For a detailed discussion of the theory underlying these shift variables see Layard et al. (1991) or for a simple taxonomy of explanations see Coulton and Cromb (1994).

8 Notwithstanding the use of the term ‘long-run’ in the Engle - Granger sense in relation to the empirical analysis, the term is used in this theoretical section in an economic modelling sense that focuses on the passage of time. Under this interpretation, therefore, all I(0) real variables have obtained their long-run values (i.e. $\Delta U = 0$ and $p - p^e = 0$ in this model) and the long-run relationship between the markup and inflation holds.
( \omega_i = 0 ) and income shares are independent of the level of productivity in the long-run then the long-run markup, \( \overline{mu} \), collapses to:

\[
\overline{mu} = p - w + \phi = \omega_0 + \omega_1 \hat{z}_p - \omega_6 \Delta \hat{p}.
\] (4)

Equation (4) shows that under the above restrictions the long-run relationship between the markup and inflation is independent of wage pressure shocks \( z_w \) but dependent on the competitive environment captured by \( z_p \). With \( \omega_6 = 0 \) as in the standard model, the markup is independent of inflation in the long-run. In the general model, \( \omega_6 > 0 \) and the markup in the long-run is negatively related with the rate of inflation.10

2.1 The Statistical Properties of Inflation

Equation (4) gives useful insight into the possible integration properties of the data. Abstracting for the moment from structural breaks, it may be seen from (4) that the order of integration of the markup must match the order of integration of inflation assuming that the

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9 Normal cost markup and kinked demand curve models suggest the price level is largely insensitive to demand fluctuations. See Hall and Hitch (1939), Sweezy (1939), Layard et al. (1991), Carlin and Soskice (1990), Coutts, Godley and Nordhaus (1978), Tobin (1972), Bils (1987). For labour and firms to maintain stable income shares in the long-run and for these shares not to continually rise or fall with trend productivity, the coefficient on productivity in the long-run markup equation \((\omega_5 \gamma_i + \omega_1 \gamma_5)/(\gamma_1 + \omega_1)\)

10 This specification is not strictly true for it implies that the markup approaches zero as inflation tends to an infinite rate. However, over a small range of inflation the log linear relationship estimated in this paper may be a good approximation. Russell (1998) deals more generally with this issue by specifying the cost of inflation in the form \( \omega_6 \left[ \Delta \hat{p} / (\Delta \hat{p} + \sigma) \right] \) where \( \sigma \) is trend productivity growth.

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The choice between options (a) and (d) as the way to proceed with the empirical investigation can be made on practical and conceptual levels. Shifts in the mean rate of inflation over the

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11 The implications for the markup of structural breaks in inflation also apply to the exogenous unmodelled processes of competition. However, in order to be succinct we consider only the cases that relates to the structural breaks in inflation.
period reflect changes in the target rates of inflation by the monetary authorities. Therefore, understanding the ‘true’ statistical process of inflation depends, in part, on how we characterise the behaviour of the monetary authorities in response to inflation shocks and the nature of the shocks themselves.

Focussing first on option (a) in Table 1. If the authorities hold a series of unique inflation targets that are independent of the inflation shocks then inflation will follow a stationary process with shifting mean. If one were able to identify the timing of every shift in the target rate of inflation then a dummy variable could be introduced to capture each shift in the target. The maximum number of dummies would be one less than the number of observations in the sample investigated. In practice one would introduce enough dummies to ‘render’ inflation a stationary series. Given the well-known low power of unit root tests and tests of breaks in series, it is likely the series would be ‘rendered’ stationary with the inclusion of a small number of dummies. However, in practice this approach is unsatisfactory as it is unlikely that the number of dummies would be identical to the ‘true’ number of shifts in the target rates of inflation by the monetary authorities. On a conceptual level this approach is also unsatisfactory due to the lack of economic interpretation of the dummies and the model structure it entails.

An alternative way to proceed is to focus on option (d) and characterise the monetary authorities as at least partially adjusting their inflation target in response to the inflation shocks in each period. In this case, inflation is likely to follow a non-stationary statistical process. Given that the Australian monetary authorities have responded to both

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12 The term ‘target’ is used loosely and does not imply that the monetary authorities explicitly state a target rate of inflation. Instead, the ‘target’ refers to the revealed preference of the authorities following shocks to the ‘general’ level of inflation. If the authorities were not satisfied with the ‘general’ level of inflation, they would have adjusted monetary policy to achieve a ‘general’ rate of inflation with which they were satisfied.
unemployment and inflation when setting monetary policy over most, if not all, of the period in question, the second characterisation of the monetary authorities appears the most relevant.

While acknowledging the possibility that the ‘true’ statistical process of inflation may be stationary about a frequently (but unknown) shifting mean, this paper proceeds to investigate the relationship between inflation and the markup by allowing for the possibility that either or both series are integrated.

3 ESTIMATING THE LONG-RUN RELATIONSHIP

We propose an imperfect competition model of the markup based on (4) where firms desire in the long-run a constant markup of price on unit costs net of the cost of inflation\(^\text{13}\) Short-run deviations in the markup are the result of shocks and the economic cycle\(^\text{14}\) For an open economy, costs include those for labour and imports. Assuming that the competitive environment is unchanged, the long-run markup equation analogous to (4) can be written:15

\[
mu = p - \delta u_c - (1 - \delta) p_m = q - \lambda \Delta p
\]  

\(^{13}\) Implicitly the long-run markup equation assumes that the unit cost of capital is a component of the markup and that the short-run cyclical effects of real interest rates on the cost of capital can be ignored in estimating the long-run coefficients if real interest rates are I(0).

\(^{14}\) The short-run impact of the economic cycle on the markup will depend in part on the specification of the underlying production function (see Rotemberg and Woodford (1999)). However, the cyclical variations in the markup could be taken as essentially short-run influences and conceptually will not affect the long-run estimates or their interpretation.

\(^{15}\) The form of the long-run price equation is a dynamic generalisation of that estimated in de Brouwer and Ericsson (1998).
where $ulc$ is unit labour costs, $pm$ is the price per unit of imports, and $q = \omega_0 + \omega_3 z_p$ is the ‘gross’ markup. The inflation cost coefficient $\lambda$ is greater than zero and $0 \leq \delta \leq 1$. The coefficients $\delta$ and $1 - \delta$ are the long-run price elasticities with respect to unit labour costs and import prices respectively. Long-run homogeneity is imposed with these coefficients summing to one. That is, for a given rate of inflation, an increase in either unit labour costs or import prices will see prices fully adjust in the long-run to leave the markup unchanged. Equation (5) collapses to the standard imperfect competition markup model of prices when $\lambda = 0$. In the more general case when $\lambda \neq 0$, inflation imposes costs on firms and the markup net of inflation costs is reduced.

The remainder of this section seeks to estimate the long-run markup equation using quarterly Australian data allowing for the possibility that the levels of prices and costs are I(2). The heart of the empirical analysis of the paper is to model three core variables as an I(2) system. The core variables are the logarithms of prices, unit labour costs and unit import prices, denoted $p_t$, $ulc_t$, and $pm_t$, and are defined on a national accounts basis as the private consumption deflator, the Australian Treasury’s measure of non-farm unit labour costs and

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16 To obtain the equation in this form with import prices included explicitly (rather than a component of $z_p$) requires us to modify (1) slightly so that the firm’s desired markup, $mu$, is $p - \delta w - (1 - \delta) pm$ and $z_p$ no longer includes import prices. It can then be shown that the $mu$ is a linear function of the inverse of the real wage $w - p$ and the reductions (3) and (4) can be made to give (5). Finally, noting that $mu = (p - w) - (1 - \delta) (pm - w)$ and that $pm - w$ is a linear function of the same variables as those determining $p - w$, (5) is therefore justified.

17 Note that without linear homogeneity $q$ does not represent the ‘gross’ markup of prices on costs.

18 For a detailed analysis of the estimation of I(2) systems the reader is referred to Haldrup (1998), Johansen (1995a, b) and Paruolo (1996). Engsted and Haldrup (1999) and Juselius (1998) also provide useful empirical applications.
the imports implicit price deflator respectively. The analysis is conditioned on a number of predetermined variables that are assumed to be integrated of order 0 and are described in due course. Nevertheless, the ‘important’ assumption is that the three core variables in the system are integrated of order 2.19

This ‘important’ assumption poses some quite interesting and econometrically tricky modelling challenges. However, working in I(2) space allows us to consider the scenario where the core variables cointegrate as the markup of price on labour and import costs, \( m_u \), and where the markup is I(1). In this scenario, taking a linear combination of the core variables leads to a reduction in the order of integration by only 1. In addition there are two other interesting possibilities for cointegration. First, the I(2) core variables may cointegrate directly to a stationary variable. That is, the markup, \( m_u \), is I(0). Second, if the markup is I(1), a linear combination of \( m_u \) with the differences of the core variables may lead to an I(0) variable. The second possibility is referred to in the I(2) literature as polynomial cointegration or multicointegration.20

19 The data used in the empirical analysis is an updated version of that used in Cockerell and Russell (1995) with 3 extra quarterly observations. The variables were tested for unit roots using PT and DF-GLS tests from Elliott, Rothenberg and Stock (1996). These univariate tests show that the predetermined series are best described as I(0) processes. The price level is I(2) while unit labour costs and import prices may be I(1). However, the evidence from the analysis of the systems reported in the paper below indicates the core variables are best described as I(2) processes and we proceed on this assumption. This assumption is strongly supported by univariate unit root tests of the relative prices, given by the markup of prices on unit labour costs and the markup of prices on unit import costs, which are shown to be clearly I(1) which could only be achieved if all the core variables are I(2) given the clear support for the hypothesis that prices are I(2). The results of the univariate tests are available from the authors.

The second possibility is of particular interest and allows us to investigate directly our main theoretical proposition that there is a relationship between the markup, $m_u$, and inflation in the long-run, thereby emphasising the empirical and theoretical relevance of the $I(2)$ analysis. The polynomially cointegrating relationship is interpreted as the long-run relationship between inflation and the markup. The $I(2)$ framework, therefore, enables us to estimate relationships not allowed for by the more standard $I(1)$ framework.

Consider a second-order vector autoregression of the core variables, $x_t$, of dimension $n \times 1$:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + \Phi D_t + \mu + \epsilon_t$$  \hspace{1cm} (6a)

where $\mu$ is a constant term that may be unrestricted and $D_t$ is a vector of predetermined variables on which the empirical analysis is conditioned. Equation (6a) may be written:

$$\Delta^2 x_t = -\Gamma \Delta x_{t-1} + \Pi x_{t-1} + \Phi D_t + \mu + \epsilon_t$$  \hspace{1cm} (6b)

where $\Pi = \Pi_1 + \Pi_2 - I_n$ and $\Gamma = I_n + \Pi_2$.

The predetermined variables may or may not enter the cointegrating space depending on the restrictions imposed during estimation of the system and may include seasonal or intervention step or spike dummies. The variable $\epsilon_t$ is a $n$-dimensional vector of errors assumed to be Gaussian with mean vector $0$ and variance matrix $\Sigma$. The parameters $(\Pi_1, \Pi_2, \Pi, \Phi, \mu, \Sigma)$ are assumed to be variation free. The VECM has been restricted to two lags without any loss of generality since one can consider extensions to any order of the lag structure without altering any of the basic arguments.
In our specific empirical model, \( n = 3 \) and \( x_t \) is the vector of core variables defined earlier.

The predetermined variables, \( D_t \), are set out in Table 2.

<table>
<thead>
<tr>
<th>( \Delta ) Unemployment</th>
<th>First difference of the logarithm of the unemployment rate lagged one quarter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta ) Tax</td>
<td>Calculated as the first difference of the log of the variable ((1 + \text{tax}/100)) where tax is defined as non-farm indirect tax plus subsidies as a proportion of non-farm GDP at factor cost.</td>
</tr>
<tr>
<td>( \Delta ) Petrol Prices</td>
<td>First difference of the log of petrol prices.</td>
</tr>
<tr>
<td>Strikes</td>
<td>Strikes measured as working days lost as a proportion of employed full and part-time persons. The variable is adjusted for a shift in the mean in the March quarter of 1983.</td>
</tr>
</tbody>
</table>

Notes: See Cockerell and Russell (1995) for more details concerning the series and their sources.

For a system to be I(2) requires not only that the long-run matrix \( \Pi = \alpha \beta' \) is of reduced rank but that \( \alpha'_\perp \Gamma \beta'_\perp \), is also of reduced rank \( s \geq 2 \). This latter matrix is, therefore, expressible as \( \alpha'_\perp \Gamma \beta'_\perp = \xi \eta' \) where \( \xi \) and \( \eta \) are matrices of order \((n - r) \times s\) with \( s < n - r \) and the matrices indexed by \( \perp \) represent the orthogonal complements of the corresponding matrices and are each of rank \( p - r \).

Having met this requirement the I(2) system can be decomposed into I(0), I(1) and I(2) directions with dimensions \( r \), \( s \) and \( n - r - s \) respectively. Moreover, the \( r \) cointegrating relationships are further decomposable into \( r_0 \) directly cointegrating relationships where the levels of the I(2) variables cointegrate directly to an I(0) variable and \( r_1 \) polynomially cointegrating relationships where the levels cointegrate with the differences of the levels to give an I(0) variable. Thus:
\[ \beta' x_t \sim I(0) \text{ where } \beta_0 \text{ is } n \times r_0 \text{ with rank } r_0; \tag{7a} \]

\[ \beta_1' x_t + \kappa' \Delta x_t \sim I(0) \text{ where } \beta_1 \text{ and } \kappa \text{ are } n \times r_1; \tag{7b} \]

\[ r_0 + r_1 = r. \tag{7c} \]

It is possible of course for either \( r_0 \), \( r_1 \) or both to be zero. In general, however, the algebra of the processes dictates that the number of polynomially cointegrating relationships equals the number of I(2) common trends in the system. Therefore; \( r = r_0 + r_1 \) and \( r_1 = n - r - s = -s_2 \). If \( s_2 \) equals zero, or equivalently \( n - r = s \), the I(2) system collapses to the I(1) case.

### 3.1 Reduction from I(2) to I(1): Estimating the I(2) System

We proceed now to presenting the results of estimating our I(2) system described in the previous section. When the system is estimated without any restrictions except for the exclusion of quadratic trends, Table 3 shows that \( r = 1 \) and \( n - r - s = 1 \).\[22\] Since the number of I(2) trends in the model equals the number of polynomially cointegrating relationships, the arithmetic implies that the only cointegrating relationship detected above must be of the polynomially cointegrating variety and confirms the empirical relevance of option (d) in

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21 Technically we need to check further rank condition(s) to rule out the possibility that the system is I(3).

Since both statistically and economically this might be regarded as an extremely unlikely case we assume that the conditions which rule out I(3) behaviour hold in our analysis.

22 In common with much of the existing empirical analysis of I(2) processes, our main restrictions on the nuisance parameters is that the constant is restricted in such a way that quadratic trends are disallowed in the data and there is no trend in the cointegrating space. See Engsted and Haldrup (1998), Haldrup (1998), Juselius (1998).
Thus $p_t$, $ulc_t$, and $pm_t$ cointegrate from I(2) to I(1) and this must further cointegrate with the first differences of the core variables to provide the so-called ‘dynamic’ error correction (ECM) term.

Table 3: Estimating $r$ and $s$ in the I(2) System

<table>
<thead>
<tr>
<th>$n - r$</th>
<th>$r$</th>
<th>$Q(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>229.06</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>84.66</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>18.40</td>
</tr>
<tr>
<td>$n - r - s$</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: Statistics are computed with 2 lags of the core variables. The estimation sample is March 1972 to June 1995 with 94 observations and 83 degrees of freedom. The shaded cell indicates acceptance at the 10 per cent level of significance. The 90 % and 95 % critical values for the case of no pre-determined variables from Paruolo (1996) are reported in the table below. The 95 % critical values are in italics.

Critical Values for the Joint Trace Test $Q(s, r)$

<table>
<thead>
<tr>
<th>$n - r$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$n - r - s$</td>
<td>3</td>
</tr>
</tbody>
</table>

The results presented in Table 3 provide formal justification of the existence of I(2) trends in the data. However, given the doubts pertaining to the use of asymptotic critical values, in particular the sensitivity of these values to the inclusion of nuisance parameters and predetermined variables, we undertook a sensitivity analysis to determine the robustness of
our findings. The cointegration results are essentially the same if the analysis is repeated with all the predetermined variables excluded. Further evidence may be provided graphically by looking at the cointegrating combination $\beta'_i x$, which looks more stationary if one controls for the differences of $x_t$ and this may be verified formally by testing for the integration properties of the ‘static’ versus the ‘dynamic’ error correction term.

We also note that the five largest roots in modulus of the characteristic polynomial are 1.0000, 1.0000, 0.9923, 0.4760, 0.2447 and 0.0400. This is exactly as we would expect under the maintained null hypothesis of one cointegrating vector ($r=1$), one I(1) common trend ($s=1$) and one I(2) common trend ($n-r-s=1$). This is a remarkably robust finding and not altered by numerous respecifications of the model to allow for various combinations of predetermined variables and restrictions on nuisance parameters and lags in the core variables. Therefore, we proceed under the maintained assumption of one I(1) trend and one I(2) trend.

The normalised cointegrating vector $\beta'_i$ and system diagnostics are reported in Table 4. The restriction of linear homogeneity is accepted with a p-value of 0.18. Therefore, $\beta'_i$ with the linear homogeneity restriction imposed represents the markup of price on labour and import costs. Consequently, the markup in the I(2) system is defined as:

$$mu_t = p_t - 0.868 uc_t - 0.132 pm_t .$$

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23 Examples of ‘nuisance’ parameters may include trends and constants. The indices are also sensitive to whether the ‘nuisance’ parameters are unrestricted or restricted to the cointegrating space. Rahbek et al. (1999) propose methods of making inference not depend on the trend.
Furthermore, the polynomially cointegrating relationship under this restriction given by the I(2) analysis is:

\[ mu_t + (2.284, 2.252, 2.490)^{'} \Delta x_t. \]

In the notation adopted in (7b), \( mu_t \equiv \beta_1^{'} x_t \) and \( (2.284, 2.252, 2.490)^{'} \equiv \kappa^{'} \).

### Table 4: Normalised Cointegrating Vector \( \beta_1^{'} \)

<table>
<thead>
<tr>
<th></th>
<th>( p_t )</th>
<th>( ulc_t )</th>
<th>( pm_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestricted</td>
<td>1</td>
<td>-0.818</td>
<td>-0.154</td>
</tr>
<tr>
<td>Linear Homogeneity Imposed</td>
<td>1</td>
<td>-0.868</td>
<td>-0.132</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.078)</td>
<td>(0.078)</td>
</tr>
</tbody>
</table>

Notes: Likelihood ratio tests of (a) linear homogeneity is accepted \( \chi^2 = 1.79 \), p-value = 0.18; (b) exclusion of quadratic trend is accepted, \( \chi^2 = 0.79 \), p-value = 0.38; and (c) joint test of linear homogeneity and no quadratic trend is accepted, \( \chi^2 = 2.57 \), p-value = 0.28. Standard errors reported in brackets.

### System Diagnostics for the Restricted Model

Tests for Serial Correlation

- Ljung-Box (23) \( \chi^2 (195) = 194.69 \), p-value = 0.49
- LM(1) \( \chi^2 (9) = 11.24 \), p-value = 0.26
- LM(4) \( \chi^2 (9) = 9.12 \), p-value = 0.43

Test for Normality: Doornik-Hansen Test for normality: \( \chi^2 (6) = 12.71 \), p-value = 0.05

Since in the steady state \( \Delta p_t = \Delta ulc_t = \Delta pm_t \), this implies that the long-run relationship between the markup and the ‘general’ rate of inflation is \( mu_t = -1.433 - 7.026 \Delta p_t \), where the dynamics and, more importantly, the impact of the predetermined variables are ignored in calculating the long-run relationship. This may however be regarded as an approximation since over the period studied the core variables may not all grow at the same rate.
3.2 Estimating the I(1) System

In order, therefore, to estimate the long-run relationship between the markup and price inflation alone, which is the primary area of our concern, we estimate an I(1) system given by

\[
\begin{pmatrix}
\text{mulc} \\
\text{rer} \\
\Delta p
\end{pmatrix}
= \begin{pmatrix}
p_i - ulc_i \\
p_i - pm_i \\
\Delta p_i
\end{pmatrix}.
\]

This may be justified as follows. Estimation of the I(2) system showed that the I(0) and I(1) directions of the data are given by the vectors

\[
\beta'_1 = \begin{pmatrix} 132.0, 868.0, 11 \end{pmatrix}^\prime \\
\beta'_2 = \begin{pmatrix} 1, 659.0, 440.02 \end{pmatrix}^\prime
\]

when the homogeneity restriction is imposed and accepted. The restriction on the vector \( \beta'_3 = (1,1,1) \) providing the I(2) direction is also accepted. These vectors are orthogonal to each other and, in particular, the first two vectors lie in the space orthogonal to \( \beta'_3 \). A basis for this space is given by the matrix

\[
H = \begin{pmatrix}
1 & 1 & 0 \\
-1 & 0 & 1 \\
0 & -1 & 1
\end{pmatrix}.
\]

Thus

\[
\begin{pmatrix}
H'x_i \\
\beta'_1 \\
a'x_i
\end{pmatrix}
\]

provides the transformation to I(1) which keeps all the cointegrating and polynomially cointegrating information. Hence if we take \( a \) to be \( (1,0,0) \), then the trivariate system given by

\[
\begin{pmatrix}
p_i - ulc_i \\
p_i - pm_i \\
\Delta p_i
\end{pmatrix}
\]

that the I(2) common trend in the data enters with equal weight in the three components of the I(2) system. The transformation above implies that the markup of price on unit labour costs, \( \text{mulc} \), and the ‘real exchange rate’, \( \text{rer} \), are both I(1) variables. Estimating this trivariate,

\[24\] The term \( \text{rer} \) may loosely be referred to as the ‘real exchange rate’ given the similarity with the relative price of traded and non-traded goods used by Swan (1963) in his classic article.
system and transforming \( mulc \) and \( rer \) linearly gives the long-run relationship between the markup, \( mu \), and inflation, \( \Delta p \)  

Turning first to the number of cointegrating relationships in the I(1) system. The existence of a single cointegrating relationship is clearly established in Table 6. Further evidence of a single cointegrating relationship is provided by the first five roots in modulus of the companion matrix being given by 1.0000, 1.0000, 0.6541, 0.4197, 0.0431. 

<table>
<thead>
<tr>
<th>Null Hypothesis ( H_o : r = )</th>
<th>Eigenvalues</th>
<th>Estimated Trace Statistic ( Q(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2502</td>
<td>34.14 (26.70)</td>
</tr>
<tr>
<td>1</td>
<td>0.0669</td>
<td>7.07 (13.31)</td>
</tr>
<tr>
<td>2</td>
<td>0.0060</td>
<td>0.56 (2.71)</td>
</tr>
</tbody>
</table>

Notes: Statistics are computed with 2 lags of the core variables. The estimation sample is March 1972 to June 1995 with 94 observations and 83 degrees of freedom. The shaded cell indicates acceptance at the 10 per cent level of significance. Critical values shown in curly brackets \{\} are from Table 15.3 of Johansen (1995b). 

Estimates of the I(1) system are reported in Table 6. The ECM calculated from the cointegrating matrix \( \beta' \) is given by \( ECM_i = mulc_i + 0.110 rer_i + 7.993 \Delta p_i \) from which the implied estimated markup is \( mu_i = p_i - 0.901 ulc_i - 0.099 pm_i \) and the long-run relationship between the markup and inflation is \( \overline{mu} = -1.487 - 7.201 \overline{\Delta p} \). 

---

25 We are very grateful to Hans Christian Kongsted for this analysis. 

26 As with estimating the I(2) system, re-estimating the system without any predetermined variables and a range of lags in the core variables does not alter the main finding of the long-run relationship between the markup and inflation.
Table 6: I(1) System Analysis of Inflation and the Markup  
March 1972 – June 1995

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>lag</th>
<th>Markup Equation</th>
<th>‘Real Exchange Rate’ Equation</th>
<th>Price Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Δ Markup</td>
<td>Δ RER</td>
<td>Δ Inflation</td>
</tr>
<tr>
<td><strong>Loading Matrix α</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error Correction Term</td>
<td>1</td>
<td>-0.106</td>
<td>0.064</td>
<td>-0.051</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.0)</td>
<td>(1.1)</td>
<td>(-4.3)</td>
</tr>
<tr>
<td><strong>Short-run Matrices Γi</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ Markup</td>
<td>1</td>
<td>-0.069</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.2)</td>
<td>(0.5)</td>
<td>(-2.2)</td>
</tr>
<tr>
<td>Δ Real Exchange Rate</td>
<td>1</td>
<td>0.036</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.6)</td>
<td>(0.4)</td>
<td>(-0.3)</td>
</tr>
<tr>
<td>Δ Inflation</td>
<td>1</td>
<td>-0.267</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.634)</td>
<td>(0.6)</td>
<td>(-1.9)</td>
</tr>
<tr>
<td><strong>Predetermined Variables D_t</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>-0.175</td>
<td>0.111</td>
<td>-0.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.0)</td>
<td>(1.1)</td>
<td>(-4.3)</td>
</tr>
<tr>
<td>Δ Unemployment</td>
<td>1</td>
<td>0.040</td>
<td>-0.023</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.8)</td>
<td>(-0.6)</td>
<td>(-3.9)</td>
</tr>
<tr>
<td>Δ Tax</td>
<td>0</td>
<td>0.750</td>
<td>-1.665</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.1)</td>
<td>(-2.8)</td>
<td>(2.1)</td>
</tr>
<tr>
<td>Δ Petrol Prices</td>
<td>0</td>
<td>0.008</td>
<td>-0.158</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.3)</td>
<td>(-3.1)</td>
<td>(4.0)</td>
</tr>
<tr>
<td>Strikes</td>
<td>1</td>
<td>-0.064</td>
<td>-0.150</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.4)</td>
<td>(-3.4)</td>
<td>(3.2)</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td></td>
<td>0.31</td>
<td>0.25</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Notes: Reported in brackets are t-statistics. \( ECM_t \equiv mulc_t + 0.110 \, rer_t + 7.993 \, \Delta p_t \).

**System Diagnostics**

**Tests for Serial Correlation**

- Ljung-Box (23) \( \chi^2 (195) = 188.82 \), p-value = 0.61
- LM(1) \( \chi^2 (9) = 12.86 \), p-value = 0.17
- LM(4) \( \chi^2 (9) = 8.41 \), p-value = 0.49

**Test for Normality** Doornik-Hansen Test for normality: \( \chi^2 (6) = 13.27 \), p-value = 0.04
Excluding the insignificant variables in Table 6 on a 5 per cent t-criterion the final form of the inflation and markup equations in the system can be represented as:

\[ \Delta \text{mulc}_t = \mu_1 - \alpha_1 (\text{mulc} + \theta_1 \text{rer} + \theta_2 \Delta p)_{t-1} + \phi' D_t + \epsilon_{1t} \] (8a)

\[ \Delta \text{rer}_t = \phi_2' D_t + \epsilon_{2t} \] (8b)

\[ \Delta^2 p_t = \mu_3 - \alpha_3 (\text{mulc} + \theta_1 \text{rer} + \theta_2 \Delta p)_{t-1} + \gamma_1 \Delta^2 p_{t-1} + \delta_1 \Delta \text{mu}_{t-1} + \phi_3' D_t + \epsilon_{3t} \] (8c)

The estimated system as represented by (8a), (8b) and (8c) describes an economy where disequilibrium from the long-run relationship is corrected by changes in the rate of inflation and the markup of prices over unit labour costs. This implies that adjustment occurs in the goods and labour markets as well as by the monetary authorities. In contrast, the ‘real exchange rate’ is weakly exogenous to the remainder of the system. Adjustment to the long-run relationship is therefore not due to changes in import prices.

Table 7 shows the similarity between the estimates from the I(2) and I(1) systems of the long-run coefficients on the core cost variables. The estimates imply that for a given rate of inflation, a 10 per cent increase in unit labour costs with no change in the level of import prices will lead to around a 9 per cent increase in prices in the long-run leaving the markup on total costs unchanged. Alternatively, a simultaneous 10 per cent increase in labour and import costs will see prices increase by 10 per cent in the long-run.

<table>
<thead>
<tr>
<th>Method of Estimation</th>
<th>Unit Labour Costs</th>
<th>Import Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>System I(2)</td>
<td>- 0.868</td>
<td>- 0.132</td>
</tr>
<tr>
<td>System I(1)</td>
<td>- 0.901</td>
<td>- 0.099</td>
</tr>
</tbody>
</table>
The presence of the long-run, or cointegrating, relationship between inflation and the markup subtly alters the interpretation of linear homogeneity in this model of prices. In standard models where inflation and the markup are not related in the long-run, an increase in costs will be fully reflected in higher prices and the markup will be unchanged in the long-run irrespective of any change in the rate of inflation. With inflation and the markup negatively related in the long-run, higher costs are only fully reflected in higher prices in the long-run leaving the markup unchanged if the rate of inflation in the long-run also remains unchanged. An increase in costs that is associated with an increase in inflation in the long-run will not be fully reflected in higher prices as the markup of prices on costs falls with the higher inflation.

**Graph 4: The Estimated Markup**

![Graph 4: The Estimated Markup](image)

Notes: The markup is calculated as: $mu_t = p - 0.901ulc_t - 0.099 pm_t$

The estimated markup from the I(1) analysis is shown in Graph 4. In a ‘standard’ empirical model of prices where it is assumed that inflation is stationary then the markup, which is the
‘static’ error correction term, should also be stationary. From the graph and from the detailed systems analysis above it is clear that the markup is not stationary. However, in the preferred specification where it is shown that inflation is I(1) and cointegrates with the markup the ‘dynamic’ error correction term is a linear combination of inflation and the markup and this is shown in Graph 5. Graphically it appears that the error correction term is stationary and this can be verified by formal testing.

**Graph 5: The Error Correction Term of the I(1) Analysis**

![Graph 5](image)

Notes: The ECM is calculated as: \[ ECM_t = \text{mulc}_t + 0.110 \times \text{rer}_t + 7.993 \times \Delta p_t \]

---

27 This can also be confirmed by univariate unit root tests.

28 This graphical analysis of the ‘static’ and ‘dynamic’ ECMs in Graphs 4 and 5 replicates almost exactly the graphs derived from the I(2) analysis when the path of the cointegrating vector among the levels of the core variables is plotted without and with a dynamic adjustment from the differenced core variables.
4 INFLATION AND THE MARKUP IN THE LONG-RUN

Table 8 sets out the two system estimates of the long-run relationship between the markup and inflation. The last column of the table provides the respective estimates of the fall in the markup that is associated with a 1 percentage point increase in annual inflation. Both estimates indicate that the markup is around 1 ¾ per cent lower in the long-run if inflation is 1 percentage point higher.

Table 8: Long-Run Relationship Between Inflation and the Markup

<table>
<thead>
<tr>
<th>Method of Estimation</th>
<th>Long-Run Relationship</th>
<th>Decrease in the Markup Associated with a 1 Percentage Point Increase In Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>System I(2)</td>
<td>( \mu u = -1.433 - 7.026 \Delta p )</td>
<td>1.8 %</td>
</tr>
<tr>
<td>System I(1)</td>
<td>( \mu u = -1.487 - 7.201 \Delta p )</td>
<td>1.8 %</td>
</tr>
</tbody>
</table>

Notes: A percentage point increase in annual inflation is equivalent to an increase in \( \Delta p \) of 0.25 per quarter.

The I(1) system estimate of the long-run relationship between the markup and inflation is shown as the solid line, \( LR \), in Graph 6.\(^{29}\) Also shown as a scatter plot are combinations of the estimated markup and actual annualised quarterly inflation. The negative relationship between inflation and the markup is evident from the graph.

One interpretation of the negative relationship is that it is a short-run relationship and generated by a combination of supply shocks and slow price and wage adjustment. Consequently the large positive wage shocks in the early 1970s led to a fall in the markup and an increase in inflation. This interpretation should be questioned on conceptual and empirical levels. Firms were free to set prices during nearly all of the sample examined. If the standard model with slow adjustment was sufficient to explain the relationship found in the data then

\(^{29}\) The long-run relationship in Graph 6 assumes that the predetermined variables are at their long-run or mean values and that the second difference of inflation and the first difference of \( mule \), and \( rer \), are zero.
the markup should not have persisted at low levels for 10-15 years following the shocks. If the markup had recovered rapidly then no relationship with inflation would have been observed in the data.

**Graph 6: The Markup and Inflation**

Markup defined as: \( m_{t} = p - 0.901ulc_{t} - 0.099 pm_{t} \) with 100 equals the period average.

Long-run relationship shown as solid line, LR, and defined as: \( \overline{mu_{t}} = -1.487 - 7.201 \Delta p_{t} \).

**Inflationary Periods**

(Solid symbols in the graph represent the average markup and inflation for the period.)

<table>
<thead>
<tr>
<th>Period</th>
<th>Symbol</th>
<th>Average Markup</th>
<th>Average Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 72 – March 73</td>
<td>Diamonds</td>
<td>102.6</td>
<td>5.6 %</td>
</tr>
<tr>
<td>June 73 – March 83</td>
<td>Circles</td>
<td>94.3</td>
<td>15.0 %</td>
</tr>
<tr>
<td>June 83 – December 90</td>
<td>Triangles</td>
<td>102.5</td>
<td>6.9 %</td>
</tr>
<tr>
<td>March 91 – June 95</td>
<td>Squares</td>
<td>106.9</td>
<td>2.0 %</td>
</tr>
</tbody>
</table>
On an empirical level if the interpretation was correct then the observations should be spread evenly along the entire long-run curve. The scatter plot in the graph separates the realisations of the markup and inflation into four inflationary periods and these are indicated by different symbols. Shown as solid black symbols on the graph are the combinations of the average markup and the average rate of inflation for each of the inflationary periods.

What we see in this graph is the realisations of the markup and inflation are confined to different sections of the curve depending on the general level of inflation. The longest inflationary period is shown as circles for the period June 1973 - March 1983. During this period of 10 years the markup never returns to its pre June 1973 level shown by the diamond symbols. This period of persistent ‘high’ inflation that averages 15.0 per cent is associated with persistently ‘low’ markups. Moreover, the two periods with similar average markups (the triangles and diamonds) are associated with similar average rates of inflation even though the periods they represent are separated by 10 intervening years. Finally, the period with the lowest average inflation has the highest average markup. It is clear that the realisations of the markup and inflation are not spread evenly along the long-run curve and therefore we reject the ‘supply shock / slow adjustment’ explanation of the relationship between inflation and the markup.

5 CONCLUSION

This paper set out to investigate the proposition that inflation and the markup may be negatively related in the long-run. It was argued that this proposition imposed a definite modelling strategy on the empirical investigation. It was found that the levels of prices and costs are best characterised as I(2) statistical process which was accommodated by estimating an I(2) system using techniques developed by Johansen (1995a, b).
We find that a linear combination of the I(2) levels of prices and costs cointegrate to the markup which is I(1). It is also found that a linear combination of the markup and inflation also cointegrate. The analysis, therefore, supports the proposition by identifying a long-run relationship where higher inflation is associated with a lower markup and *vice versa*.

The lower markup is interpreted within the imperfect competition model employed in this paper as the cost to firms of higher inflation. Importantly, the fall in the markup associated with an increase in inflation appears to be economically significant with a 1 percentage point increase in inflation associated with nearly a 1 ¾ percent decrease in the markup.
REFERENCES


Simon, J. (1999). Markups and Inflation, Department of Economics Mimeo, MIT.


