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CYCLICAL UNCERTAINTY AND PHYSICAL INVESTMENT DECISIONS

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Abstract

The paper studies the interaction between cyclical uncertainty and investment in a stochastic real option framework where demand shifts stochastically between two different states, each with different rates of drift and volatility. In our setting the shifts are governed by a two-state Markov switching model with constant transition probabilities. The magnitude of the link between cyclical uncertainty and investment is quantified using simulations of the model. The chief implication of the model is that recessions are important catalysts for waiting. In other words, our model shows that macroeconomic risk acts as a deterrent to present investments.

Keywords: Business Cycles, Real Options, Investment, Markov Switching, Tobin's *q*, Uncertainty
JEL-Classification: D81, D92, E32

1. Introduction

Corporate investment opportunities may be represented as a set of real options to acquire physical capital. As argued by Dixit and Pindyck (1994, p. 3) “most investment decisions share three important characteristics, investment irreversibility, uncertainty and the ability to choose the optimal timing of investment”. Managers are aware that investment is an opportunity and not an obligation. This causes them to behave as if they own option-rights. Moreover they know that, due to partial irreversibility, the exercise of their option rights reduces flexibility.[1] As a result, the optimal time to kill the option is well after the point at which expected discounted future cash flow equals the cost of investment and firms may prefer a “wait-and-see” attitude even when they are risk-neutral. In volatile environments in which new information is arriving, the best tactic may be to “keep options open” and await new information rather than commit to an investment today.[2] This appealing modelling approach can thus enrich theory by clarifying issues concerning the “when” of investments.

In the real options literature it is widely assumed that the present values of cash flows generated by the capital stock are uncertain and that their evolution can be described by stochastic processes. Consequently, the literature on investment under uncertainty uses options-based models and option pricing techniques to study investment decisions. An appropriate identification of the optimal exercise strategies for real options plays a crucial part in the maximization of a firm´s market value. So far, however, the real options literature provides relatively little insight into the impact of business cycles on the investment decisions of firms. Most of the time, authors assume that the entire (exogenous) uncertainty in the economy can be described by a geometric Brownian motion process which is unsystematic across firms.[3] It is, however, much more realistic to model an economy which is subject to macroeconomic shocks and business cycle fluctuations. The impact of business cycles on investment activity is well-documented. For example, Stock and Watson (1996, 1998) find that the U.S. investment in equipment and non-residential structures is procyclical. This link between investment and business cycles remain under-studied in the real options literature. Consequently, our objective in this paper is to enrich the stream of literature on real options by incorporating the impact

[1] Graham and Harvey (2001) show that about 26 percent of the companies involved in the study always or almost always incorporate real options when evaluating investment projects.

[2] Note, however, that some investments may not fall into this category. For example, a firm cannot wait in a “winner take all (or most)” competitive situation. The literature on real options has been developing rapidly over the past decade. Reviews of this burgeoning literature are provided in Amran and Kulatilaka (1999), Copeland and Antikarov (2001), Copeland and Tufano (2004), Coy (1999) and Dixit and Pindyck (1994). See Mikosch (1998) for a (non-rigorous) treatment of stochastic calculus with finance in view.

[3] A standard assumption in the real options literature is that investment does not resolve uncertainty; it is time that resolves uncertainty. Clearly, this assumption will not be valid for certain investments in which the firm gains the critical information because it has invested. For example, R&D investments will give the firm information about the likelihood of a product´s success. Roberts and Weitzman (1981) present a model of sequential investment in which each investment gives the firm more information and the option of further investment.

of business cycle fluctuations.[4] We assume that demand shifts stochastically between two different states, each with different rates of drift and volatility. In other words, the setting assumes that the shifts are governed by a two-state Markov switching model with constant transition probabilities. Moreover, the firms are not aware of future business cycle turning points but they know the probability distribution.[5] We maintain the standard assumption that investors are risk-neutral.[6]

The layout of the paper is as follows. In section 2 we develop a stylized options-based model of investment under cyclical uncertainty. Section 3 contains an in-depth numerical analysis and interpretation of our results. The final section 4 of the paper summarizes some key findings and provides comments which have policy implications.

2. A Model of (Partially) Irreversible Investment

Here we present the basic model, including technical details and derivations. Our starting point is Abel and Eberly´s (1994) model of irreversible investment, which is a flexible and tractable example of the options-based models, and can be readily generalized to include cyclical uncertainty. We place standard assumptions on the production function of the representative firm to guarantee that the firm´s problem is well-behaved. The Cobb-Douglas production function is given by

$$(1) \qquad Y = K_t^{\alpha} N^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K is the capital stock, N is the employment level, and α is a parameter determining the shares between capital and labour in production. The employment N is taken as given at any point in time, giving rise to strict concavity of the production function. It is assumed that the firm faces an isoelastic demand function

$$(2) \qquad p = Y^{(1-\psi)/\psi} Z, \ \ \psi \geq 1,$$

[4] The paper closest to this paper is the one by Driffill et al. (2003). Dixit (1989), Leahy (1993) and Caballero and Pindyck (1996) have also provided models in which the only investment decision of firms is whether or not to enter, and in some cases, to exit from the industry. Instead of focussing on entry and exit decisions, we analyze the cyclical investment strategies of firms.

[5] Alternatively, Bagliano and Bertola (2004, p. 113 and pp. 244-245) have also discussed a stylised model of factor demand under perfectly predictable cycles by assuming that the cyclical index is given by a deterministic trigonometric function of time which repeats itself every $\pi = 3.1415$ units of time.

[6] Risk aversion may be accommodated by assuming that the firm is a mean-variance optimizer, attempting to maximize the utility function $\Pi^* = \{E(\Pi) - \varepsilon\sigma\Pi\}$, where Π are profits, σ is the standard deviation of profits, and risk aversion is captured by allowing $\varepsilon > 0$.

where p denotes the price, Y is output, Z denotes the demand shock, and ψ is an elasticity parameter that takes its minimum value of 1 under perfect competition.[7] Therefore, current profits, measured in units of output, are defined as

$$(3) \qquad \Pi = Z_t K_t^{\alpha_1} N^{\alpha_2} - C(I_t) - wN$$

where $\alpha_1 = \alpha/\psi$ and $\alpha_2 = (1-\alpha)/\psi$, w represents real wages, I_t is gross investment, and $C(\cdot)$ are the total investment expenditures denoted by the following functions:

$$(4) \qquad C(I_t) = \begin{cases} a_K + p_K^+ I_t + \frac{1}{2}\gamma I_t^2 & \text{for } I_t > 0 \\ 0 & \text{for } I_t = 0 \\ a_K + p_K^- I_t + \frac{1}{2}\gamma I_t^2 & \text{for } I_t < 0 \end{cases}$$

Fixed costs a_K are non-negative costs of investment that are independent of the level of investment. However, a firm can avoid these fixed costs by setting investment to zero. Purchase (resale) costs are the costs of buying (selling) capital. Let p_K^+ (p_K^-) be the price per unit of investment good at which the firm can buy (sell) any amount of capital. We assume that $p_K^+ \geq p_K^- \geq 0$.[8] Adjustment costs, $\gamma I_t^2/2$, are continuous and strictly convex in I, and γ is a positive parameter. Considering the depreciation of capital, the adjustment of capital over time is denoted by

$$(5) \qquad \frac{dK}{dt} = I - \delta K ,$$

where δ represents the depreciation rate.

[7] We have ignored behavioural assumptions regarding market rivalry, which in turn would necessitate some kind of game-theoretic analysis to take account of the strategic interactions among the firms, results of which are in turn heavily dependent on assumptions regarding the information sets available and the type of game being played. Leahy (1993) has, however, shown that the assumption of myopic firms who ignore the impact of other firms´ actions results in the same critical boundaries that trigger investment as a model in which firms correctly anticipate the strategies of other firms.

[8] Thus, we relax the assumption that investment be irreversible. Instead we assume that reversibility is a continuous rather than a dichotomous concept. The assumption of complete irreversibility is given by $p_K^- = 0$. Investments that are largely reversible include those that do not depreciate, those that have many uses, or those that are traded in efficient secondary markets. Often, however, as buyers in second hand markets are unable to evaluate the quality of an item they will offer a price lower than the market one. This “lemons” problem then becomes the cause of partial irreversibility of many investments even when the assets are not firm or industry specific.

In the 1980s and 1990s, the pioneering work of Hamilton (1989, 1990) clearly suggested that turning points are naturally defined in nonlinear models of regime switching. The notable characteristic of such models is the assumption that the unobservable realization of the states is governed by a discrete-time, discrete state Markov stochastic process with fixed transition probabilities and state-dependent variances.[9] In other words, it is time itself and not the state of the economic environment that governs turning points.

We assume that the demand process follows the continuous-time stochastic (geometrical Brownian motion) Markov switching process

(6) $$dZ = \eta_i Z dt + \sigma_i Z d\varpi \,, \quad \text{for } i = 0, 1$$

where ϖ is a Wiener process; $d\varpi = \varepsilon\sqrt{dt}$ (since ε_t is a normally distributed random variable with mean zero and a standard deviation of unity and serially uncorrelated), η_i is the drift parameter, and σ_i^2 the variance parameter. It is assumed that if the state 0 (recession state) occurs, the drift and the variance parameters are η_0 and σ_0^2 respectively; if the state 1 (boom state) occurs, they are η_1 and σ_1^2 respectively.[10] The probabilities of changes from the state 1 (0) to the state 0 (1) are represented by ϕ and θ separately. It is expected that the value of the drift (growth of demand) of the state 1 is higher than the one of the state 0; that is, $\eta_1 > \eta_0$. The specification reflects the importance of idiosyncratic and aggregate uncertainty. The importance of idiosyncratic uncertainty is consistent with recent microeconometric research examining the factors behind productivity growth. A striking finding of this literature is the magnitude of heterogeneity across firms which imply that idiosyncratic factors in firm-level outcomes dominate the pace of investment, reallocation and job creation in an economy. Another key pattern in the behaviour of firm-level reallocation, investment and productivity

[9] There is no denying the attractions of the model, as many theories are naturally expressed in terms of regimes and the transition from one regime to another is often described by exogenous processes. Applications, however, have only become common in the last decade with the advent of greater computing power. Markov-switching models with constant transition probabilities have been applied to interest rates [Hamilton (1988)], the behaviour of GNP [Hamilton (1989)], stock returns [Cecchetti et al. (1990)], and floating exchange rates [Engel and Hamilton (1990)]. A comprehensive review of the applications of Markov-switching models in econometrics can be found in Kim and Nelson (1999). There are also theoretical reasons why business cycles may be described as a regime-switching process. Such a process can, for instance, be characterized by a rapid-innovation regime, when large changes occur, and a quietsome regime, when only minor developments take place.

[10] Recall that Hamilton (1989) has assumed state-independent variances. The baseline Markov-switching model was extended to allow for time-varying transition probabilities by Filardo (1994). The two-state Markov chain allows agents´ sentiments to switch from one state to another in a manner reminiscent of Keynes´ "animal spirits". It is important to stress that the way we model business cycles allows us to account quite naturally for different degrees of business cycle fluctuations. Although the modelling approach is given exogenously – and thus it may be considered *ad hoc* – it allows us to assess the investment reactions to cyclical uncertainty generated by different underlying conceptual models.

is that the pace varies cyclically, i.e. the data provides evidence on synchronisation/staggering of creation/ destruction.[11] Both facts provided a motivation for the modelling framework presented here. The firm chooses its optimal level of investment over time to maximise the intertemporal value of profits, subject to the capital stock accumulation [equation (5)] and the stochastic Markov switching processes [equation (6)]. Thus, the firm's profit-maximisation problem is denoted by:

(7) $$V = \max_{I_t} E\left[\int_0^{\infty}\left[Z_t K_t^{\alpha_1} N^{\alpha_2} - wN - C(I_t)\right]e^{-rt}dt\right], \text{ s.t. (5) and (6),}$$

where r is the discount rate. Applying Ito's Lemma, the stochastic nature of this optimization problem requires the solution to the following Bellman equations for the states 0 and 1:

(8) $$rV_0 = ZK^{\alpha_1}N^{\alpha_2} - wN - C(I) + V_{0K}(I - \delta K) + \eta_0 ZV_{0Z} + \frac{1}{2}\sigma_0^2 Z^2 V_{0ZZ} + \theta(V_1 - V_0),$$

(9) $$rV_1 = ZK^{\alpha_1}N^{\alpha_2} - wN - C(I) + V_{1K}(I - \delta K) + \eta_1 ZV_{1Z} + \frac{1}{2}\sigma_1^2 Z^2 V_{1ZZ} + \phi(V_0 - V_1),$$

where V_0 represents the value of the firm in the state 0 and V_1 denotes the value of the firm in the state 1. The nature of the solution of this problem is now intuitive. The investment policy that maximizes profits has a simple and intuitive form: the q-type investment function for I for the states 0 and 1 is denoted by

(10) $$p_K^{+/-} + \gamma I = q_i \Rightarrow I = \frac{q_i - p_K^{+/-}}{\gamma},$$

where $q_i = V_{iK}$ for $i = 0, 1$. In effect, the capital stock is assumed to be continuously divisible, so that investment can be undertaken up to the point at which it becomes unprofitable. By substituting (10) back into the Bellman equations (8) and (9) and rearranging we obtain

(11) $$rV_0 = ZK^{\alpha_1}N^{\alpha_2} - wN + \frac{\left(q_0 - p_K^{+/-}\right)^2}{2\gamma} - a_K - \delta q_0 K + \eta_0 ZV_{0Z} + \frac{1}{2}\sigma_0^2 Z^2 V_{0ZZ} + \theta(V_1 - V_0)$$

[11] Excellent surveys of this literature are available in Bartelsman and Doms (2000) and Haltiwanger (2000). Caballero and Engel (1993) and Caballero et al. (1995) have recently proposed frameworks to discuss the distinction between idiosyncratic and aggregate shocks, and the potentially contrasting implications of these shocks to the dynamics of aggregate variables. Their results suggest that idiosyncratic shocks tend to smooth out microeconomic rigidities, while aggregate shocks (for example, business cycle fluctuations) tend to coordinate individual firms´ actions.

and

$$(12)\qquad rV_1 = ZK^{\alpha_1}N^{\alpha_2} - wN + \frac{\left(q_1 - p_K^{+/-}\right)^2}{2\gamma} - a_K - \delta q_1 K + \eta_1 Z V_{1Z} + \frac{1}{2}\sigma_1^2 Z^2 V_{1ZZ} + \phi(V_0 - V_1).$$

Using the definitions $q_i = V_{iK}$, $q_{iZ} = V_{iKZ}$, $q_{iK} = V_{iKK}$ and $q_{iZZ} = V_{iKZZ}$ for i = 0,1 and differentiating both sides of equations (11) and (12) with respect to *K* yields

$$(13)\qquad (r+\delta)q_0 = \alpha_1 ZK^{\alpha_1 - 1}N^{\alpha_2} + \frac{\left(q_0 - p_K^{+/-}\right)q_{0K}}{2\gamma} - \delta q_{0K}K + \eta_0 Z q_{0Z} + \frac{1}{2}\sigma_0^2 Z^2 q_{0ZZ} + \theta(q_1 - q_0)$$

and

$$(14)\qquad (r+\delta)q_1 = \alpha_1 ZK^{\alpha_1 - 1}N^{\alpha_2} + \frac{\left(q_1 - p_K^{+/-}\right)q_{1K}}{2\gamma} - \delta q_{1K}K + \eta_1 Z q_{1Z} + \frac{1}{2}\sigma_1^2 Z^2 q_{1ZZ} + \phi(q_0 - q_1).$$

Note that the processes of $q_0(Z)$ in the state 0 and $q_1(Z)$ in the state 1 are regulated. In the state 0, as the *Z* hits the (dis-)investment thresholds so that the firm buys (sells) new (old) capital. However the value of q_0 is always within the boundary in equilibrium:[12] $p_K^- \le q_0 \le p_K^+$. The same logic applies to state 1 so that $p_K^- \le q_1 \le p_K^+$. Therefore we have two coupled regulated stochastic processes for q_0 and q_1.

Table 1: The *Z* Thresholds for Booms and Recessions and their Corresponding *q* Values

	State 1 (boom)		State 0 (recession)	
	Investment	Disinvestment	Investment	Disinvestment
Thresholds	Z_1^+	Z_1^-	Z_0^+	Z_0^-
The values of *q*	$q_1(Z_1^+) = p_K^+$	$q_1(Z_1^-) = p_K^-$	$q_0(Z_0^+) = p_K^+$	$q_0(Z_0^-) = p_K^-$

The optimal investment problems happen only when the values of Tobin's *q* for states 0 and 1 equals to corresponding entry/exit costs; that is, $q_0(Z_0^+) = p_K^+$ and $q_0(Z_0^-) = p_K^-$ for the state 0 and

[12] Note, however, that q_0 value can deviate away from the boundary if the adjustment cost of capital γ is not trivial.

$q_1\left(Z_1^+\right)=p_K^+$ and $q_1\left(Z_1^-\right)=p_K^-$ for the state 1. This is the famous "reluctance to invest" result. Therefore, (13) and (14) takes the following simpler forms:

$$(15) \qquad (r+\delta)q_0 = \alpha_1 ZK^{\alpha_1-1}N^{\alpha_2} - \delta q_{0K}K + \eta_0 Zq_{0Z} + \frac{1}{2}\sigma_0^2 Z^2 q_{0ZZ} + \theta(q_1 - q_0),$$

$$(16) \qquad (r+\delta)q_1 = \alpha_1 ZK^{\alpha_1-1}N^{\alpha_2} - \delta q_{1K}K + \eta_1 Zq_{1Z} + \frac{1}{2}\sigma_1^2 Z^2 q_{1ZZ} + \phi(q_0 - q_1).$$

The coupling of equations (15) and (16) leads to a four-threshold system that needs to be solved simultaneously. The solutions for q_0 and q_1 both consist of particular solutions and general solutions so that $q_0 = q_0^P + q_0^G$ and $q_1 = q_1^P + q_1^G$. It is shown in Appendix *A* that the particular solutions for q_0 and q_1 are represented by:

$$(17) \qquad q_0^P = a_0 ZK^{\alpha_1-1}N^{\alpha_2},$$

$$(18) \qquad q_1^P = a_1 ZK^{\alpha_1-1}N^{\alpha_2},$$

where

$$a_0 = \frac{\alpha_1(r+\phi+\theta+\alpha_1\delta-\eta_1)}{(r+\phi+\alpha_1\delta-\eta_1)(r+\theta+\alpha_1\delta-\eta_0)-\phi\theta}$$

and

$$a_1 = \frac{\alpha_1(r+\phi+\theta+\alpha_1\delta-\eta_0)}{(r+\phi+\alpha_1\delta-\eta_1)(r+\theta+\alpha_1\delta-\eta_0)-\phi\theta}.$$

The general solutions for q_0 and q_1 represent the net value of options and are (for details, see Appendix *B*)

$$(19) \qquad q_0^G = -A_1\left(ZK^{\alpha_1-1}\right)^{\beta_1} - A_2\left(ZK^{\alpha_1-1}\right)^{\beta_2} + A_3\left(ZK^{\alpha_1-1}\right)^{\beta_3} + A_4\left(ZK^{\alpha_1-1}\right)^{\beta_4}$$

and

$$(20) \qquad q_1^G = -B_1\left(ZK^{\alpha_1-1}\right)^{\beta_1} - B_2\left(ZK^{\alpha_1-1}\right)^{\beta_2} + B_3\left(ZK^{\alpha_1-1}\right)^{\beta_3} + B_4\left(ZK^{\alpha_1-1}\right)^{\beta_4}.$$

Where $\beta_1, \beta_2, \beta_3,$ and β_4 are the four characteristic roots of the following equation for β

(21) $$\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_0\beta-\frac{1}{2}\sigma_0^2\beta(\beta-1)+\theta\right)\times\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_1\beta-\frac{1}{2}\sigma_1^2\beta(\beta-1)+\phi\right)=\phi\theta$$

And two of the roots are positive and two negative. Assign the following order for the characteristic roots, $\beta_1 > \beta_2 > 0 > \beta_3 > \beta_4$. The relationships between A_i and B_i for i = 1, .., 4 are

(22) $$A_i\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_0\beta-\frac{1}{2}\sigma_0^2\beta(\beta-1)+\theta\right)=B_i\theta \qquad \text{for } i = 1, .., 4$$

The set of boundary conditions that applies to this optimal stopping problem is composed by the value matching conditions[13]

(23) $$q_0\left(Z_0^+, A_1, A_2, A_3, A_4\right)=q_0^P\left(Z_0^+\right)+q_0^G\left(Z_0^+, A_1, A_2, A_3, A_4\right)=p_K^+$$

(24) $$q_0\left(Z_0^-, A_1, A_2, A_3, A_4\right)=q_0^P\left(Z_0^-\right)+q_0^G\left(Z_0^-, A_1, A_2, A_3, A_4\right)=p_K^-$$

(25) $$q_1\left(Z_1^+, B_1, B_2, B_3, B_4\right)=q_1^P\left(Z_1^+\right)+q_1^G\left(Z_1^+, B_1, B_2, B_3, B_4\right)=p_K^+$$

(26) $$q_1\left(Z_1^-, B_1, B_2, B_3, B_4\right)=q_1^P\left(Z_1^-\right)+q_1^G\left(Z_1^-, B_1, B_2, B_3, B_4\right)=p_K^-$$

and the smooth-pasting conditions

(27) $$\frac{\partial q_0\left(Z_0^+, A_1, A_2, A_3, A_4\right)}{\partial Z_0^+}=0$$

(28) $$\frac{\partial q_0\left(Z_0^-, A_1, A_2, A_3, A_4\right)}{\partial Z_0^-}=0$$

(29) $$\frac{\partial q_1\left(Z_1^+, B_1, B_2, B_3, B_4\right)}{\partial Z_1^+}=0$$

(30) $$\frac{\partial q_1\left(Z_1^-, B_1, B_2, B_3, B_4\right)}{\partial Z_1^-}=0$$

Making use of the value-matching and smooth-pasting conditions, we get the boundary values that separate the space into two regions: one where it is optimal to exercise the investment option and another where it is not. There are 12 unknown variables: Z_1^+, Z_0^+, Z_1^-, Z_0^-, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 and 12 equations: (23) ~ (30) and four relationships in equation (22). In the next section, the model is parameterised and used to gauge the magnitude of the link between cyclical uncertainty and investment.

3. Model Simulations

Chapter 2 has carefully developed and discussed the main features of the model. Unfortunately, the model has no closed form solution. This means that we need to use extensive numerical illustrations to gain further insight into the results of the previous section to have a "feel" for the model. The most important goal of these simulations is to see how certain crucial aspects of the model react to changes in parameters. In order to simulate the model, we need to cross the "minefield" of calibration. As methodological issues related to calibration are not the focus of this paper, a pragmatic stance is taken. The unit time length corresponds to one year. Where possible, parameter values are drawn from empirical studies. Our base parameters which were chosen for realism are σ_0 = 0.2, σ_1 = 0.1, η_0= 0.01, η_1= 0.03, δ= 0.07, ψ= 1.5, α= 0.3, θ = 0.33, ϕ= 0.15, $p_K^+ = 1.0$, $p_K^- = 0.4$, r = 0.05 and K_0 = N_0 = 1.0. Choosing values for σ_0 and σ_1 requires care, since these parameters underpin the link between cyclical uncertainty and investment. As a guide to calibration we have used the macroeconomic fact that the standard deviation of *GDP* in the *OECD* countries during recessions was about twice as high as in boom periods. The procedure used to parameterize the switching probabilities is as follows. We set the baseline standard deviations equal to θ = 0.33 and ϕ = 0.15, respectively. θ = 0.33 implies that the expected duration of a recession is (1-0.33)/0.33 = 2.0 years, while ϕ = 0.15 implies that the expected duration of an expansion is (1-0.15)/0.15 = 5.66 years.[14] Finally, the price elasticity of demand parameter is set at Ψ= 1.50 as in Bovenberg et al. (1998).

The main output of the model consists of thresholds that bisect the firm´s decision-making space into zones where it is optimal to exercise the investment option and zones where the firm maximizes its value by leaving the option unexercised. We call these thresholds "bands of hysteresis". To get some qualitative ideas of the impact exercised by the parameters of the model and to get a sense of the magnitudes, we give here some numerical calculations of the Z thresholds for a range of parameter values. First, we consider alternative switching probabilities. The results for alternative θ´s and ϕ´s are given in figure 1 - 2 below.

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[13] The value-matching conditions here are different from Driffill et al. (2003). They use a financial explanation and approach to solve the system. In this paper, a direct mathematical approach of regulated stochastic processes of q value is used to solve the system.

[14] Business cycle fluctuations are characterized by an asymmetry in the duration of recessions and expansions – with the latter lasting on average roughly 3-5 times as long as the former. The corresponding Markov-switching parameter values in Driffill et al. (2003) are θ= 0.3245 and ϕ= 0.10509, respectively.

Figure 1: The Impact of θ Upon the Z Thresholds

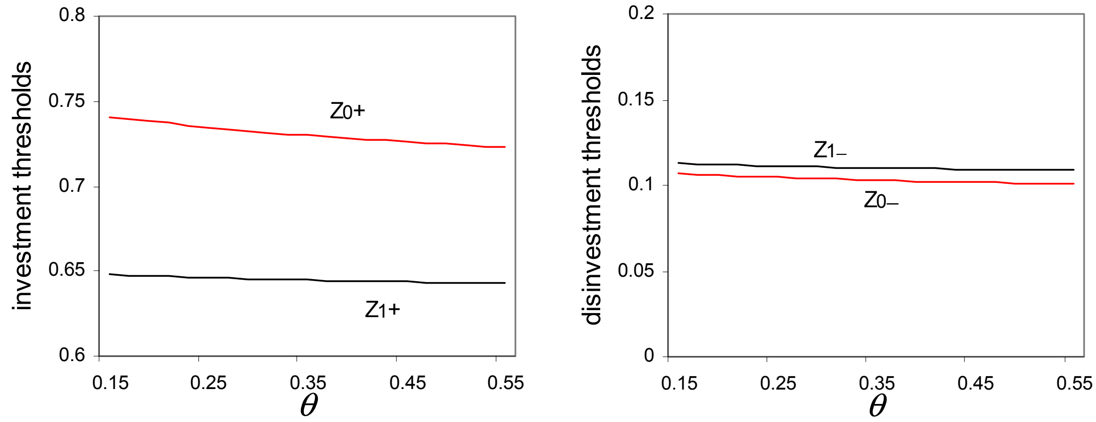
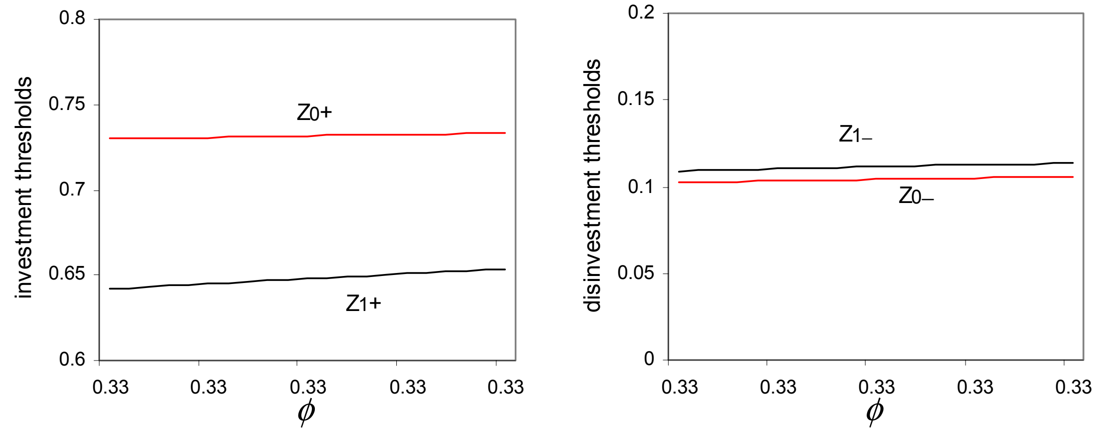


Figure 2: The Impact of ϕ Upon the Z Thresholds



A rise in θ (the probability of jumping from recession to expansion) reduces the expected duration of recessions. Therefore, the firm invests earlier and disinvests later. On the contrary, a higher value of ϕ (the probability of jumping from expansion to recession) implies that firms are more sceptical about the durability of expansions, and therefore a rising ϕ leads to a widening of the no action area since the reward for waiting is increasing.[15]

[15] It is increasingly being recognized that firms tend to share broadly similar ex-ante assessments regarding cyclical uncertainties. This "bandwagon" phenomenon or "follow-the-leader" behaviour implies that a recession may leave an economy trapped in low investment equilibrium.

Figure 3: The Impact of σ_0 Upon the Z Thresholds for $\sigma_0 = 2\sigma_1$

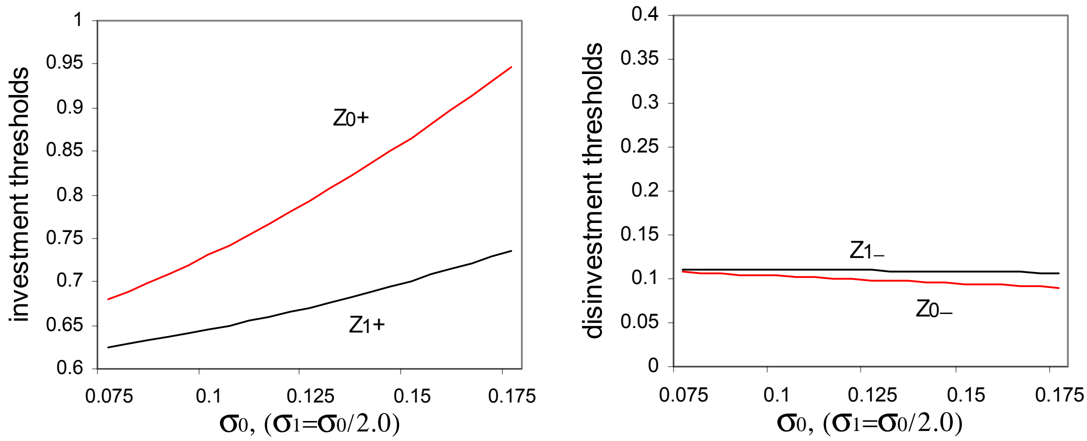
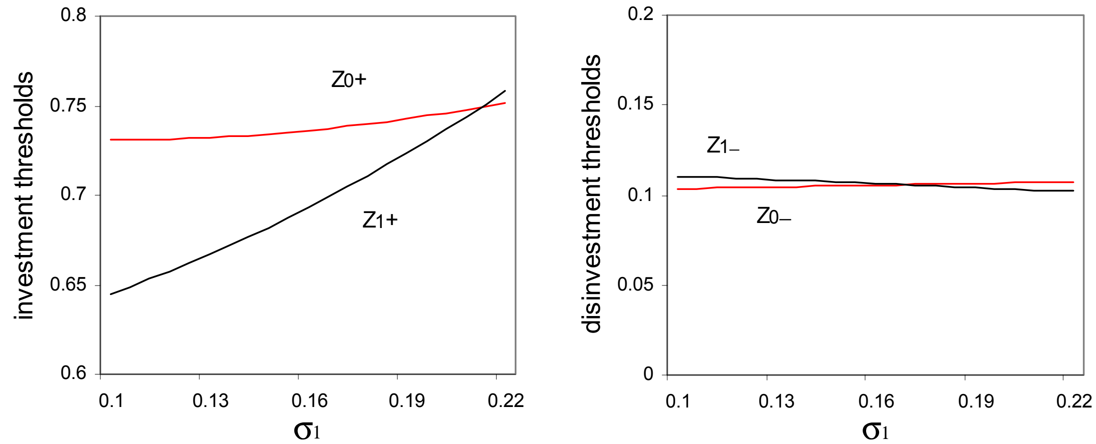


Figure 4: The Impact of σ_1 Upon the Z Thresholds

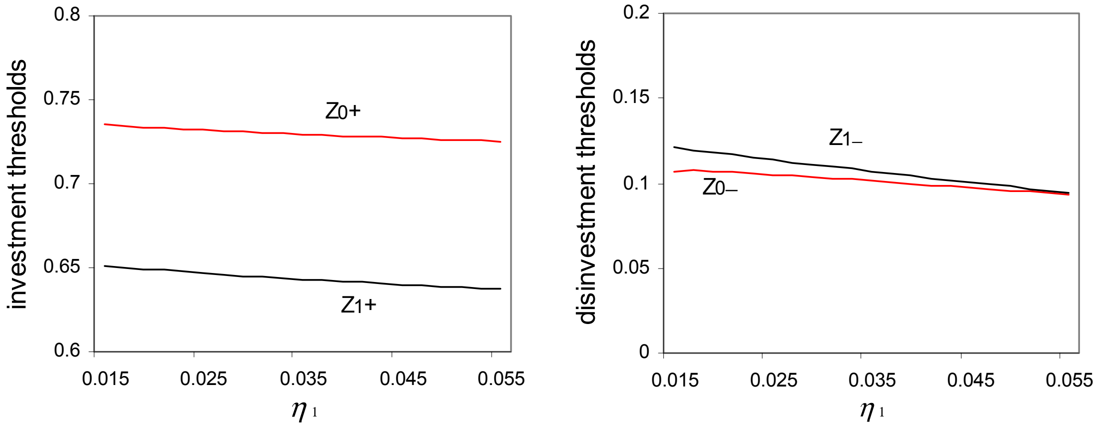


Let us now consider changes in σ_i ($i = 0,1$).[16] In other words, we analyse the sensitivity of the optimal thresholds with respect to changes in the volatility of the geometric Brownian motion in booms versus recessions. In figure 3, the ratio of σ_1/σ_0 is kept constant, while in figure 4 σ_1 is increasing for $\sigma_0 = \overline{\sigma}_0$. As in the existing literature, we find that the threshold value at which investment takes place is increasing in the "noisiness" level even though the firm is risk neutral. In more volatile environments, the best tactic is to keep options open and await new information rather than commit an investment today.[17] Figure 5 shows how the band of hysteresis depends upon the mean growth rate in boom periods. The graphs have an immediate interpretation: An increasing drift term reduces the precautionary motive for waiting over and above investing.

[16] Irrespective of what caused changes in the volatility of business cycles in the 1990s (new economy effects, improved monetary policy or inventory structure, favourable supply shocks), its effect was firmly on the second rather than the first moment of U.S. GDP growth. The empirical evidence by McConnell and Pérez-Quirós (2000) indicates that the output variance has declined in the U.S. since the 1980s (they found an output variance break in 1984:1). For an overview, see Stock and Watson (2002).

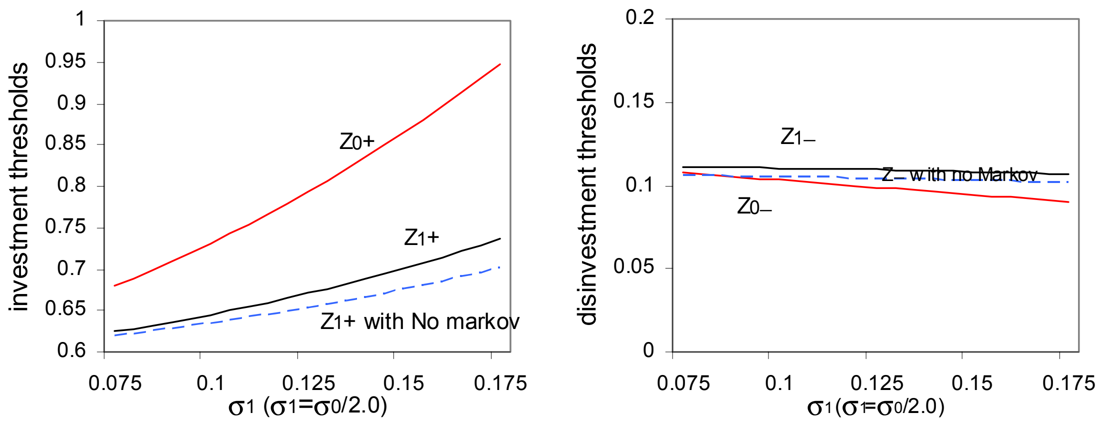
[17] Both figures also reveal that the Z^+ surfaces are much more sensitive to changes in σ than the Z^- surfaces.

Figure 5: The Impact of the Drift Term η_1 Upon the Z Thresholds



We end the threshold simulations by considering the impact of σ in the model without the Markov switching component, i.e. we also show simulation results for a counterfactual economy in which macroeconomic cycles do not exist. This allows us to compare our results with the standard real option modelling approach. The numerically calculated results in figure 6 are intuitive: the threshold values indicate that firms invest earlier and disinvest later when the economy is in the state of a permanent boom. In other words, a consequence of regime-switching is that the exercise price is different than in the no-switching case. Omitting business cycles and switching therefore leads to a badly-timed exercise decision.

Figure 6: The Z Thresholds With and Without the Markov Switching Component



One point is especially worth discussing further. We characterize the firm´s dynamic factor adjustment in terms of the model´s parameters assuming that the capital stock is inherited from that past, but Tobin´s q adjusts freely in the market. A fruitful way to summarize the implications for the dynamics of the economy is to analyse the phase-diagram. The two variables we will focus on are the capital stock, K, and Tobin´s q. With $dK/dt = I - \delta K$ and $p_K^+ + \gamma I = q$, we have

(31) $$\frac{dK}{dt} = I - \delta K = 0 \Rightarrow \frac{q - p_K^+}{\gamma} - \delta K = 0 \Rightarrow q = p_K^+ + \delta\gamma K \ .$$

Equation states that K is decreasing for $q < p_K^+ + \delta\gamma K$ because K depreciates at a rate of δ. Figure (6) also indicates that the phase diagram is partitioned into various domains, including a no-action area between p_K^- and p_K^+ where no investment or disinvestment (scrapping) should optimally be made. Rewriting (12) and (13) as an equation for dq_i/dt yields

(32) $$\frac{dq_i}{dt} = (r + \delta)q_i - \alpha_1 Z K^{\alpha_1 - 1} N^{\alpha_2} = 0, \quad i = 1, 2 .$$

The set of points satisfying (32) is downward-sloping in the (K, q) space. If the economy switches from a recession (state 0) to a boom (state 1), then the q value increases for a given value of K and shifts the $\dot{q} = 0$ line in a north-easterly direction. Another important feature is that both $\dot{q} = 0$ lines are stochastic because they are a function of Z.[18] The qualitative content of the model for a *fixed* Z ($Z = \overline{Z}$) can be summarized graphically by means of figure (7).[19]

The graph allows exemplifying the implications of recessions and expansions since the assumption of $Z = \overline{Z}$ renders the model rather simple indeed. We begin with the case where the firm is initially in the unique recession equilibrium A in the top panel of figure (7). The upward shift of the $\dot{q} = 0$ curve from $\dot{q}_0 = 0$ to $\dot{q}_1 = 0$ in an expansion implies that q jumps immediately to point B on the new saddle path for the given capital stock as indicated by the arrows in figure (7).[20] K and q then steadily move down that saddle path to the new long run equilibrium C. Thus an expansion leads to an investment boom and an increasing capital stock. Now assume that state 1 (boom period) is given initially and K and q begin at point D. A cyclical turning point and therefore a switch from state 1 (expansion) to state 0 (recession) implies that the $\dot{q} = 0$ locus shifts from $\dot{q}_1 = 0$ to $\dot{q}_0 = 0$; this is shown in the lower panel of figure (7). Intuitively, it means that the firm´s profits will be lower, and thus that the existing capital stock is less valuable. As a result, the firm finds itself stuck with an excessive stock of capital and q jumps to point E on the new saddle path. Afterwards, K and q again move along the saddle path to the new equilibrium F, which involves lower K and higher q. At point E, however, the firm is in the

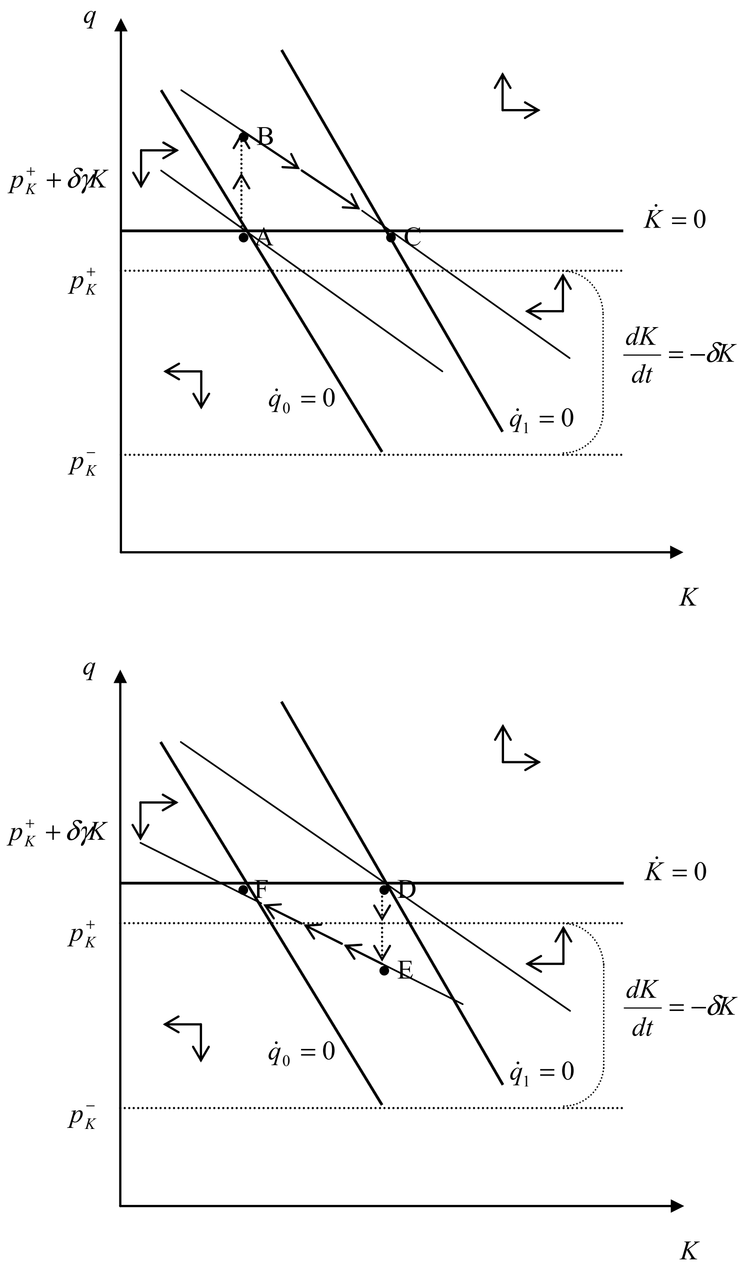
[18] When Z falls (rises), then the $dq_i/dt = 0$ curves also shift downwards (upwards). Note that these alternating and stochastically fluctuating $dq_i/dt = 0$ curves and the resulting sequences of adjustment differ from what we would get without Markov-switching.

[19] While the assumption of a fixed Z risks "contaminating" the dynamics of the model, it is nevertheless informative because it allows for some intuitive graphical analysis of the model features.

[20] The intuition behind this response is straightforward. Since K adjusts slowly, existing capital earns rents and therefore q rises.

interior of the stay-put or hysteresis zone which provides an incentive to wait and see because of the Markov nature of uncertainty and partial investment irreversibility. As a result, the reduction in the capital stock solely depends upon δ and K falls only slowly. At some point in time, however, the depreciation of K will eventually lead to q to hit the upper threshold p_K^+ and the firm will optimally start to invest until the new equilibrium F is reached.[21] The options-based framework to studying investment under uncertainty provides the rationale for these discontinuities in investment strategies.

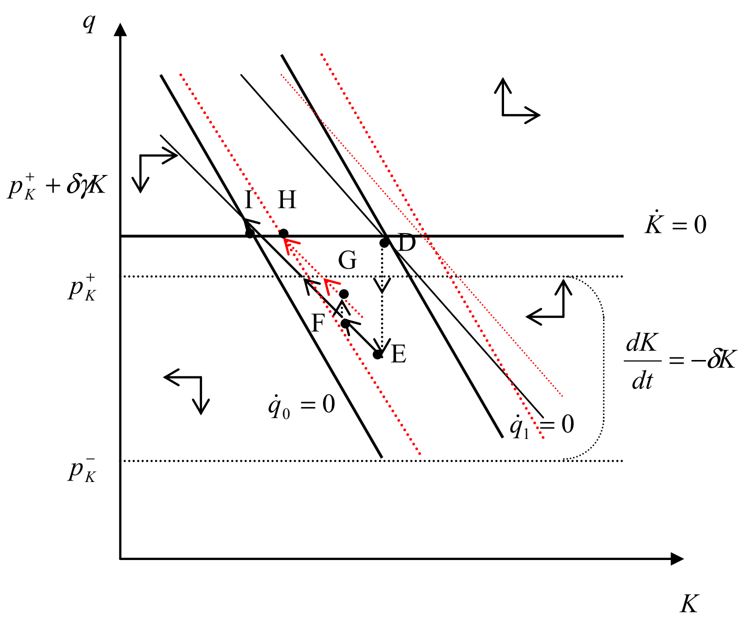
Figure 7: The Dynamics of the Capital Stock and Tobin´s q for $Z = \overline{Z}$



[21] Although the firm invests, depreciation exceeds gross investment so that the capital stock falls over time.

The main disadvantage of figure (7) is that we have assumed a fixed *Z*. Although it makes the analysis simple, this assumption is not very appealing. Next, we will therefore analyse the dynamics of *K* and *q* off the $Z = \overline{Z}$ assumption. What is happening? The more complicated dynamic forces can be demonstrated with the aid of figure (8). In *D* the economy is in an initial equilibrium. Now consider a cyclical turning point. We have already explained that a trough shifts the $\dot{q} = 0$ line from $\dot{q}_1 = 0$ (expansion) to $\dot{q}_0 = 0$ (recession). The firm jumps to point *E*. The major alteration compared to our earlier case is that both $\dot{q} = 0$ lines are fluctuating stochastically. Let´s assume that during the adjustment process an unexpected jump in *Z* occurs which shifts both $\dot{q} = 0$ curves upwards. In order to establish equilibrium, a further jump in *q* from *F* to *G* is occurring, followed by gradual (smooth) adjustment from *G* to *H*. The basic insight in comparison with the previous case of a fixed *Z* ($Z = \overline{Z}$) is, of course, that a firm may also stay in the no action area after a positive *Z*-shock.

Figure 8: The Dynamics of the Capital Stock and Tobin´s *q* for Stochastically Fluctuating *Z*



As another numerical "finger exercise" with the model, consider the *Z*-dynamics, the *K*-dynamics and *I*-dynamics as illustrated in figure (9) to (11). In order to get a clear "feel" for the dynamics of the model, we first have to specify a solution method that will lead us to generate discrete realizations of the endogenous variables, given the chosen levels of parameters.[22] Several options are available at this

[22] It might appear that the impact of uncertainty is always to lower investment expenditures. This is, however, not the case. The intuitive reason for this finding is that while uncertainty as measured by the variance parameters raises the threshold level for investment to occur (negative first-order effect), it also raises volatility, allowing *Z* to hit the thresholds more often (positive second-order effect).

point, but the structure of the model readily suggests using a sequential iterations method. It works as follows. Equation (6) is proxied by the following discrete stochastic differential equation – the Euler scheme,

$$(33) \qquad Z_{t+\Delta t} - Z_t = \eta_i Z_t \Delta t + \sigma_i \sqrt{\Delta t} Z_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \quad \text{for } i = 0, 1,$$

where the normal random variables, ε_t, are generated via the central limit theorem and the Box-Muller (1985) method for transforming a uniformly distributed random variables to a normal distribution with given mean and variance.[23] If state 0 occurs, then the demand shock Z is governed by

$$(34) \qquad Z_{t+\Delta t} - Z_t = \eta_0 Z_t \Delta t + \sigma_0 \sqrt{\Delta t} Z_t \varepsilon_t .$$

If state 1 occurs, the demand shock Z is simulated by

$$(35) \qquad Z_{t+\Delta t} - Z_t = \eta_1 Z_t \Delta t + \sigma_1 \sqrt{\Delta t} Z_t \varepsilon_t .$$

The transitions between the states are described by Poisson processes with mean durations λ_0 and λ_1. Note that the probabilities of changes from state 1 (0) to state 0 (1) are represented by ϕ and θ separately. The implies that the expected duration for the state 0 until state 1 arrives is equal to $(1-\theta)/\theta$, and the corresponding expected duration for state 1 is $(1-\phi)/\phi$. Substituting the benchmark values of $\theta = 0.33$ and $\phi = 0.15$ gives that the expected durations $\lambda_0 = 2.0$ and $\lambda_1 = 5.67$. At the beginning of the state 0 (1), an integer number of unit rate Poisson random events with mean durations λ_0 (λ_1) is generated to determine how long the state will last until the transition happens.[24] As the time passes, the term Z_t fluctuates according to the corresponding stochastic processes and K will depreciate as long as Z_t is staying within the no-action area. If Z_t hits the threshold Z_1^+ in state 1 or the threshold Z_0^+ in state 0, the firm will invest according to

$$(36) \qquad I_t = \frac{q_1(Z_t) - p_K^+}{\gamma} = \frac{q_1(Z_t) - q_1(Z_1^+)}{\gamma} \qquad \text{for state 1}$$

[23] See, for example, Press et al. for a description of the algorithm.
[24] Note that this procedure is used repeatedly after each turning point; hence the procedure is termed a sequential iterations procedure. Clearly, this is a simplification, which makes our numerical problem easier. When "0" is generated by the Poisson generator, a value "0.5" is used to replace "0" to guarantee that the transitions would not vanish immediately.

and

(37) $$I_t = \frac{q_0(Z_t) - p_K^+}{\gamma} = \frac{q_0(Z_t) - q_0(Z_0^+)}{\gamma} \qquad \text{for state 0.}$$

Note that since it is impossible to compute the coefficients A_1, A_2, A_3 and A_4, and then the homogenous solutions outside the thresholds, we use the differences between the particular solutions to proxy the value of $q_i(Z_t) - q_i(Z_1^+)$. The same approach is applied to the calculation of dis-investment. After the level of investment is determined, the corresponding capital stock is computed using the capital accumulation constraint

(38) $$K_{t+1} = K_t + I_t - \delta K_t,$$

which become the initial value of K for the time t+1, by which the new thresholds are recomputed accordingly for the time t+1. With the aid of this numerical solution principle, the adjustment paths can be simulated. What do these dynamic adjustment paths look like? Three different sample paths of the stochastic adjustment process are given in figure (9) to (11) below.[25] Superimposed on the graphs are the boom and recession phases.[26]

The figures visualize three (alternative) realizations of the demand shock Z, the four threshold variables, the sequence of expansions versus recessions (indicated by the broken vertical lines) and the corresponding optimal net investment and installed capital stock time series over 30 years. We immediately see that net investment always occurs when the firm "by accident" hits the relevant threshold.[27] Furthermore, the sample paths of I and K are "zigzagging", i.e. the overall finding is that the Markov-switching specification can indeed mimic actual cyclical movements in I and K.[28]

[25] The initial value for $Z_{t=0}$ is 0.65, Δt=0.2 and γ= 1.0. All other parameters are as in the benchmark case.
[26] It should be pointed out that Figure 9 -11 display the results for three different seeds for the random number generator, and might not represent a typical or average path..
[27] In the simulations, the firm never hits the dis-investment threshold given the resale price p_K^- = 0.4. Therefore, dis-investment never takes place although it is physically possible.
[28] Investment at the firm level is characterized by major and infrequent adjustments. Cooper et al. (1999), for example, have documented this lumpiness.

Figure 9: A Sample Path of the Demand Shock (*Z*), the *Z*-Thresholds, Installed Capital (*K*), and Optimal Investment

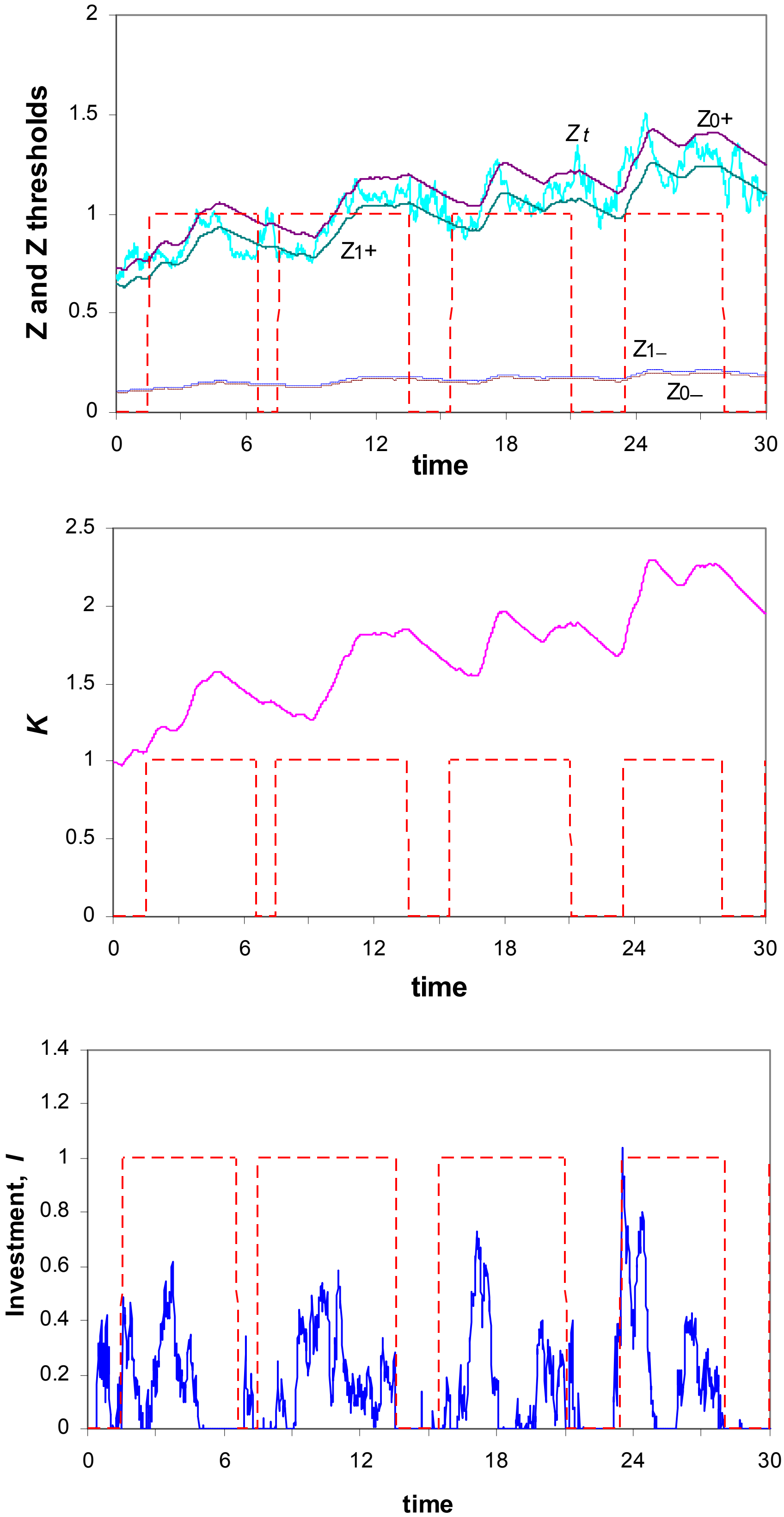


Figure 10: A Sample Path of the Demand Shock (*Z*), the *Z*-Thresholds, Installed Capital (*K*), and Optimal Investment

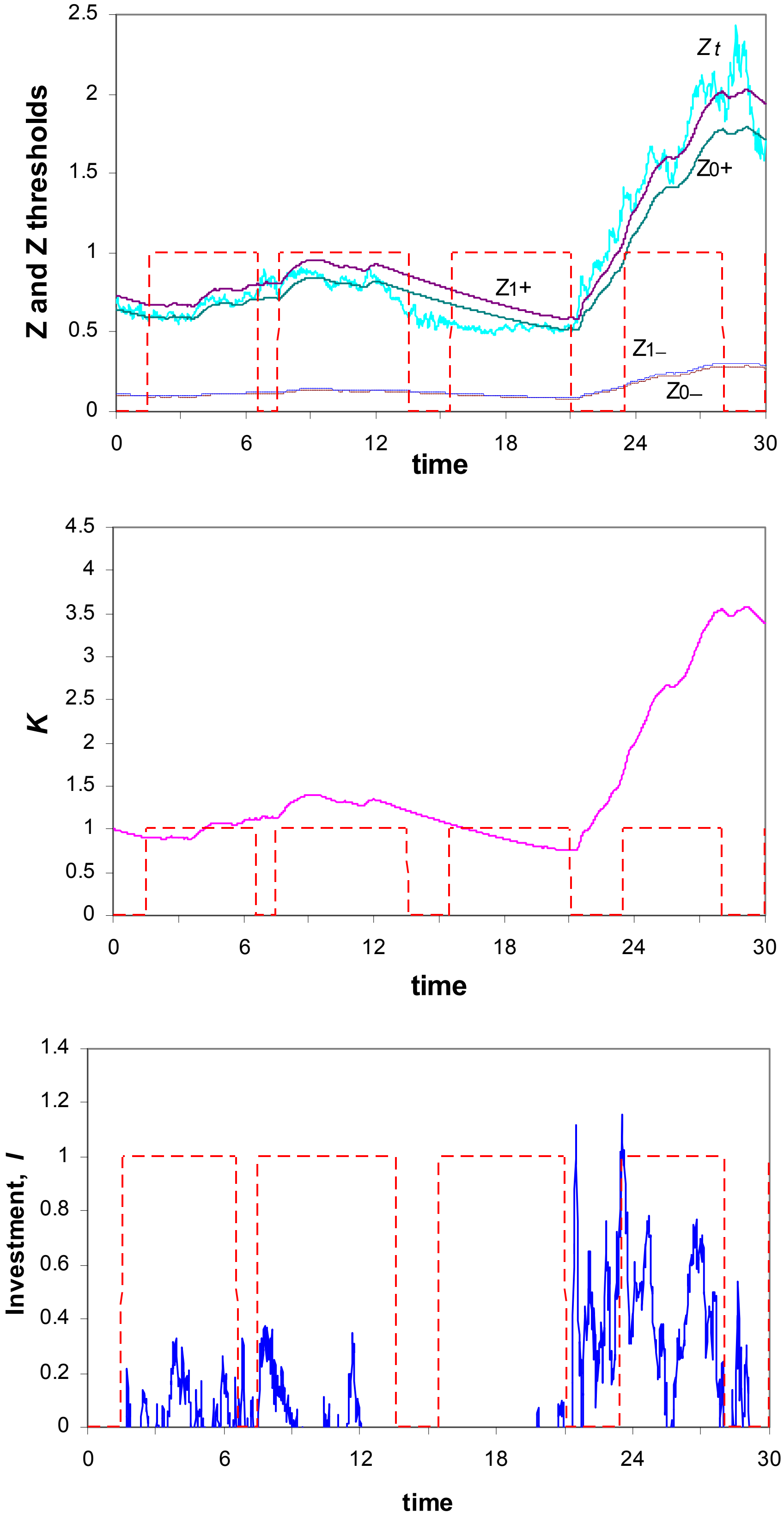
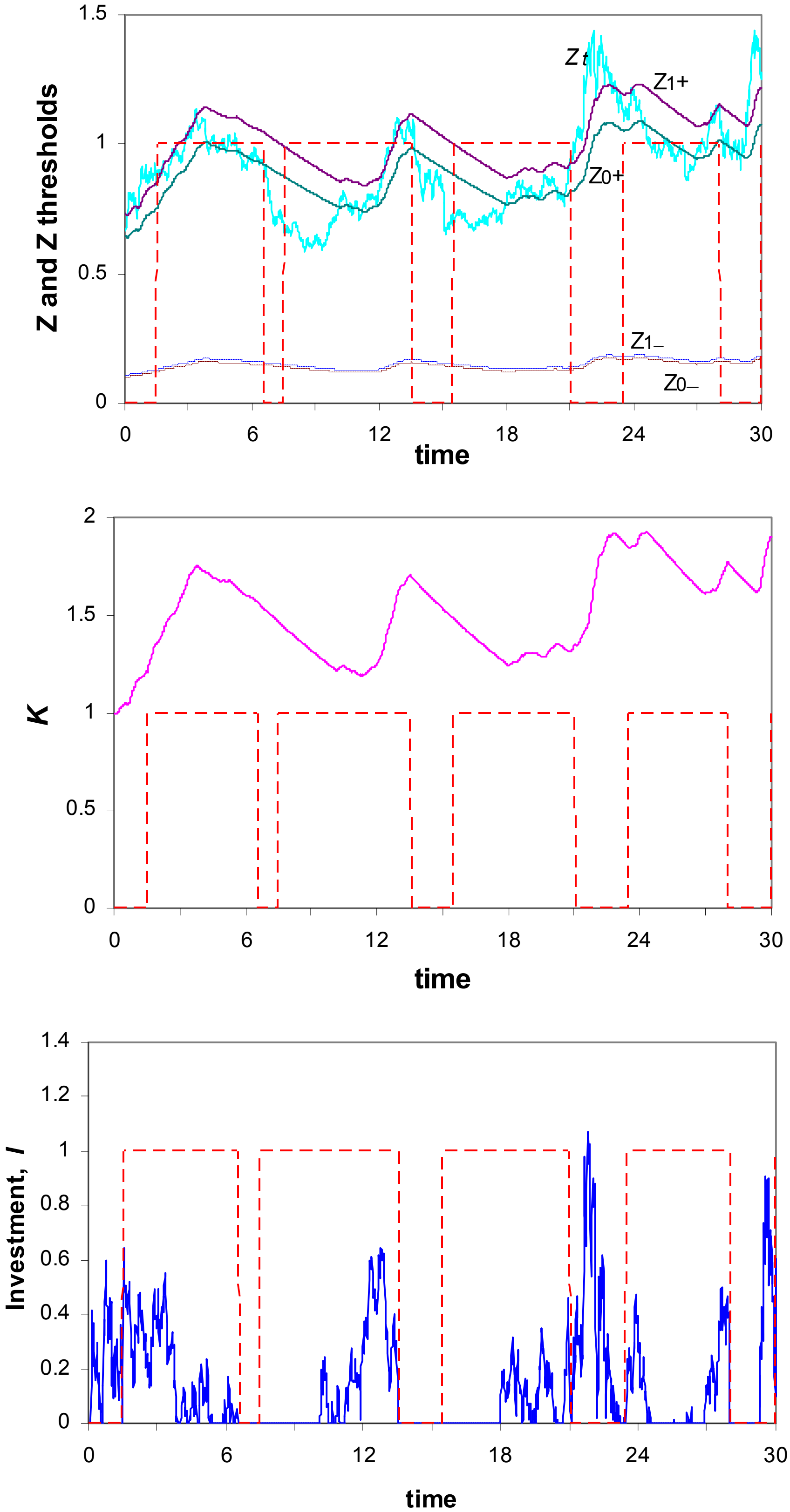
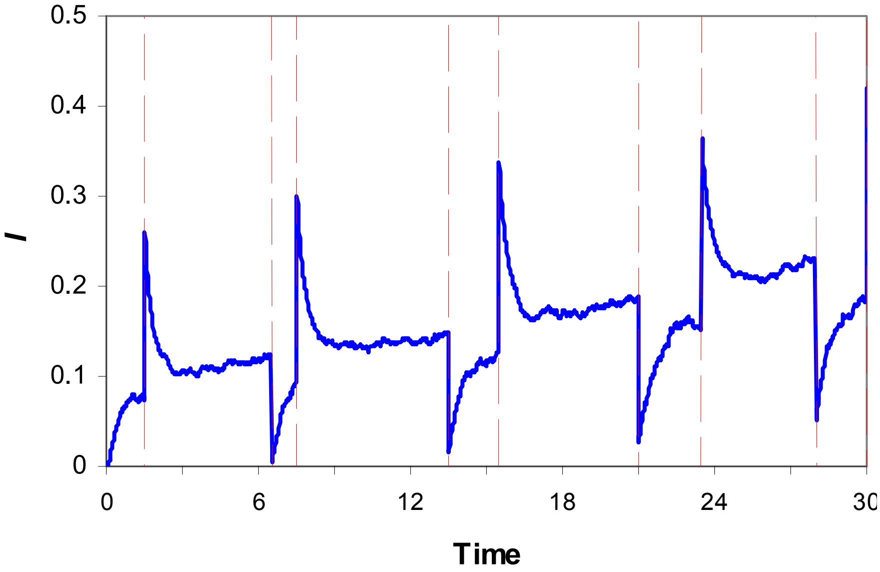


Figure 11: A Sample Path of the Demand Shock (Z), the Z-Thresholds, Installed Capital (K), and Optimal Investment



Up to now, we have interpreted the model as applying to a single firm. Suppose that we re-interpret the model at the macroeconomic level, i.e. K and I now represent economy-wide gross investment and the capital stock, respectively, and the interpretation of q is likewise altered. Unlike microeconomic data, aggregate investment series look smoother since microeconomic adjustments are far from being perfectly synchronized. The question arises as to whether aggregation eliminates all traces of infrequent lumpy microeconomic adjustment. We again focus on investment (I), and we model aggregate investment in terms of average investment of a large number of individual firms indexed by $i \in [1,1000]$.[29] The resulting dynamics of investment (I) resulting from the 1000 stochastic sample paths is given in figure 12.

Figure 12: Average Investment Dynamics in the Economy



Although the individual sample paths are far from being synchronised because of idiosyncratic Z-shocks, the economy nevertheless converges to "business cycles" and the model is therefore able to explain cyclical fluctuations in key macroeconomic variables.

4. Summary and Conclusions

This paper is an attempt at providing a unifying framework that makes explicit and clarifies thinking on the inter-linkages between cyclical uncertainty, option value and the choice and timing of investment. With the aim of parsimony in mind, but also wanting to ensure a fair degree of reality, we

[29] The procedure implies that we consider firm-level investment in isolation although real-life investment decisions are probably interrelated through market interactions and feedbacks brought about by goods markets, and factor markets, among other things. Although a rigorous analysis of aggregate dynamics should take this into account, the simulation results derived below will suffice for our purposes. See Caballero (1999) for a detailed discussion of the aggregation issue.

extend and generalize a standard model of irreversible investment by introducing Markov-switching. The Markov-switching modelling approach allows the derivation of analytical and numerical results on option pricing, taking into account that firms not only either observe or infer the current state of the system but also make predictions about future regime switches.

The chief implication of the model is that recessions are important catalysts for waiting. In other words, our model shows that macroeconomic risk acts as a deterrent to present investments. Hence, a policy maker interested in accelerating investment should aim at stabilizing business cycles.[30] Unfortunately, deriving economic policy conclusions from these results is not straightforward, considering that deliberate government action cannot directly influence some of the variables determining business cycles.[31]

[30] The welfare costs of cyclical uncertainty can be substantial. Levine and Renelt (1992) have studied the empirical importance for growth rates of a large number of political, public finance, trade and macroeconomic variables, and the concluded that the only robust empirically significant cross-country relationship is a positive relationship between investment and the growth rate of output. The 'new growth theory' further illuminates the key role of investment in the growth process. However, while the growth literature focuses on the effects of *current* productivity shocks for long run levels and growth rates, the focus in this paper is on the role of anticipated *future* shocks in investment decisions.

[31] Drawing policy conclusions is, in addition, hampered by the fact that the causality could in part be running from investment to cyclical uncertainty.

Appendix A: Particular Solutions for q_0 and q_1

Assume that the particular solutions have the following functional forms:

(A1) $q_0^P = a_0 Z K^{\alpha_1 - 1} N^{\alpha_2}$,

(A2) $q_1^P = a_1 Z K^{\alpha_1 - 1} N^{\alpha_2}$.

Plugging into equations (15) and (16), we get

(A3) $a_0(r+\delta) = \alpha_1 - \delta a_0(\alpha_1 - 1) + \eta_0 a_0 + \theta(a_1 - a_0)$

(A4) $a_1(r+\delta) = \alpha_1 - \delta a_1(\alpha_1 - 1) + \eta_1 a_1 + \phi(a_0 - a_1)$.

Rearranging and collecting terms yields

(A3) $(r + \theta + \delta\alpha_1 - \eta_0)a_0 - \theta a_1 = \alpha_1$

(A4) $-\phi a_0 + (r + \phi + \delta\alpha_1 - \eta_1)a_1 = \alpha_1$.

Therefore, by Cramer's rule it follows directly that

$$a_0 = \frac{\alpha_1(r + \phi + \theta + \alpha_1\delta - \eta_1)}{(r + \phi + \alpha_1\delta - \eta_1)(r + \theta + \alpha_1\delta - \eta_0) - \phi\theta}$$

and

$$a_1 = \frac{\alpha_1(r + \phi + \theta + \alpha_1\delta - \eta_0)}{(r + \phi + \alpha_1\delta - \eta_1)(r + \theta + \alpha_1\delta - \eta_0) - \phi\theta}.$$

Appendix *B*: Homogenous Solutions for q_0 and q_1

The homogenous parts of equations (15) and (16) in the text are represented by

(B1) $(r+\delta)q_0 = -\delta q_{0K} K + \eta_0 Z q_{0Z} + \frac{1}{2}\sigma_0^2 Z^2 q_{0ZZ} + \theta(q_1 - q_0)$,

B2) $(r+\delta)q_1 = -\delta q_{1K} K + \eta_1 Z q_{1Z} + \frac{1}{2}\sigma_1^2 Z^2 q_{1ZZ} + \phi(q_0 - q_1)$.

The problem is simplified by observing that N is constant. Let us therefore assume that the general solutions have the following functional forms:

(B3) $q_0^G = A\left(ZK^{\alpha_1 - 1}\right)^{\beta}$,

(B4) $q_1^G = B\left(ZK^{\alpha_1 - 1}\right)^{\beta}$.

Substituting into equations (B1) and (B2) respectively yields

(B5) $$A\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_0\beta-\frac{1}{2}\sigma_0^2\beta(\beta-1)+\theta\right)=B\theta,$$

(B6) $$B\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_1\beta-\frac{1}{2}\sigma_1^2\beta(\beta-1)+\phi\right)=A\phi.$$

Equations (B5) and (B6) yield the equation for β and the relationship between *A* and *B*. The equation governing β is derived by equalling the ratios A/B in equations (B5) and (B6)

(B7) $$\begin{aligned}&\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_0\beta-\frac{1}{2}\sigma_0^2\beta(\beta-1)+\theta\right)\times\\&\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_1\beta-\frac{1}{2}\sigma_1^2\beta(\beta-1)+\phi\right)=\phi\theta\end{aligned}$$

which gives four characteristic roots for β: two positive and two negative. Assign the following order for the characteristic roots, $\beta_1>\beta_2>0>\beta_3>\beta_4$. Therefore, the general solutions are shown as follows:

(B8) $$q_0^G=-A_1\left(ZK^{\alpha_1-1}\right)^{\beta_1}-A_2\left(ZK^{\alpha_1-1}\right)^{\beta_2}+A_3\left(ZK^{\alpha_1-1}\right)^{\beta_3}+A_4\left(ZK^{\alpha_1-1}\right)^{\beta_4},$$

(B9) $$q_1^G=-B_1\left(ZK^{\alpha_1-1}\right)^{\beta_1}-B_2\left(ZK^{\alpha_1-1}\right)^{\beta_2}+B_3\left(ZK^{\alpha_1-1}\right)^{\beta_3}+B_4\left(ZK^{\alpha_1-1}\right)^{\beta_4}.$$

The positive/negative signs are chosen to have positive values of A_1, A_2, A_3, A_4 ,B_1, B_2, B_3, B_4 - resulting in positive values for all option terms. The relationships between A_i and B_i for *i* = 1, .., 4 is derived by the following ratio of *A/B*:

(B10) $$A_i\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_0\beta-\frac{1}{2}\sigma_0^2\beta(\beta-1)+\theta\right)=B_i\theta,$$

or

(B11) $$B_i\left(r+\delta+\delta\beta(\alpha_1-1)-\eta_1\beta-\frac{1}{2}\sigma_1^2\beta(\beta-1)+\phi\right)=A_i\phi.$$

References:

Abel, A.B. and J.C. Eberly (1994) "A Unified Model of Investment Under Uncertainty", *American Economic Review* 84, 1369-1384.

Amran, M. and N. Kulatilaka (1999) *Real Options: Managing Strategic Investment in an Uncertain World*, Boston (Harvard Business School Press).

Bagliano, F.-C. and G. Bertola (2004) *Models for Dynamic Macroeconomics*, Oxford (Oxford University Press).

Bartelsman, E.J. and M.E. Doms (2000) "Understanding Productivity: Lessons from Longitudinal Microdata", *Journal of Economic Literature* 38, 569-594.

Bovenberg, A.L., J.J. Graafland and R.A. de Mooij (1998) *Tax Reform and the Dutch Labor Market: An Applied General Equlibrium Approach*, CPB Netherlands Bureau for Economic Policy Analysis, Research Memorandum 143, The Hague.

Box, G.E.P. and M.E. Muller (1958) „A Note on the Generation of Random Normal Deviates, *Annals Mathematical. Statistics* 29, 610-611.

Caballero, R.J. (1999) "Aggregate Investment", in: Taylor, J.B. and M. Woodford (eds.) *Handbook of Macroeconomics*, Vol. 1B, Amsterdam (Elsevier), 814-862.

Caballero, R.J. and E. Engel (1993) "Heterogeneity and Output Fluctuations in a Dynamic Menu Cost Economy", *Review of Economic Studies* 60, 95-119.

Caballero, R.J., Engel, E. and J. Haltiwanger (1995) „Plant-Level Adjustment and Aggregate Investment Dynamics“, *Brookings Papers on Economic Activity*, No. 2/1995, 1-54.

Caballero, R.J. and R.S. Pindyck (1996) "Uncertainty, Investment, and Industry Evolution", *International Economic Review* 37, 641-662.

Cecchetti, S.G., Lam, P. and N.C. Mark (1990) "Mean Reversion in Equilibrium Asset Prices", *American Economic Review* 80, 398-418.

Cooper, R., Haltiwanger, J. and L. Power (1999) „Machine Replacement and the Business Cycle: Lumps and Bumps“, *American Economic Review* 89, 921-946.

Copeland, T. and V. Antikarov (2001) *Real Options – A Practitioner´s Guide*, London (Texere Publishing).

Copeland, T. and P. Tufano (2004) "A Real-World Way to Manage Real Options", *Harvard Business Review*, March 2004, 90-99.

Coy, R. (1999) "Exploiting Uncertainty: The Real-Options Revolution in Decision-Making", *Business Week*, June 7, 118-124.

Dixit, A. (1989) "Entry and Exit Decisions Under Uncertainty", *Journal of Political Economy* 97, 620-638.

Dixit, A. and R.S. Pindyck (1994) *Investment Under Uncertainty*, Princeton (Princeton University Press).

Driffill, J., Raybaudi, M. and M. Sola (2003) "Investment Under Uncertainty with Stochastically Switching Profit Streams: Entry and Exit over the Business Cycle.", *Studies in Nonlinear Dynamics & Econometrics*: Vol. 7: No. 1, Article 1 (http://www.bepress.com/snde/vol7/iss1/art1).

Dunne, T., Roberts, M. and L. Samuelson (1988) "Patterns of Firm Entry and Exit in U.S. Manufacturing Industries", *Rand Journal of Economics* 19, 495-515.

Engel, C. and J.D. Hamilton (1990) "Long Swings in the Dollar: Are They in the Data and Do the Markets Know it?", *American Economic Review* 80, 689-713.

Filardo, A.J. (1994) "Business Cycle Phases and Their Transitional Dynamics", *Journal of Business and Economic Statistics* 12, 299-308.

Graham, J.R. and C.R. Harvey (2001) "The Theory and Practice of Corporate Finance: Evidence from the Field", *Journal of Financial Economics* 60, 187-243.

Haltiwanger, J. (2000) "Aggregate Growth: What Have We Learned From Microeconomic Evidence?", *OECD Economics Department Working Paper No. 267*, Paris.

Hamilton, J.D. (1988) "Rational Expectations Econometric Analysis of changes in Regime: An Investigation of the Term Structure of Interest Rates", *Journal of Economic Dynamics and Control* 12, 385-423.

Hamilton, J.D. (1989) "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", *Econometrica* 57, 357-384.

Hamilton, J.D. (1990) "Analysis of Time Series Subject to Changes in Regime", *Journal of Econometrics* 45, 39-70.

Kim, C.-J. and C.R. Nelson (1999) *State-Space models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications"*, Cambridge (MIT Press).

Leahy, J. (1993) "Investment in Competitive Equilibrium: The Optimality of Myopic Behaviour", *Quarterly Journal of Economics* 108, 1105-1133.

Levine, R. and D. Renelt (1992) "A Sensitivity Analysis of Cross-Country Growth Regressions", *American Economic Review* 82, 942-963.

McConnell, M.M. and G. Pérez-Quirós (2000) "Output Fluctuations in the United States: What Has Changed Since the Early 1980s", *American Economic Review* 90, 1464-1476.

Mikosch, T. (1998) *Elementary Stochastic Calculus*, Singapore/London (World Scientific Publishing).

Press, W.H., Teukolsky, S.A., Vetterling, W.T. and B.P. Flannery (1992) *Numerical Recipes in C: The Art of Scientific Computing*, 2nd edition, Cambridge (Cambridge University Press).

Roberts, K. and M.L. Weitzman (1981) "Funding Criteria for Research, Development, and Exploration Projects", *Econometrica* 49, 1261-1288.

Stock, J.H. and M.W. Watson (1996) "Evidence on Structural Instability in Macroeconomic Time Series Relations", *Journal of Business and Economic Statistics* 14, 11-30.

Stock, J.H. and M.W. Watson (1998) "Business Cycle Fluctuations in US Macroeconomic Time Series", *NBER Working Paper* 6528, Cambridge (Mass.).

Stock, J.H. and M.W. Watson (2002) “Has the Business Cycle Changed and Why?”, *NBER Macroeconomic Annual* 17, 159-218.