Non-stationary inflation and panel estimates of United States short and long-run Phillips curves
Russell, Bill

Publication date:
2007

Citation for published version (APA):
Non-Stationary Inflation and Panel Estimates of United States Short and Long-run Phillips curves

Bill Russell
NON-STATIONARY INFLATION AND PANEL ESTIMATES OF UNITED STATES SHORT AND LONG-RUN PHILLIPS CURVES

Bill Russell

21 May 2007

ABSTRACT

This paper argues that because United States inflation has been non-stationary over the past 5 decades the body of empirical research that proceeds assuming explicitly or implicitly that inflation is stationary with constant mean is largely invalid. Using 50 years of US inflation data the standard results in the Phillips curve literature are shown to be due to unaccounted shifts in the mean rates of inflation over the period. We then proceed to estimate short and long-run Phillips curves for the United States using time series panel data techniques which account for these shifts in mean.

Keywords: Phillips curve, inflation, panel data, non-stationary data
JEL Classification: C22, C23, E31

* Department of Economic Studies, University of Dundee, Dundee, DD1 4HN, United Kingdom. Email: brussell@brolga.net. Tel. 01382 384443. Fax. 01382 384691. I would like to thank Hassan Molana for his considered comments and suggestions, Steve Bond, Martin Chalkley and Colin Tilley for their detailed and helpful insights into estimating panel time series models, Issam Malki for the Bai-Perron estimates of the shifts in the mean rate of inflation, and Pierre Perron for making available the Bai-Perron programme.
1. **INTRODUCTION**

Consider estimating the model:

\[ y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 w_{t-1} + \nu_t \]  

where \( y_t \) is an unspecified time series of data, \( w_t \) is a stationary time series of data with a constant mean, and \( \nu_t \) is a random error term. With no further information concerning \( y_t \) we have no prior belief concerning an estimate of \( \alpha_1 \). However, if we believe that \( y_t \) is integrated of order 1 or trend stationary then we would expect that estimates of \( \alpha_1 \) to be insignificantly different from 1. Alternatively, if \( y_t \) is a stationary process with shifting means then the estimate of \( \alpha_1 \) will be biased towards 1. Importantly, if the shifts in mean are frequent and/or large then estimates of \( \alpha_1 \) will be insignificantly different from 1.\(^1\) These conclusions are not affected by the choice of estimator or the inclusion of more complicated dynamics such as adding to the model a lead or further lags in \( y \).

Consider now that \( y_t \) is United States inflation data for the last 50 years. Our prior beliefs concerning the estimate of \( \alpha_1 \) in equation (1) now depends on what we believe is the ‘true’ statistical process of inflation over this period. Given inflation in developed economies appears to be bounded below at around zero and above at some moderate rate it is unlikely that inflation is truly an integrated variable. It is also unlikely that inflation is trend stationary unless the trend is a proxy for a systematic unidirectional change in the central bank’s target rate of inflation.

The third alternative is that inflation is stationary with shifting means.\(^2\) The dynamics of inflation in ‘modern’ Phillips curve theories since Friedman (1968) and Phelps (1967) start with a discrete shift in monetary policy that leads to a discrete shift in the long-run rate of inflation.

\[^1\] This is a generalisation of the Perron (1989) result that large shifts in mean lead to the erroneous acceptance that the data contains a unit root. That is, the estimate of \( \alpha_1 \) in equation (1) is insignificantly different from 1. See also Banerjee and Urga (2005).

\[^2\] A fourth alternative is for inflation to be integrated of order greater than 1. It is hard to imagine how this process would be generated and so this alternative is excluded from this discussion.
inflation. In the short-run, inflation displays stationary perturbations around the long-run rate. Consequently, we may expect inflation to be a stationary process with shifts in mean where the latter represent changes in the long-run rate of inflation due to changes in monetary policy.

Return now to estimating equation (1) with the last 50 years of United States inflation data. If we believe that the data is stationary with frequent shifts in mean then we must conclude that estimates of $\alpha_t$ will be biased towards 1 unless we account for these shifts in the mean rate of inflation in the estimation process. Consequently, we might conclude that the extensive empirical literature that examines the veracity of ‘modern’ Phillips curve theories by estimating the coefficients on leads and lags in inflation in models based on equation (1) is invalid as the shifts in the mean rate of inflation are not explicitly accounted for. The estimates will be imprecise and biased towards accepting the hypothesis that the sum of the lags and leads in inflation is 1. Furthermore, once the shifts in mean are accounted for in the estimation process, the sum of the estimated coefficients on the dynamic inflation terms (i.e. the leads and lags of inflation) must be less than 1. If this is not the case then the inflation data remains non-stationary suggesting that the shifts in mean have not been properly accounted for in the estimation process. This paper empirically demonstrates these conclusions before providing estimates of the short and long-run Phillips curves that explicitly account for the shifts in the mean rate of inflation in the data.

2. **ESTIMATING ‘MODERN’ PHILLIPS CURVE MODELS OF INFLATION**

We can understand modern Phillips curve theories in terms of the Hybrid Phillips curve where inflation, $\Delta p_t$, depends on expected inflation, $E_t(\Delta p_{t+1})$, conditioned on information available at time $t$, lagged inflation, $\Delta p_{t-1}$, and a ‘forcing’ variable, $x_t$, and written:

$$\Delta p_t = \delta_j E_t(\Delta p_{t+1}) + \delta_t \Delta p_{t-1} + \delta_x x_t + \epsilon_t$$

---

3 ‘Modern’ Phillips curve theories include Friedman-Phelps expectations augmented, New Keynesian and Hybrid theories as well as the New Classical and real business cycle theories of inflation.

4 The term ‘non-stationary’ is used in this paper to mean all statistical processes other than stationary with a constant mean and includes stationary with shifting means.
The error term, \( \varepsilon_t \), is due to the random errors of agents and the shocks to inflation. The ‘forcing’ variable represents excess demand and measured in the literature in a variety of ways including the gap between the unemployment rate and its long-run level, the gap between real and potential output, real marginal costs, and labour’s income share.

In the purely backward looking Friedman (1968) and Phelps (1967) (F-P) expectations augmented Phillips curve model, agents hold adaptive expectations implying that \( \delta_f = 0 \) and \( \delta_b = 1. \) In the purely forward-looking rational expectations New Keynesian (NK) Phillips Curve models of Clarida, Gali and Gertler (1999) and Svensson (2000) \( \delta_b = 0 \) and \( \delta_f = 1. \) Finally, the more general Hybrid model of Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001) assumes that there are both backward and forward-looking price setting agents and that \( \delta_f + \delta_b = 1. \) For the long-run Phillips curve to be vertical requires \( \delta_f + \delta_b = 1 \) in all three models of inflation.

The vertical long-run Phillips curve is a central tenant of ‘modern’ Phillips curve theories of inflation since Friedman (1968) and Phelps (1967) and implies that inflation may be non-stationary with multiple long-run rates of inflation. Indeed, a large measure of Friedman’s success in establishing the existence of the vertical long-run Phillips curve was in predicting the ‘breakdown’ of the original Phillips curve identified by Phillips (1958). The ‘breakdown’ was due to changes in the expected rate of inflation associated with changes in the long-run rate of inflation and therefore concomitant with inflation being non-stationary.

If inflation is non-stationary then the empirical Phillips curve literature reveals a strange dichotomy in the economics profession. Since the work of Yule (1926), Granger and Newbold (1974, 1977), Plosser and Schewert (1978), Hendry (1980), Box and Jenkins (1976) and Phillips (1986) on ‘spurious’ regressions, applied time series economists carefully avoid estimating models with non-stationary data. The dichotomy is that even though applied time series economists are careful not to use inappropriate estimation techniques on non-stationary data.

---


6 The term Friedman-Phelps Phillips curve acknowledges the intellectual shoulders that the ‘modern’ Phillips curve literature stands on.
data, nearly all of the empirical work on the ‘modern’ Phillips curve fails to account for the shifts in the mean rates of inflation and makes use of estimation techniques that are suitable for models where the data is stationary with a constant mean.\(^7\)

For example, Gordon (1970, 1975, 1977, and 1997) and Alogoskoufis and Smith (1991) estimate versions of equation (2) where \(\delta_f = 0\). McCallum (1976) and Sumner and Ward (1983) estimate models where wage growth is the dependent variable and Roberts (1995) imposes the restriction that \(\delta_f = 1\) and estimates a model where the dependent variable is \(\Delta p_t - E_t(\Delta p_{t+1})\). A wide range of estimators are used to estimate these forms of Phillips curve models including ordinary least squares (OLS), generalised least squares, instrumental variables and full information maximum likelihood estimators. More recently, the New Keynesian and Hybrid Phillips curve work of Batini, Jackson and Nickell (2000, 2005), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001, 2005), and Rudd and Whelan (2005) estimate models using generalised method of moments (GMM) estimators to overcome the problems of correlation between expected inflation and the error term and the endogeneity of the forcing variable, \(x_t\).\(^8\)

All these estimators provide biased estimates when the shifts in mean are not accounted for in the data and will lead to the erroneous acceptance of the hypothesis that \(\delta_f + \delta_h = 1\).\(^9\) The estimation of Phillips curve models without accounting for the shifts in mean is all the more surprising given that the modern Phillips curve literature leads us to expect that inflation is stationary with shifting means.

The inability to recognise that inflation is non-stationary raises questions concerning the validity of recent empirical work on the competing Phillips curve models. This work uses the estimated values for \(\delta_f\) and \(\delta_h\) in equation (2) to choose between the competing models on

---

\(^7\) The Phillips curve literature is substantial. A search using EconLit of titles and keywords containing ‘Phillips Curve’ returns around 450 and 1000 pieces of work respectively since 1969.

\(^8\) Pesaran (1981, 1987), Stock, Wright, and Yogo (2002), Mavroeidis (2004, 2005) and Dufour, Khalaf and Kichian (2006a, 2006b) argue it is inappropriate to estimate Phillips curve models with GMM estimators when the data is integrated or near integrated. As demonstrated empirically below, once the shifts in mean are accounted for, the inflation data is not integrated and so this criticism is not relevant.

\(^9\) See footnote 1.
the basis of the veracity of the underlying ‘model-defining’ behavioural assumptions. This behavioural emphasis on the estimated values of \( \delta_f \) and \( \delta_b \) is entirely misplaced. If the size of the estimated coefficients is due to the important ‘model-defining’ behaviour of economic agents then this behaviour should not only be present over the whole sample but also during all sub-samples. In other words, if the behaviour is present and stable then the estimated coefficients should be stable and sum to 1. Without even estimating equation (2) we know that the estimated coefficients must be unstable if inflation is stationary with shifts in mean. In this case, estimates of the sum of \( \delta_f \) and \( \delta_b \) will be insignificantly different from 1 over the whole sample due to the shifts in mean. However, within each stationary episode (or inflation regime) where the mean is constant, we know that the sum of the estimated coefficients, \( \delta_f + \delta_b \), must be less than 1.\(^{10}\) If this was not the case, then the data within an individual inflation regime contains a unit root and would be non-stationary which is not possible if the data is stationary with a constant mean. Therefore, the estimates of \( \delta_f \) and \( \delta_b \) must be unstable when inflation is stationary with shifting means and do not have a behavioural interpretation unless we accept the underlying behaviour of the models is similarly unstable.

Three propositions that follow from the discussion so far are considered below. First, inflation is non-stationary. In Section 3, this proposition is established by considering what the implications are if the converse is true. That is, what are the implications if inflation is stationary with a constant mean? As the converse can be rejected as inconsistent with our understanding of the inflationary process we can conclude that inflation must be non-stationary.

The second proposition is that estimating Phillips curve models without taking into account that inflation is non-stationary leads to biased estimates of \( \delta_f \) and \( \delta_b \) and, in turn, incorrect inferences concerning the underlying behaviour of economic agents. This proposition is demonstrated in Section 4 by estimating the Phillips curve models (F-P and Hybrid) using two sets of inflation data. The models are first estimated with 50 years of actual United

\(^{10}\) Russell and Banerjee (2006) compare full sample and rolling 10 year estimates of Phillips curve models. The former provides estimates very similar to those in the literature. The later demonstrates the estimates are unstable whenever the 10 year sample passes over a shift in the mean rate of inflation.
States quarterly consumer price index (CPI) inflation data to re-establish the standard results in the literature. Both models are then estimated with inflation data that has been de-meaned to remove the multiple shifts in the mean rate of inflation. If the actual inflation data is stationary with a constant mean then the estimated coefficients on expected and lagged inflation using actual inflation will be essentially the same as those estimated using the de-meaned data. De-meaning of the inflation data will only affect the value of the estimated constant.

However, if the inflation data is stationary with multiple shifts in mean as argued here then the estimates using the non-stationary actual inflation data will differ from the estimates from the stationary de-meaned inflation data. Section 4 shows that there are large differences in the estimates from the two data sets. For example, when estimating the Hybrid Phillips curve the estimated coefficient on expected inflation is insignificantly different from zero when account is taken of the shifts in mean. Furthermore, estimates on lagged inflation are around 0.47 and significantly less than 1 by a wide margin. We can conclude, therefore, that the data is inconsistent with the hypotheses of forward looking behaviour of agents as in the New Keynesian model and with adaptive expectations as in the Friedman-Phelps model once we account for the non-stationarity in the data.

The final proposition is that the estimates of $\delta_f$ and $\delta_y$ in the Phillips curve literature are a direct result of the non-stationary properties of the inflation data. In short, the empirical results published in the standard literature over the past 35 years that do not account for the non-stationary properties of the inflation data are ‘spurious’ regressions. This is demonstrated in Section 5 by creating a ‘mean-shift’ inflation series comprising the mean rate of inflation from each inflation regime and a random variable with unit variance. By design, the ‘mean-shift’ series has no relevant information concerning actual inflation except the size and timing of the shifts in the mean rate of inflation. The Phillips curve models are then estimated with actual inflation as the dependent variable and the independent variables of expected and lagged inflation based on the ‘mean-shift’ inflation series. It is found that the estimates are similar in magnitude and significance as those reported in Section 4 and in the standard literature. This demonstrates that the significance and size of the estimated coefficients are almost entirely due to the shifts in the mean rate of inflation.
Having demonstrated that the standard empirical results are due to unaccounted shifts in the mean rate of inflation, we proceed in Section 6 to estimate a Phillips curve model of United States inflation that allows for these shifts in mean. The data is separated into eight ‘inflation regimes’ where inflation is stationary with a constant mean in each regime. Each inflation ‘regime’ can then be modelled as an individual time series of data and this allows us to estimate Phillips curve models using standard, and well understood, time series panel techniques.\(^{11}\) The estimated models allow the identification of individual short-run Phillips curves for each inflation regime and indirectly allow us to identify the long-run Phillips curve. Once the shifts in mean are accounted for, the panel estimates suggest that (i) there is still no significant evidence that expected inflation as measured in the standard literature influences inflation; (ii) lagged inflation has a coefficient significantly less than 1 which is inconsistent with the ‘strict’ adaptive expectations version of the Friedman – Phelps Phillips curve model of inflation; (iii) the long-run Phillips curve has a small and significant positive slope in line with the views expressed in Friedman’s (1977) Nobel lecture and Russell and Banerjee (2006); and (iv) there is evidence that the long-run curve is non-linear and becomes steeper with higher levels of mean inflation.

3. **PROPOSITION 1 - INFLATION IS NOT STATIONARY WITH A CONSTANT MEAN**

There are few statements that we can be confident about in macroeconomics. One of them is that inflation over the past 50 years has not been stationary with a constant mean. Leaving aside evidence from unit root tests that are notoriously unreliable due to their low power, the statement logically follows by considering what the implications are if the converse of this statement is true. If inflation is stationary with a constant mean then there is a unique long-run rate of inflation and this would imply that:

(i) The question ‘what is the long-run rate of inflation?’ is valid. Furthermore, the answer must be invariant to whether you are standing in 1950, 1974, 1989, 1995 or 2007 and if you are looking into the future or into the past. A common method of estimating the long-run rate of inflation is to simply measure the mean rate of inflation over the sample under

\(^{11}\) I am very thankful to Hassan Molana who suggested during a conversation concerning the identification of inflation regimes that, once inflation has been transformed into a stationary process with a constant mean in each inflation regime, panel data techniques are a valid estimation procedure.
consideration. For the period March 1952 to September 2004, annualised United States CPI inflation had a mean of around 3 ¾ per cent compared with a mean of around 4 ¾ per cent since March 1970 and a mean of around 2 ½ per cent for the last ten years (see Graph 1). If there is a constant long-run rate of inflation then the obvious question is which data period provides the ‘true’ long-run rate of inflation. The usual response to this argument is to say there are ‘breaks’ in the inflation series in the 1970s, 1980s and 1990s. This response simply acknowledges that there have been shifts in the mean rate of inflation. That is, the long-run rate of inflation is not constant.

(ii) Institutional arrangements have no impact on the long-run rate of inflation. For example, the targeting of inflation, money or exchange rates, the level of independence of the central bank, or the personalities of the governors of the central bank (i.e. Volker versus Greenspan versus Bernanke, or conservative versus expansionary central bankers) will have no effect on the constant long-run rate of inflation. Instead, these issues only influence how fast inflation returns to the long-run rate of inflation and not the long-run rate itself.

(iii) All the monetary economics and macroeconomics literature that describes the dynamics associated with changes in the long-run rate of money growth is ‘misplaced’ as only one growth rate of money is consistent with the long-run rate of inflation. Similarly the debate surrounding the optimum rate of inflation is meaningless if, in a practical sense, there is only one rate of inflation in the long run.12

(iv) The long-run Phillips curve in an applied sense is a single point as there is only one rate of inflation in the long run. There is also only one short-run Phillips curve as there is only one expected rate of inflation associated with the unique long-run rate of inflation. This means that economies with low inflation in the 1960s and 1990s are on the same short-run Phillips curve as during the high inflation of the 1970s and 1980s. Furthermore, given there is no change in the long-run rate of inflation then the original Phillips curve did not ‘breakdown’. Therefore, the arguments of Friedman (1968) and Phelps’ (1967) concerning the vertical long-run Phillips curve are not relevant on a practical level. Furthermore, Phillips’ (1958) original arguments are valid in terms of a stable trade-off between inflation

and the unemployment rate. Finally, any theoretical discussion of the dynamics that an economy will display during the transition between different rates of inflation in the long run is meaningless as the economy has not experienced any change in the long-run rate of inflation.

Unless we are willing to accept what is implied by a constant long-run rate of inflation we must conclude that inflation does not have a constant mean. That is, inflation is non-stationary.

Graph 1 shows United States inflation measured as the quarterly change in the natural logarithm (multiplied by 400) of the seasonally adjusted consumer price index (CPI) for the period March 1952 to September 2004. One of the striking characteristics of United States inflation is its similarity to the inflation processes in the developed and many of the developing economies over the past 50 years. After a protracted period of low inflation in the 1950s and early 1960s, inflation began to increase towards the end of the 1960s. The high inflation in the 1970s and early 1980s associated with two Organisation of Petroleum Exporting Countries (OPEC) oil price increases is then followed by a discrete reduction in inflation early in the 1980s (the ‘Volker deflation’) and then again in the early 1990s at the time of a large recession.

These visual shifts in mean inflation can be shown more formally by applying the Bai and Perron (1998, 2003a, 2003b) technique to estimate multiple breaks in the mean rate of inflation. This technique identifies 7 shifts in the mean rate of inflation and therefore 8 ‘inflation regimes’ over this 50 year period. The mean rates of inflation for each ‘inflation regime’ are shown on Graph 1 as horizontal solid thin lines. From a purely visual perspective, the Bai-Perron technique appears to have identified all the large shifts in mean inflation over the period. However, the technique may have missed two of the smaller shifts in mean inflation in 1955-1957 and 1997-1999 and possibly some small movements in mean inflation within the other inflation regimes.

---

See Appendix 1 for details and sources of the data used in this paper.

See Appendix 2 for details concerning the estimation of inflation regimes using the Bai-Perron technique.
To demonstrate that the estimates of the coefficients are biased and due to overlooked non-stationarity in the data we estimate the Hybrid and F-P Phillips curve models over the sample March 1952 to September 2004 with two series of inflation data. The first series is the actual inflation data and the second is the inflation data de-meaned for the shifts in mean inflation in each of the eight inflation ‘regimes’ reported in Graph 1. Graph 2 shows the de-meaned inflation data.

Estimating the Phillips curve models with actual and de-meaned inflation presents two possible outcomes. If the actual inflation data is stationary with a constant mean then the estimated coefficients on the inflation terms in the models estimated with actual and de-meaned inflation will be the same. De-meaning the inflation data will only affect the size of the constant. The second possible outcome is when the inflation data are stationary with shifting means. In this case, the models estimated with the de-meaned inflation data will provide unbiased estimates while the models estimated with the actual non-stationary inflation data will provide estimates where the sum of the coefficients on the explanatory inflation terms is biased towards 1.

The forcing variable, $x_t$, is the gap between the actual, $U_t$, and potential unemployment rates, $U_t^*$, and measured as the United States unemployment rate adjusted for a broken trend in June 1978. To conform to the recent Hybrid Phillips curve literature, the models are estimated using GMM with instruments of three lags of both inflation and the de-trended unemployment rate. The Hybrid Phillips curve encompasses both the F-P and NK models with a single lead and a single lag in inflation. The Friedman-Phelps model is estimated with three lags of inflation. In both models the number of lags of inflation is chosen by a 5 per cent $t$ criterion.

---

15 The New Keynesian model restricts $\delta_t = 0$ in the Hybrid model. As this restriction is comprehensively rejected by the data the New Keynesian estimates are not reported for clarity of exposition.

16 The break in trend was identified using the Perron (1998) technique.

17 The results and conclusions presented here do not depend on GMM and are robust to any form of estimation technique including ordinary least squares and two stage least squares as long as the technique is only appropriate for stationary data with a constant mean.
The estimated Hybrid and Friedman-Phelps (F-P) models using the full data sample are reported in Table 1 as models 1 and 2 respectively. The results are in line with estimates published in the literature and demonstrate (a) from above that the sum of the estimates of $\delta_f$ and $\delta_b$ is insignificantly different from 1 in both models.\(^{18}\) Estimates from the Hybrid model also identify the dominant role played by expected inflation and the forward looking agents. However, note that the models are badly mis-specified in terms of serially correlated errors. In the F-P model, the sum of the lags of inflation is insignificantly different from 1.

The results of estimating the Phillips curve models with the de-meaned inflation data are reported in columns 3 and 4 in Table 1. In the Hybrid model where the data can distinguish between the competing F-P and NK models, we now find no significant role for expected inflation and a coefficient on lagged inflation similar in size to that in the F-P model. As expected when estimating the F-P model with de-meaned data which is now stationary with a constant mean, the sum of the coefficients on lagged inflation is 0.44 and significantly less than 1. The F-P model now appears well specified while the Hybrid model that incorporates the insignificant expected inflation remains badly mis-specified.

Note that the unemployment term in the Hybrid model (Table 1, column 1) is insignificant. This is a common finding in the literature and motivates Gali and Gertler (1999) to substitute this term with labour’s income share which they find significant. We do not conduct a ‘search’ for a significant forcing variable to replace the unemployment term as once we account for the shifts in the mean rate of inflation the unemployment term is significant with the expected sign in the Phillips curve models. It appears the ‘forcing’ variable is insignificant because it is incapable of explaining the non-stationarity in the inflation data.

In summary, when the models are estimated with de-meaned data, expected inflation is insignificant in the Hybrid model and both models comprehensively reject the hypothesis that $\delta_f + \delta_b = 1$. This is in contrast with the same models estimated with the actual non-stationary inflation data where expected inflation is significant and we accept $\delta_f + \delta_b = 1$.

\(^{18}\) The results can be compared with those reported in Table 1 of Gali, et. al. (2005) or any of the Hybrid Phillips curve papers cited in Section 2.
5. **PROPOSITION 3 – THE STANDARD RESULTS ARE DUE TO SHIFTS IN MEAN INFLATION**

In the previous section we show that the standard results of the ‘modern’ Phillips curve literature disappear if the shifts in the mean rate of inflation are accounted for in the data. Some observers may feel that the results are in some way due to how the inflation data were de-meaned. This section, therefore, undertakes the opposite experiment by making use of a ‘mean-shift’ inflation series, $\Delta p_{i,MS}$, constructed as:

$$\Delta p_{i,MS} = \sum_{i=1}^{8} \left[ R_i \mu_i + R_i \text{Random} \ (0, 1) \right]$$  \hspace{1cm} (3)

where $R_i$ is a shift dummy that takes the value of zero in each inflation regime except in regime $i$ where the dummy takes the value of one, $\mu_i$ is the mean rate of inflation in regime $i$, and $\text{Random} \ (0, 1)$ is a random variable taken from a standard normal distribution with mean zero and unit variance. Graph 3 shows the first ‘mean-shift’ inflation series generated from the random distribution.

The ‘mean-shift’ inflation series is then used as the explanatory inflation series in the Phillips curve models where the dependent variable is actual inflation, such that:

$$\Delta p_i = \delta_f E_i(\Delta p_{i+1,MS}) + \delta_b \Delta p_{i-1,MS} + \delta_c \left( U - U^* \right) + \epsilon_i$$  \hspace{1cm} (4)

Note that in equation (4), the only information contained in the explanatory variables that is relevant to explaining the dependent variable (which is actual inflation) is the shift in mean contained in $\Delta p_{i,MS}$ and the forcing variable, $U - U^*$. Finally, this model is estimated 10,000 times using Monte Carlo techniques so as to recover the mean values of the estimates.

---

19 An alternative method for identifying multiple breaks in data series is the spectral density technique of Ahamada, Jouini, and Boutahar (2004) and Ben Aissa, Boutahar and Jouini (2004). However, if the Bai-Perron technique has poorly identified the shifts in the mean rate of inflation then the data would remain non-stationary after de-meaning and the estimates of $\delta_f$ and $\delta_b$ would remain biased and sum to 1. A weak indirect ‘test’ of how successful the Bai-Perron technique has identified the shifts in mean is that $\delta_f + \delta_b < 1$ in the models estimated with the de-meaned data.
The mean values of the estimates from the Hybrid and F-P versions of equation (4) are reported in columns 1 and 2 of Table 2 and are very similar to the estimates reported in the first two columns of Table 1. In the Hybrid model, the estimated coefficient on expected inflation is 1.3535 (0.7055 in Table 1) and the coefficient on lagged inflation is insignificant and –0.1718 compared with a significant estimate of 0.2946 in Table 1. The sum of the lagged coefficients in the F-P model is still insignificantly less than 1 (0.9881 compared with 0.9283 in Table 1).

This experiment points strongly to why expected inflation is a significant explanatory variable in the standard Hybrid Phillips curve literature. The mean-shift inflation series contains no relevant information for explaining actual inflation other than the size and timing of the shifts in mean inflation. The Hybrid model in column 1 of Table 2 shows that the coefficient on expected inflation is insignificantly different from 1. This can only be due to the shifts in mean contained in the mean-shift inflation series. Simultaneously, the lag in inflation is insignificant. It appears that the estimation procedure explains the shift in the mean rate of actual inflation by the expected inflation term and not the lag in inflation. Having explained the actual shift in mean inflation, the mean-shift inflation data contains no further information and so the lag in mean-shift inflation is insignificant. One might hypothesise that if the lag in inflation did contain some relevant information concerning actual inflation then the lag in inflation would also be significant in the Hybrid model. Consequently, in the standard empirical Hybrid Phillips curve literature, expected inflation is significant (and large) due to the unaccounted shifts in mean inflation while lagged inflation is significant (and small) due to the information content of the inflation data other than the shifts in mean inflation.

Finally, we estimate the Phillips curve models using only the ‘mean-shift’ inflation series so that:

\[
\Delta p_t^{MS} = \delta_f E_t(\Delta p_t^{MS}) + \delta_b \Delta p_{t-1}^{MS} + \delta_u (U - U^*) + \epsilon_i
\]  

(5)

20 The random data is generated using RATS 5.01 with a ‘seed value’ of 171293.
The coefficients $\delta_f$ and $\delta_{\delta}$ now represent the ability of the ‘mean-shift’ inflation series to explain the shifts in mean contained in $\Delta p^M_t$. The models are again estimated 10,000 times and the mean values of the estimates reported in columns 3 and 4 in Table 2. As expected, the results are very similar to those reported in columns 1 and 2 in the same table in terms of the size of the estimated coefficients and the diagnostics of the estimated models.

It appears, therefore, that the shifts in mean alone generate results very similar to those in the standard literature. Furthermore, if the shifts in mean are accounted for in the estimation processes then the standard results (i.e. $\delta_h = 1$ in the F-P model; $\delta_f + \delta_h = 1$ and $\delta_f > \delta_h$ in the Hybrid model) disappear. Consequently, we may conclude that the results reported in the ‘modern’ Phillips curve literature are dominated by, and are as a direct result of, the shifts in the mean rate of inflation that are not accounted for in the estimation of the models.

6. PANEL ESTIMATES OF UNITED STATES PHILLIPS CURVES

Having demonstrated that the standard estimation techniques are inappropriate for estimating Phillips curves when inflation is non-stationary, we now proceed to estimate short and long-run Phillips curves assuming explicitly that inflation is stationary with shifting means.\(^{21}\) Based on this assumption, we separate the data into eight inflation regimes where the mean rate of inflation is constant in each regime. We then organise the data as time series of eight individual inflation regimes. As the data are stationary with a constant mean by construction this allows us to analyse the data using standard unbalanced panel estimation techniques to simultaneously estimate the short-run Phillips curve for each of the inflation regimes.

In the model that we wish to estimate, the number of inflation regimes, $n$, is small (in our case 8) and the number of time periods, $t$, is large relative to $n$ and the regimes are unbalanced. Furthermore, although there is a time dimension within each regime, the time

---

\(^{21}\) If we acknowledge that inflation is non-stationary but instead assume the data is integrated then there are two ways to proceed. The first is that followed by King and Watson (1994) who difference the inflation data. However, if the ‘true’ statistical process is stationary with shifting means then this approach will lead to erroneous estimates as demonstrated in Appendix 3. The second is that followed by Russell and Banerjee (2006) who estimate an I(1) system to identify the long-run cointegrating relationship between inflation and the unemployment rate. While assuming that inflation is an integrated variable may be a good approximation of the ‘true’ statistical process if there are very frequent shifts in mean, this approach does not allow the estimation of short-run Phillips curves.
periods are not aligned across regimes. As such, the model does not conform neatly to the usual estimation of panel data.\textsuperscript{22} However, two broad estimators of the model present themselves. The random effects estimator assumes the coefficients of the model are not fixed parameters to be estimated but random parameters from a distribution which is mean zero and constant variance. Important assumptions are that the random effects are uncorrelated with the other explanators and that the inflation rate is a random draw from a distribution which is common across regimes. With the inflation regimes defined by different mean rates of inflation the distributions of the regimes are not common by construction. Therefore, the random effects model is conceptually inappropriate.

The fixed effects estimator accounts for the different mean rates of inflation across regimes by introducing a constant for each regime. This estimator is sometimes referred to as the ‘within estimator’ for it uses the within regime, and not the between regime, variance in the data. There are therefore as many constants as regimes and as the number of regimes increase relative to the number of time periods there is a loss of efficiency as we only have $t$ observations to estimate the $n$ constants. As the estimated fixed effects in the model have a straightforward economic interpretation and the number of regimes is small we present the fixed effects estimates below.

The panel fixed effects specification of the Hybrid Phillips curve model of equation (2) can be written:

$$
\Delta p_t^n = \phi^n + \phi_f E_t^n (\Delta p_{t+1}^n) + \phi_b \Delta p_{t-1}^n + \phi_i U_t^n + \eta_t^n
$$

(6)

where the ‘$n$’ superscript indicates the inflation regime that the data is drawn from. The unobserved regime-specific time invariant effects, $\phi^n$, allow for shifts in the mean rate of inflation across regimes and $\eta_t^n$ is a disturbance term which is independent across inflation regimes.\textsuperscript{23} The hypothesis that the coefficients $\phi_f$, $\phi_b$ and $\phi_i$ are the same across regimes

\textsuperscript{22} See Smith and Fuertes (2003) for an excellent introduction to time series panel estimation and Baltagi (2001) and Hsiao (2003) for more detailed studies.

\textsuperscript{23} On a conceptual level this assumption must hold as the time periods are not aligned across inflation regimes.
cannot be rejected by the data leading to the restricted model in equation (6) (see the notes to Table 4).

Panel estimation of Phillips curve models allows us to:

(i) Estimate the constants, $\phi^*$, or fixed effects, that allow for different mean rates of inflation in each inflation ‘regime’;

(ii) Estimate the coefficients on expected, $\phi_f$, and lagged inflation, $\phi_b$, to examine the veracity and size of the forward and backward looking behaviour of agents;

(iii) Estimate the short-run Phillips curve for each of the inflation regimes; and

(iv) Calculate the implied long-run rate of unemployment for each inflation regime where long-run inflation is assumed equal to the mean rate of inflation for that regime.

There is a large literature on the biases in estimating dynamic panels that include lagged dependent variables when $t$ is small relative to a large $n$.\(^{24}\) The problem is that an unmodelled shock to inflation in one period will simultaneously affect the estimated constant (i.e. the fixed effect) and the estimated error which violates one of the assumptions of the fixed effects modelling procedure.\(^{25}\) However, as the number of time periods increase for a given number of individuals the correlation between the fixed effect and error term declines as the shocks average out over time and any individual shock has only $1/t-1$ impact on the estimated constant.\(^{26}\)

An important question for our purposes is when does $t$ become ‘large’? A ‘rule-of-thumb’ is that $t$ is large when it is sensible (in terms of degrees of freedom) to estimate individual

\(^{24}\) For example, see Arellano and Bond (1991), Arellano and Bover (1995), Blundell and Bond (1998) and Bond (2002).

\(^{25}\) See Nickell (1981).

\(^{26}\) One way to overcome the problem of ‘dynamic panel bias’ when $t$ is very small is to difference the data so as to eliminate $\phi^*$ from equation (6). See the references cited in footnote 24. Estimating the model with the Arellano-Bond estimator does not affect the results in an economic or quantitative sense.
equations for each regimes. In our case the regimes are unbalanced with most of the regimes consistent with this rule but some of the shorter inflation regimes are not. However, we proceed to estimate the fixed effects model using all inflation regimes and note that very similar results are obtained by estimating the model with only the longer inflation regimes. This issue is returned to at the end of Section 6.1.

Finally, it is likely that inflation and the unemployment rate are determined simultaneously and so the contemporaneous unemployment rate is endogenous and not weakly exogenous in the estimated model. Furthermore, our measure of expected inflation suggests it will be correlated with the error term. We address these problems by estimating the fixed effects model using two stage least squares (2SLS) where the instruments are two lags of inflation and the unemployment rate.

6.1 Panel estimates of the United States Phillips curve

Reorganising the data as an unbalanced time series panel does not in itself affect the standard results of the Phillips curve literature. This is easily demonstrated by estimating equation (6) with the constant, $\phi^n$, restricted to be the same across all 8 inflation regimes. This is equivalent to estimating the model assuming the mean rate of inflation is the same for each inflation regime as in the standard empirical literature. Two stage least squares estimates of the Hybrid and Friedman-Phelps Phillips curve models with the constant restricted are provided in Table 3. Note the results are very similar to those reported for the respective models in the first two columns of Table 1 and in the standard Phillips curve literature. For the Hybrid model, the sum of the estimated coefficients on the lead and lag of inflation is insignificantly different to 1 ($\phi_f + \phi_b = 0.9693$) and that there is a significant role for both forward and backward looking agents. For the F-P model the sum of the estimated coefficients on the three lags in inflation is not significantly different from 1 ($\sum \phi_b = 0.9810$).
Two stage least squares panel estimates of the fixed effects Hybrid model are reported in column 1 of Table 4. A further lag in inflation and the unemployment rate are insignificant.\(^{27}\) As with the de-meaned data in Section 5 we are unable to identify a significant role for expected inflation in the inflationary process and the sum of the estimated coefficients on the inflation terms is 0.4736 which is significantly less than 1.

Excluding the insignificant expected inflation term, the estimated Friedman-Phelps model is reported in column 2 of Table 4. A further lag in inflation and the unemployment rate remain insignificant.\(^{28}\) The estimated coefficient on lagged inflation is now 0.3323 which remains significantly less than 1. The unemployment rate is significant and negative with a value of -0.3159. These estimates are similar to the F-P estimates using the demeaned inflation data of 0.5302 and –0.3259 respectively (see Table 1, column 4) and suggest that the short-run Phillips curve for each inflation regime has a significant negative slope as we might expect.

The Bai-Perron technique results in two regimes (numbers 4 and 5) hitting the minimum quarters constraint in the estimation of the inflation regimes. Consequently the means of the data in these two regimes may not be constant. To examine whether these two regimes are in some way ‘driving’ the results reported in Table 4, the models were re-estimated with these two regimes excluded. The results are not affected in any meaningful way. In the Hybrid model, expected inflation remains insignificant and both the sum of the dynamic inflation terms, \(\phi_r + \phi_b = 0.4929\), and lagged inflation, \(\phi_b = 0.3133\), are significantly less than 1. In the F-P model the coefficient on lagged inflation is significantly less than 1, \(\phi_b = 0.3133\), and the unemployment rate is significant and negative, \(\phi_U = -0.3135\). The results are very similar to those reported in Table 4 in terms of estimated coefficients and the diagnostics of the model.

### 6.2 Calculating the Implicit Long-run Phillips curve

\(^{27}\) Likelihood ratio omitted variable tests reject the inclusion of \(\Delta p_{t-2}^n\), \(F(1, 175) = 0.7902\), prob-value = 0.3753 and \(\Delta U_{t-1}^n\), \(F(1, 175) = 0.9063\), prob-value = 0.3424.

\(^{28}\) Likelihood ratio omitted variable tests reject the inclusion of \(\Delta p_{t-2}^n\), \(F(1, 184) = 0.0376\), prob-value = 0.8465 and \(\Delta U_{t-1}^n\), \(F(1, 184) = 1.0309\), prob-value = 0.3113.
Assuming that the long-run rate of inflation is equal to the mean rate of inflation in each regime, $\bar{\Delta p}$, the implied long-run unemployment rate for inflation regime $n$, $\bar{U_n}$, can be calculated from the estimates of equation (6) as:

$$\bar{U_n} = \frac{1}{\phi_n}[\bar{\Delta p} (1 - \phi_f - \phi_b) - \phi^*]$$

(7)

If the implicit long-run unemployment rates from each inflation regime lie on the long-run Phillips curve then the locus of eight combinations of long-run rates of inflation and unemployment will loosely identify the long-run Phillips curve. This allows us to examine whether or not the long-run Phillips curve is vertical as in the strict versions of the ‘modern’ Phillips curve theories or displays a negative or positive slope. We are also able to observe any non-linearity in the long-run Phillips curve.

The standard approach to testing the slope of the long-run Phillips curve assumes a null hypothesis that the curve is vertical (i.e. test if $\phi_f + \phi_b = 1$ in equation 2). Given the bias and imprecision of the estimation techniques involved the null hypothesis is usually accepted. Leaving aside the estimates are biased, the imprecision in the estimates mean that if the standard null hypothesis is accepted then it is likely that $\phi_f + \phi_b$ will also be insignificantly different from alternative null hypotheses, such as, 0.9 or 1.1. These alternative null hypotheses lead to very different conclusions concerning the slope of the long-run Phillips curve. The standard null hypothesis is therefore an a priori restriction on the empirical examination of the slope of the long-run Phillips curve - a restriction that is avoided by calculating the long-run Phillips curve from the panel estimates.

Using the implicit long-run unemployment rates and the mean rates of inflation in each regime, ordinary least squares (OLS) estimates of linear and quadratic long-run Phillips curves are provided in the top portion of Table 5. Based on the adjusted $R^2$ criteria and that
the ‘true’ long-run Phillips curve cannot be linear if it has a slope, the non-linear model appears a better description of the long-run Phillips curve.\textsuperscript{29}

The estimated linear and non-linear long-run Phillips curves in the top portion of Table 5 suggest the long-run relationship has a small positive slope. Higher mean rates of inflation are associated with higher long-run rates of unemployment. However, having identified a positive slope to the long-run Phillips curve, an important question is whether or not the slope is significantly different from being vertical? As these curves are long-run in nature, causation between the variables is less important. Consequently, it is equally valid to estimate the long-run relationship with the unemployment rate as the dependent variable and inflation as the independent variable. The OLS linear and quadratic estimates of the long-run Phillips curve with unemployment as the dependent variable are reported in the lower portion of Table 5.

The advantage of specifying the long-run Phillips curve in this way is that if the curve is vertical as in the standard literature then it is horizontal when the unemployment rate is the dependent variable. The test of a vertical long-run Phillips curve in the standard literature is equivalent to testing whether or not the independent long-run inflation terms are insignificantly different from zero. Tests of this restriction are reported in Table 5 and are strongly rejected by the data suggesting that the long-run Phillips curve in the standard sense has a significant positive slope.

6.3 A visual representation of the short and long-run Phillips Curves

Graph 4 provides a visual representation of the panel estimates of the F-P Phillips curve model of equation (6) reported in column 2 of Table 4. The thin negatively sloped lines marked SRPC 1 to SRPC 8 are the estimated short-run Phillips curves for each of the eight inflation regimes once the short-run inflation dynamics are accounted for. Each short-run

\textsuperscript{29} If the long-run Phillips curve is linear and not vertical then as mean inflation increases the rate of unemployment would eventually become negative with a negative slope or greater than 1 with a positive slope. Both outcomes make little sense and therefore the ‘true’ long-run Phillips curve cannot be linear over the full range of inflation if the long-run curve is not vertical. However, over the range of inflation experienced by the United States (and other developed economies) over the past 50 years the long-run Phillips curve may be approximately linear.
curve is drawn for the observed range of unemployment rates in that particular inflation regime. Also shown on the graph are the actual combinations of inflation and the unemployment rate for the eight inflation regimes with the data from each regime represented by a different symbol. Shown as large crosses on the graph are the combinations of the implicit long-run rates of unemployment and the mean rate of inflation for each inflation regime. The solid line with a positive slope labelled LRPC is the OLS estimate of the non-linear long-run Phillips curve reported in Table 5.

Note that the implicit long-run rates of unemployment are not the simple mean rates of unemployment for each inflation regime. Instead, the implied long-run unemployment rates lie towards, or beyond, the end of the short-run curves in most regimes. In cases where mean inflation is increasing (i.e. regimes 3, 4, and 6) the implied long-run unemployment rate is towards the right hand end of the short-run Phillips curve and towards the left hand end when mean inflation is shifting down (regimes 5, 7, and 8). This is exactly as predicted in modern theories of the Phillips curve since Friedman and Phelps.

7. Some Implications of These Results

This paper argues that it is legitimate to model inflation as a stationary process with shifting means. This is in contrast with the standard empirical literature that usually proceeds assuming (either explicitly or implicitly) that inflation is a stationary process with a constant mean. When the statistical properties of inflation are investigated in the standard literature it is usually with reference to unit root tests. If the inflation data is found not to contain a unit root the researcher concludes the data is stationary with a constant mean. When a unit root is found in the inflation data the researcher concludes that the data is integrated but usually does not question the source of the non-stationarity. Both these conclusions are erroneous since ‘modern’ Phillips curve theories since the ‘breakdown’ of Phillips’ original curve point to inflation being stationary with shifting means. In other words, if inflation is stationary with shifting means then this would explain the unit root commonly found in the data.

It is very surprising that the standard empirical Phillips curve literature does not model inflation as a stationary process with shifting means. Furthermore, the standard finding that the estimated coefficients on the dynamic inflation terms sum to 1 is taken as important evidence in the standard literature that the underlying theories are correct. Instead, this
same finding should alert the researcher that (i) the inflation data is non-stationary; (ii) the estimation technique is inappropriate; and (iii) the estimates are biased and imprecise. Once we acknowledge that inflation is non-stationary and identify correctly the source of the non-stationarity meaningful estimates of Phillips curves are possible.

The empirical results reported above suggest that if we account for shifts in the mean rates of inflation then the standard results of the Phillips curve literature over the past 35 years disappear. An important finding above is that there is no significant role for expected inflation in the Hybrid model (as measured in the literature) once we account for the shifts in mean inflation between inflation regimes (see column 3 of Table 1 and column 1 of Table 4). As demonstrated in Section 5 (see column 1 of Table 2), it appears that the lead in inflation in the standard Hybrid model is ‘dragging’ inflation either up or down as the mean rates of inflation change between inflation regimes. Once these changes in mean are accounted for in the estimation process there is no role for the lead in inflation to play in inflation dynamics.

If we then exclude the insignificant expected inflation term from the Hybrid model, the F-P model indicates that lagged inflation is significantly less than 1 by a wide margin.\textsuperscript{30} Furthermore, we can identify a small and significant positive slope to the long-run Phillips curve.

It appears, therefore, that the consistent finding that $\delta_f + \delta_h = 1$ in the empirical Phillips curve literature over the past 35 years is due to a combination of ignoring the non-stationary properties of the inflation data and the use of inappropriate estimation techniques. One might conclude that the estimated results reported in the ‘modern’ Phillips curve literatures are ‘spurious’ in the sense of Yule (1926) and Granger and Newbold (1974).

That the long-run Phillips curve has a positive slope may unsettle some observers who would argue that the finding simply reflects the impact of supply shocks on both inflation and unemployment. Supply shocks, such as the OPEC oil price increases in the 1970s, simultaneously increase both inflation and the rate of unemployment leading to a positive

\textsuperscript{30} In the F-P model estimated with de-meaned inflation, the sum of the estimated coefficients on lagged inflation (Table 1, column 4) is 0.5302 which is nearly 6 standard errors less than 1. Similarly, the panel estimates suggest the coefficient is 0.3323 (Table 4, column 2) which is around 11 standard errors less than 1.
correlation between the variables. This is a persuasive argument for a short-run positive relationship. However, the nature of the series differ in an important way. Increases in unemployment are likely to be highly persistent due to lags in retraining, the poor mobility of workers, and the slow adaptation of capital to the new ‘post-shock’ economic environment. This is in stark contrast with the impact of supply shocks on inflation which should be short-term and transitory in an economy such as the United States where there are few or no price controls. Consequently, the supply shocks may introduce a positive bias in the estimates of the short-run Phillips curve but not in the long-run Phillips curve.

The long-run estimates reported in Table 5 can be compared with those of Russell and Banerjee (2006). They also argue that the ‘true’ statistical process of inflation is stationary with frequent shifts in mean but that this can be approximated by an integrated process. Using the same data as in this paper they estimate the long-run US Phillips curve as $\Delta p_t = \delta + 2.714U_t$, which displays a very similar positive slope to the linear long-run Phillips curve derived from the panel estimates and reported in Table 5 of $\Delta p_t = -8.1755 + 2.1190U_t$.

It appears that what is important when identifying the long-run Phillips curve is that the data is non-stationary. This observation encapsulates why the standard empirical approach to estimating Phillips curves is internally inconsistent. Consider a period of inflation where the data is stationary with a constant mean. If we estimate the Phillips curve model using the standard approach then the dynamic inflation terms must sum to less than 1 and we can calculate a unique long-run unemployment rate. With a constant mean, the data contains information about only one long-run rate of unemployment. Furthermore, the estimated Phillips curve must be a short-run Phillips curve as there is only one long-run rate of inflation and only one expected rate of inflation. To identify the long-run Phillips curve the data must contain a number of long-run (i.e. expected) rates of inflation and therefore the inflation data must exhibit shifts in mean and be non-stationary. The internal inconsistency of the standard approach is that for the method of estimation to be appropriate the inflation data must have a constant mean. Therefore, the standard approach cannot identify, nor comment on, the slope of the long-run Phillips curve as it can only reveal one combination of long-run rates of inflation and unemployment. Alternatively, if the inflation data is non-stationary so that the long-run Phillips curve can be identified then the standard estimation approach is not
An important question is whether these empirical results are in some way dependent on how the Bai-Perron technique identified the eight inflation regimes. Consider the case where the ‘true’ number and dates of the inflation regimes differ from those estimated using the Bai-Perron technique. The identified inflation regimes will then contain some residual non-stationarity and there will be an upward bias in the estimates of $\delta_f$ and $\delta_b$. As the Bai-Perron technique is unlikely to have identified exactly the ‘true’ inflation regimes we might conclude that the estimates provided above of $\delta_f$, $\delta_b$ and $\delta_f + \delta_b$ are the upper bounds of estimates based on the ‘true’ inflation regimes. Therefore, finding that (i) expected inflation plays no significant role in the inflation dynamics; (ii) $\delta_b$ is significantly less than 1; and (iii) $\delta_f + \delta_b$ is significantly less than 1 are not the result of incorrect identification of the inflation regimes by the Bai-Perron technique. Instead, given the Bai-Perron technique is unlikely to have identified the ‘true’ number and dates of the inflation regimes, this makes it more (rather than less) difficult to overturn the standard results of the empirical Phillips curve literature which rely on some non-stationarity in the data.

These results do not invalidate the expectations behaviour that underpins the respective Phillips curve models. All that is invalidated is the assumption that all economic agents behave in the ways set out in the respective theories. It is the universality of the behaviour that follows from modelling the representative agent that should be questioned. Some economic agents may look forwards and some may look backwards. But is a heroic (and very narrow) assumption that all agents behave in only one way in the Friedman-Phelps, and New Keynesian models or in only one of two ways as in the Hybrid model.

It would be hard to verify through scientific observation that economic agents only indulge in Friedman-Phelps and New Keynesian pricing behaviour. If instead agents behave in a variety of ways when adjusting prices, and maybe change pricing behaviour depending on the economic environment, then we may expect that the estimated coefficients will not conform to the strict versions of any of the standard ‘modern’ Phillips curve theories. In an important sense, these results open the way for a richer modelling of how agents adjust prices in terms of explaining the many ways that agents actually behave. Simultaneously, the results break
the strict behavioural interpretation of the estimated coefficients on expected and lagged inflation.

Finally, is the estimated slope of the long-run Phillips curve important? The estimated non-linear long-run Phillips curve (as shown in Graph 4 and Table 3) suggests that the increase in mean inflation during the 1970s from around 4 ½ to 11 per cent per annum was associated with an increase in the long-run rate of unemployment of around 2 ½ percentage points. Shifts in unemployment of this magnitude for moderate increases in inflation would appear to be important in both economic and social senses.
8. References


Econometrics, vol. 33, pp. 311-40.


APPENDIX 1  DATA APPENDIX

The consumer price index (CPI) and unemployment rate data are seasonally adjusted and obtained directly from the United States of America, Bureau of Labour Studies (BLS). The monthly data for the period March 1952 to November 2004 was downloaded on 25 November 2004. The quarterly data is the average of the monthly data. The mnemonics in Table A1 are those from the BLS database.

Table A1: Sources and details of the data manipulation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI Inflation</td>
<td>The monthly CPI is the US city average, all items, 1982-84=100, ID: CUSSR0000SA0. CPI inflation is the change in the natural logarithm of the quarterly CPI multiplied by 400 to give the annualised rate.</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>The unemployment rate is the number of people over 16 years of age as a percentage of the non-institutionalised civilian population, ID: LNS14000000.</td>
</tr>
</tbody>
</table>

The unemployment rate appears to have an increasing linear trend up to the middle to late 1970s and then a slight declining linear trend thereafter. Perron (1998) unit root test confirms this and identifies a shift in the constant and break in trend in June 1978. The de-trended unemployment rate, \((U-U^*)\), is obtained by regressing the unemployment rate on a constant, a ‘shift’ dummy for June 1978 to September 2004, trend, a truncated trend that is zero up to and including March 1978 and then increasing in unit steps between June 1978 and September 2004, and a ‘spike’ dummy for June 1978.
APPENDIX 2  IDENTIFYING THE SHIFTS IN THE MEAN RATE OF INFLATION

The Bai and Perron (1998, 2003a, 2003b) approach minimises the sum of the squared residuals to identify the dates of $k$ breaks in the inflation series and, thereby, identify $k+1$ ‘inflation regimes’. The estimated model is:

$$\Delta p_t = \gamma_{k+1} + \tau_t$$ (A2.1)

where $\Delta p_t$ is United States CPI inflation and $\gamma_{k+1}$ is a series of $k+1$ constants that estimate the mean rate of inflation in each of $k+1$ inflation regimes and $\tau_t$ is a random error. The final model is chosen using the Bayesian Information Criterion. The model is estimated using quarterly data for the period March 1952 to September 2004. The results of the estimated model are reported in the table below. Note that Graph 1 shows the estimated inflation regimes multiplied by 400 to be consistent with annualised inflation data. The Bai-Perron programme written in Gauss was kindly made available by Pierre Perron personal internet site.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dates of the ‘Inflation Regimes’</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\gamma_1$</td>
<td>March 1952 to September 1955</td>
<td>0.001040</td>
<td>0.000894</td>
</tr>
<tr>
<td>2</td>
<td>$\gamma_2$</td>
<td>December 1955 to December 1966</td>
<td>0.004687</td>
<td>0.000533</td>
</tr>
<tr>
<td>3</td>
<td>$\gamma_3$</td>
<td>March 1967 to September 1972</td>
<td>0.011323</td>
<td>0.000746</td>
</tr>
<tr>
<td>4</td>
<td>$\gamma_4$</td>
<td>December 1972 to March 1975</td>
<td>0.023194</td>
<td>0.001131</td>
</tr>
<tr>
<td>5</td>
<td>$\gamma_5$</td>
<td>June 1975 to September 1977</td>
<td>0.014975</td>
<td>0.001131</td>
</tr>
<tr>
<td>6</td>
<td>$\gamma_6$</td>
<td>December 1977 to March 1981</td>
<td>0.027216</td>
<td>0.000956</td>
</tr>
<tr>
<td>7</td>
<td>$\gamma_7$</td>
<td>June 1981 to June 1990</td>
<td>0.010038</td>
<td>0.000588</td>
</tr>
<tr>
<td>8</td>
<td>$\gamma_8$</td>
<td>September 1990 to September 2004</td>
<td>0.006329</td>
<td>0.000482</td>
</tr>
</tbody>
</table>
APPENDIX 3  DIFFERENCING INFLATION DATA

Perron (1989) shows that trend stationary processes with breaks are easily mistaken for unit root processes. The Perron argument can be generalised to the case of a stationary process with shifting means and can be demonstrated by constructing a random variable with shifts in mean and then estimating an autoregressive model. We can demonstrate this ‘generalisation’ of the Perron argument in the following way.

Consider the series $z_t$ in Graph A1 which is a random variable with unit variance and a mean of 3 between periods 0 and 70, 13 between 71 and 140, and 3 between 141 and 210. The numbers chosen (very) loosely represent United States inflation over the 50 years of quarterly data used in this paper. More shifts in mean can be incorporated to make the series look more like United States inflation but this does not change the following argument.

Graph A1: Constructed series $z$ of a random variable with shifts in mean

An augmented Dickey Fuller (adf) univariate unit root test of $z_t$ incorporating 4 lags and an intercept provides a test statistic of $-1.426$. With critical values of 1% = -3.463, 5% = -2.876 and 10% = -2.574 one might mistakenly conclude that $z_t$ is an integrated variable of order 1.
If on the basis of the adf test we incorrectly difference the data and estimate an AR(2) model using ordinary least squares (OLS), such that, \( \Delta z_t = \delta + \alpha \Delta z_{t-1} + \beta \Delta z_{t-2} + \kappa_t \) where \( \Delta z_t = z_t - z_{t-1} \) we obtain the following results:

\[
\Delta z_t = -0.0024 - 0.3378 \Delta z_{t-1} - 0.1268 \Delta z_{t-2}
\]

\((-0.0)\) \((-3.6)\) \((-1.6)\)

where the number of observation is 207, \( \bar{R}^2 = 0.09 \) and the Durbin-Watson Statistic is 1.99. The sum of the estimated coefficients, \( \alpha + \beta = -0.4647 \), with a \( t \)-statistic of \(-3.2\) and a significance level of 0.0016.

The problem demonstrated here is that if we misinterpret the shifts in mean as due to an integrated process and \textit{difference} the data then we will incorrectly conclude that there is a significant autoregressive process generating the data. This is in stark contrast with what we know is the true data generating process which is stationary with shifting means. This general result is not affected by the inclusion of further lags or a lead in \( z \) nor are they affected by estimating the models with 2SLS, GMM or full information maximum likelihood estimators.
Table 1: Demonstrating Proposition Two

<table>
<thead>
<tr>
<th>Actual Inflation Data</th>
<th>De-meaned Inflation Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model ⇒ Independent Variable ↓</td>
</tr>
<tr>
<td>Δp_{t+1}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Δp_{t-1}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Δp_{t-2}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Δp_{t-3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( (U - U^*') _t )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of Inflation Terms</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
</tr>
<tr>
<td>S.E.E</td>
<td></td>
</tr>
<tr>
<td>J-Test probability</td>
<td></td>
</tr>
<tr>
<td>LM(1) test probability</td>
<td></td>
</tr>
<tr>
<td>LM(4) test probability</td>
<td></td>
</tr>
<tr>
<td>ADF test residuals</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors reported as { }, t-statistics reported as ( ), and F test probability values as [ ]. Models 1 and 2 are estimated using actual inflation data. Models 3 and 4 are estimated with inflation data de-meaned for the 8 inflation ‘regimes’ shown in Graph 1 and reported in Appendix 2. Details concerning the de-trending of the unemployment data and the de-meaning of the inflation data are provided in the Data Appendix 1 and 2 respectively. Sample is 208 observations for the period March 1952 to September 2004.

The models are estimated by GMM in RATS 5.01 with three lags of both inflation and the unemployment rate as instruments. The J-test is the Hansen (1982) test for instrument validity. Rejection of the J-Test implies the instruments are invalid. LM(1) and LM(4) are Lagrange Multiplier tests for first and fourth order serial correlation of the residuals respectively where the null hypothesis is no serial correlation. ADF test is the augmented Dickey-Fuller unit root test of the residuals where the 1 and 5 per cent critical values are – 2.576 and - 1.941 respectively.
### Table 2: Demonstrating Proposition Three

<table>
<thead>
<tr>
<th>Dependent Variable ⇒</th>
<th>Actual Inflation $\Delta p_t^1$</th>
<th>‘Mean-shift’ Inflation $\Delta p_t^{MS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model ⇒ Independent Variable</td>
<td>Hybrid</td>
</tr>
<tr>
<td>$\Delta p_t^{MS}$</td>
<td></td>
<td>1.3535</td>
</tr>
<tr>
<td></td>
<td>(3.3)</td>
<td>(5.8)</td>
</tr>
<tr>
<td>$\Delta p_t^{MS}$</td>
<td>-0.1718</td>
<td>0.2602</td>
</tr>
<tr>
<td></td>
<td>(-0.5)</td>
<td>(3.9)</td>
</tr>
<tr>
<td>$\Delta p_t^{MS}$</td>
<td></td>
<td>0.3624</td>
</tr>
<tr>
<td></td>
<td>(6.0)</td>
<td></td>
</tr>
<tr>
<td>$\Delta p_t^{MS}$</td>
<td></td>
<td>0.3655</td>
</tr>
<tr>
<td></td>
<td>(6.1)</td>
<td></td>
</tr>
<tr>
<td>$(U - U_t^2)$</td>
<td>-0.2851</td>
<td>-0.4491</td>
</tr>
<tr>
<td></td>
<td>(-1.8)</td>
<td>(-4.7)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.6690</td>
<td>0.0284</td>
</tr>
<tr>
<td></td>
<td>(-1.3)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>Sum of Inflation Terms</td>
<td>1.1817</td>
<td>0.9881</td>
</tr>
<tr>
<td></td>
<td>(0.1534)</td>
<td>(0.0555)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.35</td>
<td>0.77</td>
</tr>
<tr>
<td>J-Test probability</td>
<td>[0.2223]</td>
<td>[0.4369]</td>
</tr>
<tr>
<td>LM(1) test probability</td>
<td>[0.0000]</td>
<td>[0.0000]</td>
</tr>
<tr>
<td>LM(4) test probability</td>
<td>[0.0000]</td>
<td>[0.0000]</td>
</tr>
<tr>
<td>ADF test residuals</td>
<td>-5.1</td>
<td>-5.8</td>
</tr>
</tbody>
</table>

The models are estimated with 209 observations re-estimated 10,000 times using Monte Carlo simulation techniques. The variable, $\Delta p_t^{MS}$, is constructed as the mean for each of the regimes plus a random variable taken from a normal distribution which is mean zero and with unit variance. The results reported are the means of the estimates from the Monte Carlo simulation of the model estimated in RATS 5.01 with a ‘seed’ of 171193. Standard errors reported as $\{}$, $t$-statistics reported as $\()$, and F-test probability values as $[\]$. The dependent variable in models 1 and 2 is the actual inflation rate. The dependent variable in models 3 and 4 is the “mean-shift” inflation variable.

The models are estimated by GMM in RATS 5.01 with three lags of both inflation and the unemployment rate as instruments. Details of the tests are provided in the notes to Table 1.
Table 3: ‘Fixed Constant’ Panel Estimates of United States Phillips Curves

**Hybrid Model**

\[
\Delta p_t^n = 0.5309 + 0.4882 \Delta p_{t+1}^n + 0.4811 \Delta p_{t-1}^n - 0.0704 U_t^n
\]

\( (1.6) \quad (4.2) \quad (4.3) \quad (-1.1) \)

\( R^2 = 0.85, \) Durban-Watson statistic 2.88. Hypothesis tests: \( \Delta p_{t+1}^n + \Delta p_{t-1}^n = 0 \) is rejected, \( F_{(1,183)} = 922.5585, \) p-val = 0.0000, and \( \Delta p_{t+1}^n + \Delta p_{t-1}^n = 1 \) is accepted \( F_{(1,183)} = 0.9224, \) p-val = 0.3381. Instruments: two lags of inflation and the unemployment rate.

**Friedman-Phelps Model**

\[
\Delta p_t^n = 1.812 + 0.6280 \Delta p_{t-1}^n - 0.0803 \Delta p_{t-2}^n + 0.4233 \Delta p_{t-3}^n - 0.1980 U_t^n
\]

\( (2.7) \quad (8.1) \quad (-0.9) \quad (5.0) \quad (-2.7) \)

\( R^2 = 0.69, \) Durban-Watson statistic 2.03. Hypothesis tests: \( \sum_{i=1}^{3} \Delta p_{t-i}^n = 0, \) is rejected, \( F_{(1,182)} = 287.1956, \) p-val = 0.0000, and \( \sum_{i=1}^{3} \Delta p_{t-i}^n = 1 \) is accepted \( F_{(1,182)} = 0.2568, \) p-val = 0.6130. Instruments: three lags of inflation and two lags of the unemployment rate.

Phillips curve models are estimated with 187 observations in 8 cross-sections using EViews 5.1. Reported as ( ) are \( t \)-statistics. Models estimated with 2SLS and with the constant restricted to be the same across all 8 inflation regimes such that \( \phi^1 = \phi^2 = \ldots = \phi^8 \).
Table 4: Fixed Effects Panel Estimates of United States Phillips Curves

<table>
<thead>
<tr>
<th>Model ⇒ Independent Variable ⇧</th>
<th>Inflation Δ$p_{t}$</th>
<th>Inflation Δ$p_{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 Hybrid</td>
<td>2 F-P</td>
</tr>
<tr>
<td>Δ$p_{t+1}$</td>
<td>0.1376 (0.6)</td>
<td></td>
</tr>
<tr>
<td>Δ$p_{t}$</td>
<td>0.3360 (5.4)</td>
<td>0.3323 (5.5)</td>
</tr>
<tr>
<td>$U_{t}$</td>
<td>-0.2643 (-2.3)</td>
<td>-0.3159 (-3.9)</td>
</tr>
<tr>
<td>Regime 1</td>
<td>1.2458 (2.1)</td>
<td>1.5050 (3.1)</td>
</tr>
<tr>
<td>Regime 2</td>
<td>2.400 (2.5)</td>
<td>2.9630 (6.1)</td>
</tr>
<tr>
<td>Regime 3</td>
<td>3.5974 (2.5)</td>
<td>4.4935 (8.4)</td>
</tr>
<tr>
<td>Regime 4</td>
<td>7.0605 (2.5)</td>
<td>8.6598 (10.0)</td>
</tr>
<tr>
<td>Regime 5</td>
<td>5.1537 (2.5)</td>
<td>6.2960 (7.4)</td>
</tr>
<tr>
<td>Regime 6</td>
<td>7.6389 (2.4)</td>
<td>9.6695 (10.7)</td>
</tr>
<tr>
<td>Regime 7</td>
<td>3.9048 (2.5)</td>
<td>4.8105 (7.2)</td>
</tr>
<tr>
<td>Regime 8</td>
<td>2.8041 (2.5)</td>
<td>3.34486 (6.7)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.75</td>
<td>0.60</td>
</tr>
<tr>
<td>AR(1)</td>
<td>[0.000]</td>
<td>[0.509]</td>
</tr>
<tr>
<td>AR(2)</td>
<td>[0.004]</td>
<td>[0.097]</td>
</tr>
<tr>
<td>AR(3)</td>
<td>[0.004]</td>
<td>[0.003]</td>
</tr>
<tr>
<td>AR(4)</td>
<td>[0.313]</td>
<td>[0.256]</td>
</tr>
<tr>
<td>F-Tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_f + \phi_b = 0$</td>
<td>[0.040]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$\phi_f + \phi_b = 1$</td>
<td>[0.023]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$\phi_f = \phi_b = \phi_U = 0$</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>$\phi^n = 0$</td>
<td>[0.541]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

Reported as ( ) and [ ] are t-statistics and F-test probability values respectively. Estimated Hybrid and F-P models have 8 cross-sections and 187 and 195 usable observations respectively. Instruments: two lags of inflation and the unemployment rate in both models. AR(1) to AR(4) are the Arellano-Bond tests of first to fourth order serial correlation in the residuals. Models estimated in levels with 2SLS using Stata/SE 8.2 and Eviews 5.1. Tests of coefficient constancy: Hybrid model, $\Delta p_{t+1}$, F(7, 169) = 0.75, [0.6306], $\Delta p_{t}$, F(7, 169) = 0.26, [0.9686], $U_{t}$, F(7, 169) = 0.61, [0.7496]. F-P model, $\Delta p_{t}$, F(7, 178) = 0.36, [0.9252], $U_{t}$, F(7, 178) = 1.68, [0.1154].
Table 5: Estimates of the Long-run Phillips Curve

**Inflation as the Dependent Variable**

*Linear:*

\[
\Delta p = -8.1765 + 2.1190 \tilde{U}, \quad R^2 = 0.91
\]

\[\begin{array}{ccc}
\text{Coefficient} & \text{t-stat} & \text{DF} \\
-8.1765 & -4.6 & 176.5 \\
2.1190 & 7.6 & \\
\end{array}\]

The estimated coefficients on \( \tilde{U} \) is zero is rejected, \( F_{(1,6)} = 57.4170 \), prob-value = 0.0003.

*Non-linear:*

\[
\Delta p = 2.5189 - 1.5445 \tilde{U} + 0.2964 \tilde{U}^2, \quad R^2 = 0.94
\]

\[\begin{array}{ccc}
\text{Coefficient} & \text{t-stat} & \text{DF} \\
2.5189 & 0.4 & \\
-1.5445 & -0.7 & 1.6 \\
0.2964 & \\
\end{array}\]

The estimated coefficients on \( \tilde{U} \) and \( \tilde{U}^2 \) are jointly zero is rejected, \( F_{(1,6)} = 37.8068 \), prob-value = 0.0010.

**Unemployment Rate as the Dependent Variable**

*Linear:*

\[
\tilde{U} = 4.0754 + 0.4273 \Delta p, \quad R^2 = 0.91
\]

\[\begin{array}{ccc}
\text{Coefficient} & \text{t-stat} & \text{DF} \\
4.0754 & 12.4 & \\
0.4273 & 7.6 & \\
\end{array}\]

The estimated coefficient on \( \Delta p \) is zero is rejected, \( F_{(1,6)} = 57.4170 \), prob-value = 0.0003.

*Non-linear:*

\[
\tilde{U} = 3.7601 + 0.6014 \Delta p - 0.0156 \Delta p^2, \quad R^2 = 0.92
\]

\[\begin{array}{ccc}
\text{Coefficient} & \text{t-stat} & \text{DF} \\
3.7601 & 7.0 & \\
0.6014 & 2.5 & \neg \neg \neg \\
-0.0156 & -0.8 & \\
\end{array}\]

The estimated coefficients on \( \Delta p \) and \( \Delta p^2 \) are jointly zero is rejected, \( F_{(1,6)} = 26.9817 \), prob-value = 0.0021.

Notes: Numbers in brackets are t statistics.
Graph 1: United States Quarterly CPI Inflation, Seasonally Adjusted, March 1952 – September 2004

Notes: Horizontal thin lines indicate the mean rates of inflation in the eight inflation regimes (see Appendix 2 for details).
Graph 2: United States ‘De-meaned’ Inflation, Seasonally Adjusted, March 1952 – September 2004
Graph 3: The First Generated ‘Mean-Shift’ Inflation Series
Graph 4: United States Phillips Curves

Non-Linear Long-run Phillips Curve

Inflation = 0.2964 ue^2 - 1.5445 ue + 2.5189

R^2 = 0.938