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ARDL: An R package for ARDL models and cointegration

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Abstract

This paper presents the ARDL package for the statistical language R, demonstrating its main functionalities in a step by step guide. Some of its main advantages over other related R packages are the intuitive API, and the fact that includes many important features missing from other packages that are essential for an in depth analysis. Additionally, it is designed in such a way that it can be combined with other packages for post regression diagnostics and tests. These characteristics are shown through an example, where we showcase part of the application demonstrated in the seminal work of Pesaran, Shin, and Smith (2001).

Keywords: R, R package, ARDL, bounds test, cointegration

JEL Classification: C22 , C87 , Y20

1 Introduction

Pesaran et al (2001) (henceforth PSS) introduced the bounds test for cointegration based on the previous work of Pesaran and Shin (1999) using the ARDL model as a platform for the test. Since then, the ARDL framework and the bounds test are used constantly by practitioners who seem to adopt every new advancement of the initial framework. A recent example combining various techniques, is Wu et al (2022) who applied bootstrap ARDL with a Fourier function.

047 This paper provides a smooth introduction to the ARDL package in R (R
048 Core Team, 2023) and its main features and capabilities. It fills the gap between
049 PSS and the recent study of Natsiopoulou and Tzeremes (2022) in which the
050 validity and accuracy of the ARDL package is verified. The first one introduces
051 the bounds test whereas the other proves that the ARDL package is capable to
052 perform this type of modeling and tests. There are many recent publications
053 using the ARDL package (Allen and McAleer, 2021; Baruah, 2022; Bertelli et al,
054 2022; Maravalle and Pandiella, 2022; Qiu et al, 2021), since it has already
055 gained trust within the econometrics R users community.

056 Regarding proprietary software like EViews, although they are generally
057 considered more user-friendly, they lack flexibility compared to programming
058 languages such as R. Additionally, these software platforms are often slow to
059 adopt the latest advancements in research and can be prohibitively expensive
060 for many users.

061 On the other hand, open-source software do not provide any guarantees
062 regarding the quality of results, and it is the responsibility of the user to verify
063 the code. The problem lies in the fact that not everyone is an expert in the
064 field, making it challenging to technically validate the code's implementation.
065 Many practitioners simply seek reliable software they can trust.

066 During the development of the ARDL package, we tested various exist-
067 ing open-source packages in R. However, when comparing the open-source
068 packages to other proprietary software options, we encountered inconsistent
069 results, with the R packages producing outcomes that differed not only among
070 themselves but also when compared to the proprietary software alternatives.
071 Moreover, we observed that these open-source packages were developed follow-
072 ing diverse programming practices. For instance, some packages present results
073 in tabulated form, similar to the approach seen in menu-driven software.

074 Hence, these shortcomings served as the driving force behind the devel-
075 opment of the ARDL package. This package has enhanced existing software by
076 providing dependable and accurate results while maintaining a user-friendly
077 experience that facilitates seamless interaction with other packages. The pur-
078 pose of this paper is to serve as a step-by-step guide for users, offering them a
079 starting point similar to a user manual found in proprietary software.

080 We demonstrate the main functionalities of the ARDL package (Natsiopoulou
081 and Tzeremes, 2023, version 0.2.4) using the data from the PSS study. This
082 way, researchers and authors interested in applying these methodologies, have
083 a clear and step by step guide where they can utilize the codes and adjust
084 them to their research needs.

085

086 **2 ARDL, ECM and Bounds test**

087

088 Below we present the ARDL equation and the conditional error correction
089 model (ECM) which are equivalent representations of the same model, and
090 they are the building blocks of the bounds test for cointegration which is also

091

092

presented. These are the most important equations that one needs in order to follow the code examples below¹.

An ARDL model, estimated using ordinary least squares (OLS), is a linear model that comprises two key components: the autoregressive part (AR) and the distributed lags (DL) of the independent variables. In the AR part, the dependent variable is considered in lagged form, while in the DL part, the independent variables are included in both their levels and lagged forms. Accordingly, an ARDL model denoted as $ARDL(3, 0, 2)$ would include, up to three lags of the dependent variable, one independent variable considered at the level, and another independent variable included in both its level and lagged forms up to two lags.

ARDL formula:

$$y_t = c_0 + c_1 t + \sum_{i=1}^p b_{y,i} y_{t-i} + \sum_{j=1}^k \sum_{l=0}^{q_j} b_{j,l} x_{j,t-l} + \epsilon_t \quad (1)$$

An ECM utilizes the first difference of the dependent variable, regressed on the first lags of both the dependent and independent variables. The remaining regressors in the model consist of the lags of the first differences of both the dependent and independent variables.

ECM formula:

$$\Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{j=1}^k \pi_j x_{j,t-1} + \sum_{i=1}^{p-1} \psi_{y,i} \Delta y_{t-i} + \sum_{j=1}^k \sum_{l=1}^{q_j-1} \psi_{j,l} \Delta x_{j,t-l} + \sum_{j=1}^k \omega_j \Delta x_{j,t} + \epsilon_t \quad (2)$$

Where $\psi_{j,l} = 0 \quad \forall \quad q_j \leq 1$, $\psi_{y,i} = 0$ if $p = 1$, and $x_{j,t-1}$ and $\Delta x_{j,t}$ cancel out becoming $x_{j,t} \quad \forall \quad q_j = 0$

The hypothesis test for the bounds F-test is based on the ECM and the terms c_0 and c_1 are included only in cases 2 and 4, respectively, where this essentially means that they enter the long-run relationship. The hypothesis is the following:

$$\begin{aligned} \mathbf{H}_0 : \pi_y = \pi_1 = \dots = \pi_k = c_0 = c_1 = 0 \\ \mathbf{H}_1 : \pi_y \neq \pi_1 \neq \dots \neq \pi_k \neq c_0 \neq c_1 \neq 0 \end{aligned} \quad (3)$$

3 The ARDL package

Below we present some examples that demonstrate the intuitive design of the package which follows common R practices in contrast to other arbitrary ways of object treatment and presentation of results. Reviewing the ARDL package it is observed that apart from providing a direct API, it also possesses two main

¹For a detailed presentation of the mathematical formulas that the ARDL package uses, please see the official package manual <https://cran.r-project.org/web/packages/ARDL/ARDL.pdf>

139 properties. First, it provides a unified framework in order to allow the devel-
140 opment of new features and expand the package without changing package’s
141 philosophy alongside with the API that the users are already familiar with.
142 Based on this design, users are able to make small or big changes to the exist-
143 ing functions and expand or adjust them to their needs. Second, it allows the
144 objects returned by the package to be used with other packages (using regu-
145 lar objects, not only console printed results) for post estimation tests or for
146 any other econometric analysis. As a result, the ARDL package does not pro-
147 vide post estimation tests out of its core concept (e.g. normality, stationarity,
148 autocorrelation and other tests).

149 On the other hand, there exist other related R packages such as `dLagM`,
150 `dynamac`, and `ardl.nardl` (Demirhan, 2020; Jordan and Philips, 2022;
151 Otoakhia, 2023) that share certain similarities with the ARDL package. However,
152 these packages lack important functionalities that prevent them from perform-
153 ing the examples presented in this paper. Notably, `dynamac` and `ardl.nardl`
154 do not offer auto ARDL procedures, while `dLagM` and `ardl.nardl` lack the crucial
155 feature of fixed regressors, such as dummy variables. Furthermore, `dLagM`
156 and `dynamac` do not provide long-run multipliers. These limitations prevent
157 the construction of complex models. Specifically, we conducted an error cor-
158 rection model estimation, which is discussed later in subsection 3.1 (denoted
159 as `m3`). However, for this estimation, the dummy variables were omitted for
160 simplicity. The outcomes derived from the `ardl.nardl` package deviate from
161 those produced by the other R packages and `EViews`. Furthermore, there are
162 disparities in the bounds F-test, a crucial aspect of this analytical approach,
163 across the three aforementioned packages and even when compared to propri-
164 etary software like `EViews`. To illustrate, in a particular model, we performed
165 estimations using the ARDL package and `EViews`, yielding F-statistics of 2.87.
166 In contrast, the `ardl.nardl` package and the `dLagM` package generated values
167 of 2.53 and 3.44, respectively. In a different model, ARDL and `EViews` com-
168 puted the F-statistic as 4.13, with lower and upper bounds at 2.56 and 3.49,
169 respectively. In comparison, the `dynamac` package yielded values of 5.12, 3.1,
170 and 3.87 for the same parameters².

171 In addition, it is important to highlight some additional features found
172 in the aforementioned packages. For instance, `dLagM` enables the estimation
173 of models such as Almon and Koyck, `dynamac` offers simulation capabilities
174 by introducing shocks to independent variables, and `ardl.nardl` provides an
175 option to estimate (Nonlinear ARDL) NARDL models. While these function-
176 alities may be valuable to certain users, we have not compared them as they
177 extend beyond the core concept of ARDL modeling and the bounds test. There-
178 fore, we cannot make definitive claims regarding these features. However, it is
179 worth mentioning that the ARDL package is continuously evolving, and future

181 ²Please note that we have not included the code and results from this analysis in the paper
182 to maintain its focus as an introduction to the ARDL package rather than a software comparison
183 study. However, we have shared all the code and results with the anonymous reviewers of this
184 paper, and they are also available upon request.

updates will include supplementary options, including the NARDL model, to enhance its functionality.

Furthermore, we present illustrative flowcharts that depict the relationships between the functions driving the construction of ARDL and ECM models in Figure 1, as well as the broader functionalities offered by the package in Figure 2. These visual aids serve as a cohesive visualization of the underlying processes and connections within the ARDL package. By offering this comprehensive visual guide, we aim to provide users with a holistic overview that reinforces the package's coherence and user-friendly approach. With these flowcharts in place, readers can better appreciate the interplay of functions, thus facilitating a more comprehensive grasp of the package's capabilities.

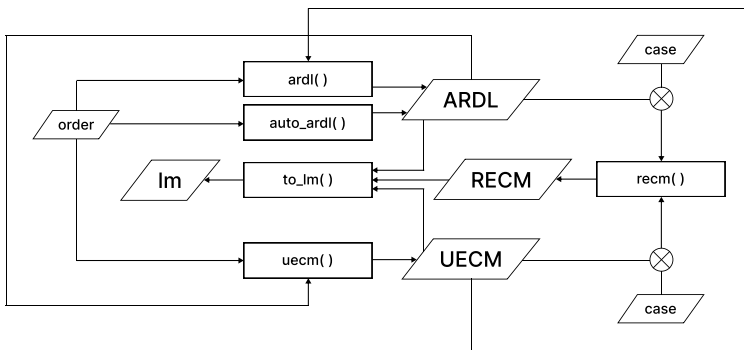


Fig. 1 Flowchart for building ARDL and ECM models using the ARDL R package. Rectangles represent functions, parallelograms stand for inputs or outputs depending on the direction of the arrow, and crossed circles show what two or more objects are used simultaneously as inputs. The model objects are shown in bold.

3.1 ARDL and EC models

The first functions we will need are `ardl()` and `uecm()`, where they estimate ARDL and ECM models respectively. These have different structures but are equivalent, thus, we will continue the analysis using the ECM form, as in PSS. We can estimate a conditional ECM model in two ways, either by `uecm(ardl())` or directly by `uecm()`.

In the subsequent examples, we aim to model real wage (w) as a function of various factors, including labor productivity ($Prod$), unemployment rate (UR), wedge effect ($Wedge$), and union power ($Union$). Our initial focus is on determining the suitable $ARDL(p, q_{Prod}, q_{UR}, q_{Wedge}, q_{Union})$ order, where p

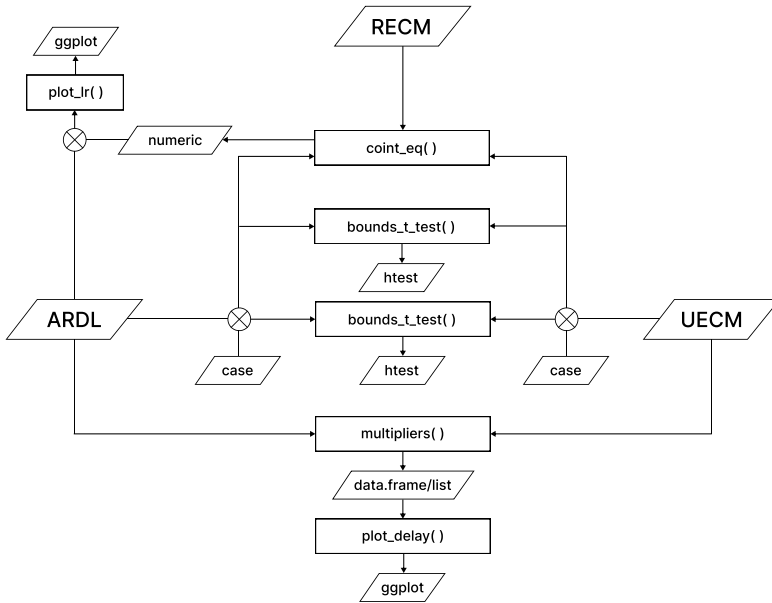


Fig. 2 Flowchart for the functionalities of the ARDL R package.

Rectangles represent functions, parallelograms stand for inputs or outputs depending on the direction of the arrow, and crossed circles show what two or more objects are used simultaneously as inputs. The model objects are shown in bold

represents the number of lags utilized for the dependent variable, q_{Prod} denotes the number of lags for the variable *Prod*, and so forth.

Specifically, we estimate each model with the same number of lags as when we estimate a VAR model (i.e. $ARDL(p, p, p, p, p)$). In order to implement such a setting, we set `order = p` which is equivalent to `order = c(p, p, p, p, p)`, where p is a positive integer.

Then we set the dataset PSS2001 and define the formula with the dependent and the independent variables, just like in a usual `lm()` model. Furthermore, we add two dummy variables that will take part in the short-run but not in the long-run relationship. This setting can be implemented by adding the variables after the symbol `|`. Additionally, we estimate models with linear trend, with, and without constant. To exclude the constant term we add `-1` just like in `lm()` and for the trend we can work in a similar manner as in `dynlm()`³ (Zeileis, 2019).

PSS note that we should not compare models with different lags as they are not compatible, so we need to ensure that all regressions have the same number of observations. Therefore, we keep out the first 8 observations to be

³`dynlm()` is the package that drives every regression of the ARDL package

used for the lags. As a result, the remain of the analysis uses the sample period from 1972:Q1 to 1997:Q4. This can be executed by adding `start = c(1972, 01)`, which is also a feature from `dynlm()`.

We run the three forms of equations (without constant, with constant, and with constant and trend) and we estimate each one of them using `order = 1`, `order = 2`, ..., `order = 7` denoting the models as $ARDL(1, 1, 1, 1, 1)$, $ARDL(2, 2, 2, 2, 2)$, ..., $ARDL(7, 7, 7, 7, 7)$, respectively⁴.

```
install.packages("ARDL")
library(ARDL)
```

```
m <- uecm(ardl(w ~ Prod + UR + Wedge + Union -1| D7475 + D7579,
              order = 3, data = PSS2001, start = c(1972, 01)))
```

```
mc <- uecm(w ~ Prod + UR + Wedge + Union |D7475 + D7579,
            order = 5, data = PSS2001, start = c(1972, 01))
```

```
mt <- uecm(w ~ Prod + UR + Wedge + Union
            + trend(w, scale = FALSE) |D7475 + D7579,
            order = 7, data = PSS2001, start = c(1972, 01))
```

The returned object is a usual `dynlm` regression model caring some extra information.

Observing the results, PSS argue that the case of no constant or with a trend are not reasonable choices for the data, and they also note that the terms $\Delta Prod_{t-1}$, ..., $\Delta Prod_{t-p}$ are not statistically significant neither individually nor jointly, and we should exclude them from the equation, meaning that $q_{Prod} = 1$.

Below the model is re-estimated as:

```
m1 <- uecm(w ~ Prod + UR + Wedge + Union |D7475 + D7579,
            order = c(5, 1, 5, 5, 5), data = PSS2001,
            start = c(1972, 01))
```

After analyzing the results, we can determine the optimal $ARDL(p, q_{Prod}, q_{UR}, q_{Wedge}, q_{Union})$ order using the `auto_ardl()` function, which provides precise control over the model formulation. With the help of the `auto_ardl()` function, we can systematically evaluate numerous (or all) potential models based on a specified criterion, typically the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC). However, it's important to note that the `auto_ardl()` function allows the user to select any other suitable criterion.

We will use this functionality here, since PSS use a slightly different version of AIC, which has the property to assign a higher value to the best model, in contrast with the usual formula of criteria like AIC and BIC. This function is

⁴For space saving reasons we present only three models.

predefined in the package as $\text{AIC_pss} = \ln(\hat{\mathcal{L}}) - K$, where $\hat{\mathcal{L}}$ is the maximized likelihood function $\mathcal{L}(\theta|X)$, and K is the number of estimated coefficients. Then we set `selection_minmax = "max"` so the best model to be the one with the highest value.

PSS suggest that 6 lags should be enough, so we restrict the `auto_ardl()` function to search only for up to 6 lags for each variable using `max_order = 6`.

In our analysis we include only one lag for the variable *Prod*, so we keep $q_{Prod} = 1$ fixed using `fixed_order = c(-1,1,-1,-1,-1)`, setting 1 in the place of q_{Prod} and -1 for the other variables for which we do not wish to set fixed lags.

Then, we can choose a starting order for the step-wise searching regression algorithm (e.g `starting_order = 5`) to avoid local optimum areas or set `grid = TRUE` to evaluate all possible combinations of orders. Although, note that the full search option is considerably more time consuming.

```
formula_c <- w ~ Prod + UR + Wedge + Union |D7475 + D7579
```

```
m2 <- auto_ardl(formula_c, data = PSS2001, start = c(1972, 01),
               max_order = 6, fixed_order = c(-1,1,-1,-1,-1),
               starting_order = 5, selection = "AIC_pss",
               selection_minmax = "max")
```

```
m2$best_order
```

w	Prod	UR	Wedge	Union
6	1	5	4	5

```
m3 <- m2$best_model
```

The optimization algorithm manages to find the global optimum model that the full search would have found, which is the *ARDL*(6,1,5,4,5). The object `m2` contains the best order, the best model itself and a list of the top orders. Now we can save this model directly, without having to estimate it again using `ardl()` with `order = c(6,1,5,4,5)`.

Last, we can get the RECM which is the ECM where we have restricted the lagged variables in levels to form a single term, the error correction term (ECT). At this point we have to decide if the constant term is going to be part of the short-run relationship or the long-run (to be included in the ECT). For this example we use `case = 3` which leaves the intercept unrestricted and thus it is part of the short-run relationship. The model can be estimated in a similar manner as we have previously calculated the UECM, for example:

```
summary(recm(m3, case = 3))
```

```

Time series regression with "zooreg" data:
Start = 1972 Q1, End = 1997 Q4

Call:
dynlm::dynlm(formula = full_formula, data = data, start = ←
  start,
  end = end)

Residuals:
      Min       1Q   Median       3Q      Max
-0.0175842 -0.0049998  0.0004342  0.0050892  0.0207731

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.619143   0.112409   5.508 4.28e-07 ***
d(L(w, 1))   -0.417479   0.090852  -4.595 1.59e-05 ***
d(L(w, 2))   -0.327934   0.096694  -3.391 0.001083 **
d(L(w, 3))   -0.523383   0.095549  -5.478 4.84e-07 ***
d(L(w, 4))   -0.133031   0.082024  -1.622 0.108767
d(L(w, 5))   -0.196693   0.075506  -2.605 0.010951 *
d(Prod)      0.315304   0.087619   3.599 0.000553 ***
d(UR)        0.003404   0.007024   0.485 0.629303
d(L(UR, 1))  0.015527   0.009777   1.588 0.116207
d(L(UR, 2))  0.003035   0.010193   0.298 0.766678
d(L(UR, 3))  0.028483   0.009982   2.853 0.005504 **
d(L(UR, 4))  0.026883   0.010119   2.657 0.009521 **
d(Wedge)    -0.297128   0.050195  -5.920 7.68e-08 ***
d(L(Wedge, 1)) -0.048266   0.051716  -0.933 0.353486
d(L(Wedge, 2)) -0.093438   0.053388  -1.750 0.083921 .
d(L(Wedge, 3)) -0.187797   0.053657  -3.500 0.000764 ***
d(Union)    -0.968727   0.713954  -1.357 0.178646
d(L(Union, 1)) -2.914759   0.808739  -3.604 0.000543 ***
d(L(Union, 2)) -0.021200   0.860420  -0.025 0.980404
d(L(Union, 3)) -0.101096   0.748143  -0.135 0.892849
d(L(Union, 4)) -1.994719   0.654656  -3.047 0.003131 **
D7475       0.029309   0.005085   5.764 1.48e-07 ***
D7579       0.016846   0.004598   3.664 0.000445 ***
ect         -0.229220   0.042411  -5.405 6.53e-07 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.00805 on 80 degrees of freedom
(0 observations deleted due to missingness)
Multiple R-squared: 0.6745, Adjusted R-squared: 0.581
F-statistic: 7.209 on 23 and 80 DF, p-value: 1.288e-11

```

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Usually we are interested in the coefficient of ECT which shows the speed of adjustment to the long-run equilibrium, which in this case is -0.229 .

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3.2 Bounds test

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```
f1 <- bounds_f_test(m1, case = 3, alpha = 0.05)
f1$tab
```

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	statistic	Lower-bound I(0)	Upper-bound I(1)	alpha	p.value
F	5.234323	2.894242	4.027319	0.05	0.007630228

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Moreover, a typical `hstest` object is returned, which includes some additional attributes, like the `tab`, which creates a data.frame summarizing the most useful information. The F-statistic is greater than the upper bound, so we can reject the H_0 of no cointegration⁵. Of course, the same conclusion can be reached through the p-value which is less than the *alpha* value.

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```
set.seed(2023)
f2 <- bounds_f_test(m1, case = 3, alpha = 0.05, exact = TRUE)
f2$tab
```

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	statistic	Lower-bound I(0)	Upper-bound I(1)	alpha	p.value
F	5.234323	2.946428	4.15907	0.05	0.01093268

453

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```
t1 <- bounds_t_test(m1, case = 3, alpha = 0.05)
t1$tab
```

458

459

460

⁵For double checking the results see PSS p.312

⁶It is important to set a seed beforehand so that our results to be reproducible.

statistic	Lower-bound	I(0)	Upper-bound	I(1)	alpha	p.value
t	-3.996164		-2.864981		-4.000246	0.05
						0.05044917

Our findings suggest that the t-statistic (-3.996164) is slightly smaller (in absolute terms) than the upper bound (-4.000246) which is consistent with the p-value being slightly greater than the alpha level $\alpha = 5\%$. In addition, we observe that the bounds as reported in PSS are rounded in the second decimal, which makes the comparison less precise⁷ and in this case, the results are inverted. However, we can still get the traditional critical value bounds for comparison.

```
t1$PSS2001parameters
```

Lower-bound	I(0)	Upper-bound	I(1)
	-2.86		-3.99

3.3 Long-run relationship and multipliers

The long-run relationship can be obtained based on the chosen model $ARDL(6, 1, 5, 4, 5)$, which is saved in the object `m3`. Therefore from the ARDL package we can obtain the long-run multipliers and the corresponding p-values, which appears as equation 31 in PSS.

```
multipliers(m3)
```

	Term	Estimate	Std. Error	t value	Pr(> t)
1	(Intercept)	2.7010826	0.24148486	11.185308	1.000458e-17
2	Prod	1.0633250	0.04999031	21.270624	9.772143e-34
3	UR	-0.1046091	0.03367004	-3.106891	2.658463e-03
4	Wedge	-0.9428506	0.26498172	-3.558172	6.474164e-04
5	Union	1.4806705	0.31061073	4.766965	8.809324e-06

From the estimated long-run multipliers we can compute the long-run relationship (cointegrating equation) and compare it graphically with the dependent variable.

```
ce <- coint_eq(m3, case = 2)
plot_lr(m3, coint_eq = ce, show.legend = TRUE)
```

The user also has the option to calculate short-run (impact), delay and interim multipliers. This is as simple as adding `type = "sr"` or `type = 0` for short-run or `type = h` for delay and interim multipliers, where `h` is a positive integer.

⁷See PSS p.312 or PSS Table CII(iii)

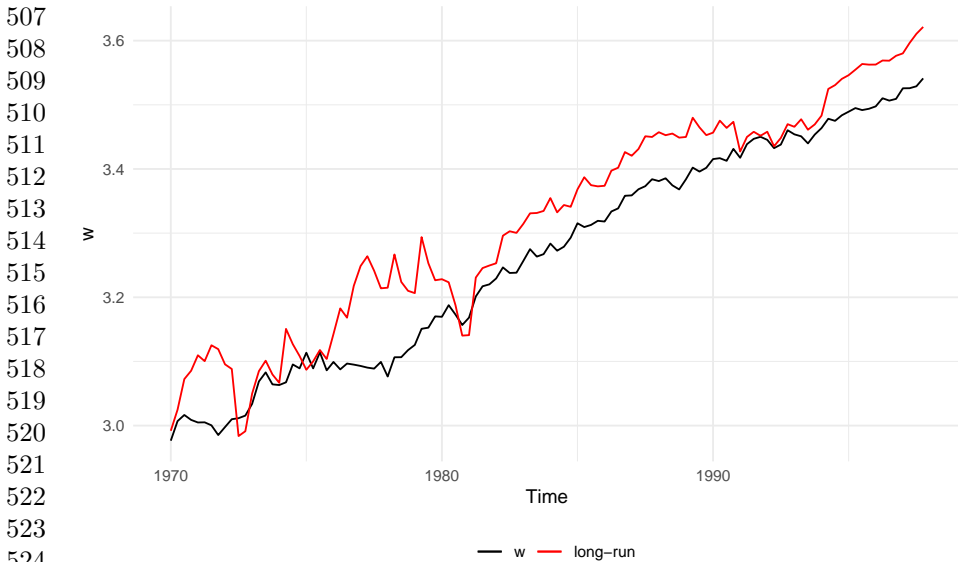


Fig. 3 Long-run relationship

In the following example, we compute the delay multipliers for 15 periods (quarters). These delay multipliers indicate the impact on the dependent variable in period $t+s$, resulting from an instant shock to an independent variable in period t . Figure 4 illustrates the responses of real wage (w) following the shock for each independent variable.

```
delay15 <- multipliers(m3, type = 15, se = TRUE)
plot_delay(delay15, interval = 0.95)
```

3.4 Post-estimation testing

After estimating an ARDL or ECM model using the `ARDL` package, it is essential to perform post-estimation testing to assess the validity of the model's assumptions and identify potential misspecifications. In this subsection, we present various post-estimation tests available through other packages, which can be used to examine the model's robustness and reliability.

In the following examples, we employ the `lmtest` package (Zeileis and Hothorn, 2002) to conduct post-estimation tests for serial correlation using the Breusch-Godfrey test, for heteroskedasticity using the Breusch-Pagan test, and for functional form misspecification using Ramsey's RESET test.

It's worth noting that when attempting to apply the RESET test to the `m3` model which was created using a data set of class `zoo`, we encounter an error. However, this is an uncommon occurrence since creating a model with a data set of another class, typically `data.frame` or `ts`, would not cause any issues. In such cases, a simple solution would be to recreate the model using the `uecm()`

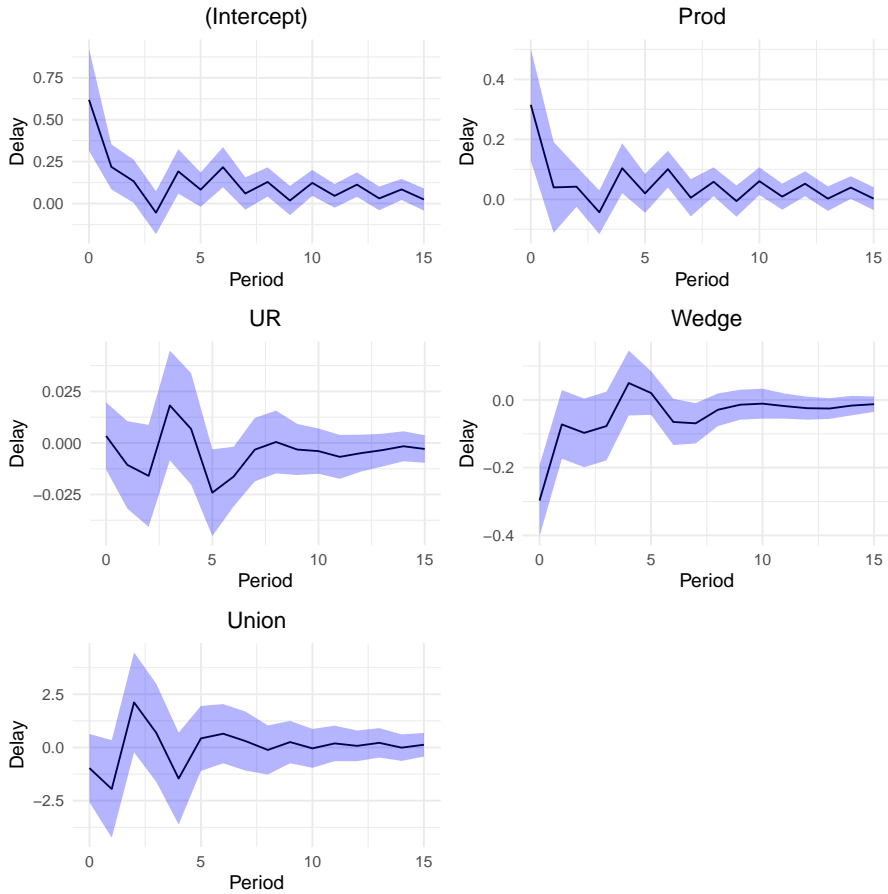


Fig. 4 Delay multipliers of model m3 for 15 periods with 95% CI

function while changing the class of the data set, for example, using `data = as.ts(PSS2001)`.

Alternatively, we can leverage the `to_lm()` function, which provides a convenient way to convert a model of class `ardl`, `uecm`, or `recm` into a regular `lms` model, as if it were created using the `lm()` function. The primary usage of the `to_lm()` function is to facilitate the conversion of the model's class, particularly because certain functions from other packages require models of `lm` class to function properly. In this case, we use the `data_class = "ts"` parameter to convert the class of the data used in the `lm` model.

```
library(lmtest) # for bgtest(), bptest(), and resettest()
library(tseries) # for jarque.bera.test()
library(strucchange) # for efp(), and sctest()
```

```
bgtest(m3, order = 4)
```

```

599 Breusch-Godfrey test for serial correlation of order up to 4
600
601 data: m3
602 LM test = 8.7447, df = 4, p-value = 0.06781

```

```

603
604
605 bptest(m3)

```

```

606
607 studentized Breusch-Pagan test
608
609 data: m3
610 BP = 41.627, df = 27, p-value = 0.03581
611

```

```

612
613 m3_lm1 <- to_lm(m3, data_class = "ts")
614 resettest(m3_lm1, power = 2)

```

```

615
616 RESET test
617
618 data: m3_lm1
619 RESET = 1.9196, df1 = 1, df2 = 75, p-value = 0.17
620

```

621 Next, we utilize the `tseries` (Trapletti and Hornik, 2023) and
 622 `strucchange` (Zeileis et al, 2002) packages to perform additional post-
 623 estimation tests. Specifically, we employ the Jarque-Bera test from the `tseries`
 624 package to assess normality and the CUSUM test from the `strucchange`
 625 package to identify structural changes.

626 During this process, we encounter an issue with the `efp()` function, which
 627 cannot handle variable names with special functions such as `d()` and `L()`. This
 628 limitation is not uncommon among many functions. To address this, we set the
 629 parameter `fix_names = TRUE` of the `to_lm()` function to convert the variable
 630 names accordingly.

631 It is important to note that, in none of the cases, did we explicitly require a
 632 model of class `lm`. However, we found the `to_lm()` function to be instrumental
 633 in resolving these issues and assisting us in conducting these post-estimation
 634 tests effectively. Its versatility proves beneficial in addressing various challenges
 635 that may arise during the testing process.

636 By employing these post-estimation testing functions from different pack-
 637 ages, we gain valuable insights into the model's assumptions and identify
 638 potential sources of misspecification, further ensuring the reliability and
 639 robustness of the models.

```

640 jarque.bera.test(residuals(m3))

```

```

641
642 Jarque Bera Test
643
644

```



```
data: residuals(m3) 645
X-squared = 0.013978, df = 2, p-value = 0.993 646
```

```
m3_lm2 <- to_lm(m3, fix_names = TRUE) 648
fluctuation <- efp(m3_lm2$full.formula, data = m3_lm2$model) 649
sctest(fluctuation) 650
```

Recursive CUSUM test 652

```
data: fluctuation 653
S = 0.58002, p-value = 0.4527 654
```

```
plot(fluctuation) 658
```

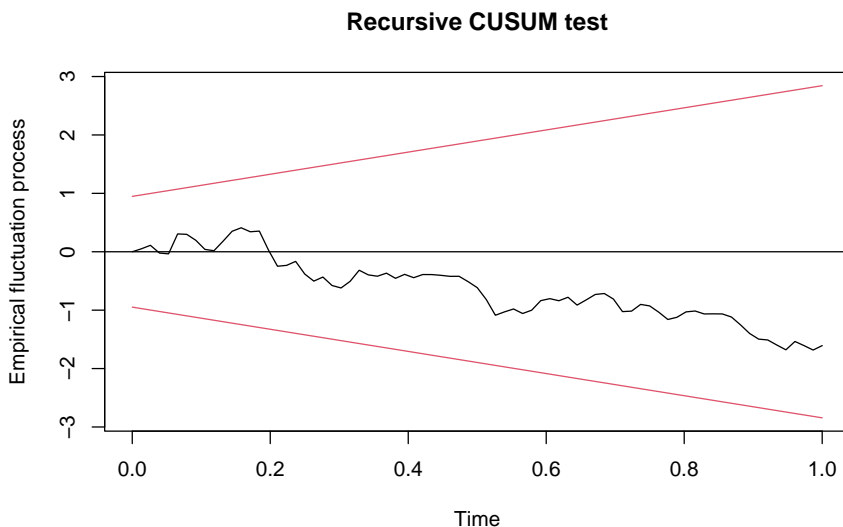


Fig. 5 Recursive CUSUM test

4 Conclusion

This paper serves as a comprehensive step-by-step guide, showcasing the core functionalities of the ARDL package, a versatile tool developed in the R language. In addition to explaining the package's capabilities, we provide insightful examples that end-users can readily adopt and tailor to suit their unique research requirements.

691 Throughout these illustrative examples, we highlight the user-friendly API
692 of the ARDL package, which enables effortless estimation of even the most
693 intricate models. The package's flexibility becomes evident as it easily accom-
694 modates the calculation of complex designs, making it a valuable asset for
695 researchers seeking reliable and robust results.

696 By presenting this guide, we aim to empower researchers and practitioners
697 to harness the full potential of the ARDL package, facilitating seamless model
698 estimation and analysis. Whether it be for economic forecasting, policy evalu-
699 ations, or academic research, the ARDL package equips users with an accessible
700 and efficient toolkit to enhance their data analysis endeavors.

701 Ultimately, our hope is that this paper contributes to the advancement of
702 empirical economic analysis and stimulates further exploration and utilization
703 of the ARDL package in diverse research settings. As we continue to improve
704 and expand the capabilities of the ARDL package through future updates, we
705 anticipate its continued relevance and impact in the field of economic modeling
706 and analysis.

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710

711 Declarations

712 **Conflict of interest.** All authors have stated that there are no conflicts of
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