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Models of Firm Dynamics and the Hazard Rate of Exits: Reconciling Theory and Evidence using Hazard Regression Models

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Abstract

This paper considers empirical work relating to models of firm dynamics. It is shown that a hazard regression model for firm exits, with a modification to accommodate age-varying covariate effects, provides an adequate framework accommodating many of the features of interest in empirical studies on firm dynamics. Modelling implications of some of the popular theoretical models are considered and a set of empirical procedures for verifying theoretical implications of the models are proposed. The proposed hazard regression models can accommodate negative effects of initial size that increase to zero with age (active learning model), negative initial size effects that may increase with age, but stay permanently negative (passive learning model), conditional and unconditional hazard rates that decrease with age at higher ages, and adverse effects of macroeconomic shocks that decrease with age of the firm. The methods are illustrated using data

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on quoted UK firms. Consistent with the active learning model, the effect of initial size is significantly negative for a young firm and falls to zero with age. The hazard function conditional on size, other firm- and industry-level characteristics, and macroeconomic conditions decreases with age only at higher ages, but shows the weaker property of Increasing Mean Residual Life over its entire life-duration. Instability in exchange rates affects survival of very young firms strongly, and the effect decreases to insignificant levels for older firms.

Key words: Firm exit, Learning, Firm Dynamics, Non-proportional hazards, Hazard regression models
JEL classification: C14, C34, C41, C52, D83, L16, L25

1 Introduction

The literature on industrial organisation proposes several theoretical models of the dynamics of firm behaviour that incorporate heterogeneity among firms, different sources of uncertainty (either firm-specific or idiosyncratic) and exit/ entry outcomes[1]. Two of these models are popular: the "passive learning" model (Jovanovic, 1982; Lippman and Rumelt, 1982)[2] and the "active learning" (also known as active exploration) model (Ericson and Pakes, 1995; Pakes and Ericson, 1998). Consistent with evidence that firms make their entry investments unsure of their success, both these models assume that new firms make their entries unsure of their quality and use "noisy" cost and profit signals to learn about their true efficiency or productivity levels. However, while the passive model assumes that this state variable remains constant over the lifetime of the firm, the active model considers a firm that can change the level of its stochastic state variable through potentially quality-enhancing investments. There are many other theoretical contributions to this literature, including models proposed by Lambson (1991, 1992), Hopenhayn (1992) and Asplund and Nocke (2003). The empirical implications of these models are similar to each other in some respects, and different in others. Understanding these empirical implications is important for understanding the nature of dynamics in different industries, as well as their

[1] Caves (1998) provides an extensive survey of the theoretical and empirical literature on turnover and mobility of firms. See also Sutton (1997) and Cabral (1997).

[2] See also Hopenhayn (1992) and Cabral (1993).

market structure, attrition, and response to possible changes in policy or other environmental conditions.

Motivated by a large body of empirical literature on the pattern of firm entry and exit as well as gross job flows, there has been a large and growing literature in modelling firm and industry dynamics[3], developing tests for alternative theoretical models, and relating the findings of empirical studies to theoretical models of firm dynamics.

In particular, Pakes and Ericson (1998) have studied the empirical implications of the passive and active learning models in great depth and proposed quite general nonparametric tests of alternative models of firm dynamics; their empirical study (on a eight-year panel of Wisconsin firms) suggests that the passive learning model fits the retailing sector well, while manufacturing shows patterns that suggest active learning. Abbring and Campbell (2004) propose alternative tests after accounting for heterogeneity across firms' pre-entry scale decisions and transitory shocks observed only by entrepreneurs, and they do not find any evidence of entrepreneural learning for Texan bars. As emphasized by Caves (1998), while these tests provide valuable insights into the nature of firm dynamics in an industry, they "suffer in that passive and active learning are not mutually exclusive: opportunities for both could be abundant in one industry, scarce in another".

In addition to the efficiency of the firm, whether time-varying or time-invariant, several contributions have emphasized the importance of industry and macroeconomic shocks in determining mobility and exit outcomes of firms. In particular, the role of the industry is discussed, for example, in Lambson (1991), Klepper (1996) and Asplund and Nocke (2003), and factors such as the position of the industry on the industry-life cycle (or vintage of capital), sunk costs and market size are found to be important. Similarly, the impact of the aggregate economy has also been studied, and both demand shocks and aggregate instability in the macroeconomic environment are found to be important determinants of survival of firms (Bergin and Bernhardt, 1999; Bhattacharjee *et al.*, 2002, 2003).

As an alternative to developing separate econometric frameworks applicable to individual models of firm and industry dynamics, this paper takes the view that many of the empirical implications of these models can be studied

[3]For a survey of the empirical literature, see for instance Siegfied and Evans (1994), Caves (1998), Cabral (1997), and Davis and Haltiwanger (1999).

using a single, flexible hazard regression model of firm exits. Hence, in addition to suggesting tests for the passive and active learning models, our aim here is to propose a simple empirical framework that encompasses a majority of the stylised facts about firm dynamics noted in the literature, and that enables study of the empirical implications of alternative theoretical models of firm dynamics.

We achieve this by making a simple modification to the Cox proportional hazards (PH) model (also called the Cox regression model) (Cox, 1972) to allow for age-varying effects of initial efficiency of the firm (initial value of the state varible), current measures of firm efficiency and macroeconomic factors. The age-varying effects can be completely flexible and unconstrained or constrained to decrease[4] with age, where suggested by relevant economic theory. This is in addition to the flexibility in the shape of the hazard function that the Cox PH model offers.

The model has the flexibility to admit effects from the initial size of the firm (a proxy for the initial value of the state variable) that are decreasing with age, as predicted by the active learning model. In addition, these models can accommodate empirical regularities such as age-varying effects of macroeconomic conditions (Bhattacharjee *et al.*, 2002, 2003) and hazard rates that are decreasing with age (particularly for older firms) even after controlling for observed heterogeneity in initial and current firm-level measures of efficiency (Agarwal and Audretsch, 2001; Bhattacharjee *et al.*, 2002, 2003)[5]. Thus, hazard regression models of firm exit incorporating the possibility of age-varying covariate effects provide useful empirical models of firm dynamics. These models can be used not only to test alternative theories, but also for modelling default probabilities, understanding changes in market structures in different industries and under different economic environments, and more generally for studying the nature of firm-specific differences in outcome paths.

This paper demonstrates the use of the Cox regression model with flexible age-varying covariate effects to understand firm dynamics, and to investigate

[4]Throughout this paper, 'decreasing' means non-increasing, and 'increasing' means non-decreasing.

[5]Both the active and the passive learning model would imply baseline hazard rates that may rise with age initially, but should decrease with age for older firms (Pakes and Ericson, 1998). This would be true, in particular, when the heterogeneity across firms' pre-entry scale decisions are not entirely observed and incorporated in the regression model.

the extent to which such models can aid study of the empirical implications of alternative theoretical models. Section 2 discusses alternative models of firm dynamics and their empirical implications. In Section 3, we set up flexible hazard regression models and inference techniques that can be used to study various implications of these models. Section 4 demonstrates an illustration of the methods using data on bankruptcies of UK quoted firms, and Section 5 collects conclusions.

2 Alternative models of firm dynamics and their empirical implications

The large body of empirical evidence on entrants' growth and failure rates[6] strongly suggest a stochastic process in which firms make their entry investments unsure of their success and do not initially position themselves at a unique optimal size. Consistent with this observation, most theoretical models of firm dynamics assume that any firm enters an industry unsure of its true quality and over time, learns about its quality through noisy information provided by its stream of realised earnings, costs and profits.

The starting point of the recent literature on stochastic dynamic industry equilibria with heterogenous firms is the seminal paper by Jovanovic (1982). In this model of "passive learning" (see also Lippman and Rumelt, 1982; Hopenhayn, 1992; and Cabral, 1993), the potential entrant into a perfectly competitive industry with heterogeneous but time-invariant efficiency levels is assumed to know the distribution of the state variable across all firms, but not its own realisation. Upon paying a (nonrecoverable) entry fee, it starts to receive noisy information on its true efficiency. Firms which learn that they are efficient grow and survive, while firms that obtain consistently negative information decline and eventually leave the market. The model produces a rich array of empirical predictions on the relationship between firm growth and survival on the one hand and firm age and size on the other. However, all firms eventually learn their efficiency level, and so there is no firm turnover in the long run.

By contrast, in "active learning" models such as Ericson and Pakes (1995)

[6]See, for example, Jovanovic (1982), Dunne *et al.* (1988, 1989), Davis and Haltiwanger (1992), Siegfried and Evans (1994), Caves (1998) and Pakes and Ericson (1998).

(see also Olley and Pakes, 1996; and Pakes and Ericson, 1998), entrants invest in uncertain but expectedly profitable innovations or cost reductions. Here, firms entering a stochastic dynamic oligopoly have efficiency varying over time due to stochastic market changes, their own investment decisions and those of other market participants. The firm grows if successful, shrinks or exits if unsuccessful. The passive learning model by Jovanovic (1982) differs from the active learning model in that the stochastic process generating the size of a firm is non-ergodic; Pakes and Ericson (1998) use this difference to develop empirical tests to distinguish between the two classes of models.

Hopenhayn (1992) considers a perfectly competitive industry. The main prediction of his model is that firm turnover is negatively related to entry costs. Due to the absence of the price competition effect, however, market size has no effect on entry and exit rates. An extension of the model to an imperfectly competitive market with monopolistic competition is considered in Asplund and Nocke (2003); the model generates implications of sunk costs and market size on firm exits and the size distribution of surviving firms. Bergin and Bernhardt (1999) consider business cycle effects in a similar model of perfect competition.

Lambson (1991) considers a model with atomistic price takers, where there are no idiosyncratic shocks but instead common shocks to input price (and demand). In equilibrium, firms may choose different technologies and hence be affected differently by the common shocks. The model predicts that the variability of firm values is negatively related to the level of sunk costs.

Like most of the literature in this area (for example, Jovanovic, 1982; Lambson, 1991; Hopenhayn, 1992; and Ericson and Pakes, 1995), we consider a single homogeneous industry and focus on the relationships between age, size, growth, exits and entries of firms within this industry. The large literature on empirical industrial organisation collects several such observable relationships; Pakes and Ericson (1998) relate some of the most important stylised relationships (R1, R2, R3a, R3b, R4a and R4b) with theoretical models of firm dynamics:

- $R1$ Conditional on age, the hazard rate is decreasing in current size.
- $R2$ The size distribution of the firms that survive from a cohort of firms increases, in a stochastic dominance sense, with age.

$R3$ Hazard rate (unconditional and conditional on size):

(a) The hazard rate is decreasing in age conditional on size (current size and/or initial size). Sometimes, the hazard rate is decreasing in age at older ages.

(b) The unconditional hazard rate may also decrease with age, at least at older ages.

$R4$ Effect of initial size:

(a) The initial value of the state variable may also matter; hazard rate may decrease in initial size (proxy for efficiency).

(b) The effect of initial size may persist even at an older age.

Pakes and Ericson (1998) show that the first two relationships ($R1$ and $R2$) hold for both the passive and the active learning models. The third relationship states that younger firms experience higher hazard rates, and that the hazard rate declines with age. Empirical studies have shown consistent evidence of declining hazard rates at higher ages, though the hazard rate for entrants is sometimes observed to be increasing with age. This is true for both unconditional hazard rates and hazard rates conditioned on initial size. Dunne *et al.* (1989) and related studies have advanced the view that a monotonically decreasing hazard function provides evidence in favour of the passive learning model. However, Pakes and Ericson (1998) show that the passive learning model does not necessarily predict hazard rates falling from the outset. They could rise at first, if ill-fated firms need some experience to be sure of their unfitness. For similar reasons, relating to the distinction between the passive and active learning models, the third relationship does not necessarily hold for the passive learning model, at least at higher ages; this need not, however, be true for the active learning model.

The fourth relationship ($R4a$ and $R4b$) is the most crucial for distinguishing between the active and passive learning models. Both $R4a$ and $R4b$ hold for the passive learning model, since this model does not allow the firm an opportunity to change its profitability distribution through investment. However, $R4b$ does not hold for the active learning model; the profitability distribution depends on investment and evolves over time, with the result that the relationship between the hazard rate of exit and initial size diminishes with age, and finally dies out. Pakes and Ericson (1998), and more recently Abbring and Campbell (2004), use this difference to construct empirical tests to distinguish between the active and passive learning models.

Other models of firm dynamics are also consistent with some of these stylised relationships. The model of Hopenhayn (1992) satisfies $R1$ but not $R4$, while Lambson's (1991, 1992) model satisfies the $R4$, and not $R1$. This, in a sense, reinforces the view of Caves (1998) that tests of persistence of the impact of initial size do not necessarily validate specific theoretical models, such as the passive or the active learning model. The aim of this paper is to propose an econometric model that encompasses all of the above stylised relationships, and indeed enables evaluation of these relationships through suitable parameter restrictions.

3 Hazard regression models and empirical studies of firm dynamics

In this section, we propose hazard regression models allowing for age-varying covariate effects as an econometric framework for empirical studies of firm dynamics. Further, the proposed models can be suitably modified to accommodate restrictions on age-varying covariate effects and decreasing baseline hazard rates. This framework encompasses the relationships (discussed in Section 2) implied by theoretical models and observed in empirical studies. In addition, the proposed model will enable study of inter-industry differences in firm dynamics, and understanding the impact of the macroeconomic environment on business exit.

There are three not-entirely-unrelated advantages of using hazard regression models for this and similar work in industrial organisation. First, these models have been widely used in empirical industrial organisation (see Caves (1998) and Bhattacharjee *et al.* (2003) for reviews) and their interpretation is well understood. Through their active use in applied economic research (not only in empirical industrial organisation but also labour economics and development economics), hazard regression models are now well-established tools in the toolbox of applied econometricians[7].

Second, hazard models explicitly incorporate the timing of alternative outcomes. This facilitates accounting for censoring or other kinds of sample

[7]See Greene (1995) for a brief introduction to duration models, Lancaster (1990) for an elaborate treatment and Neumann (1997) and van den Berg (2001) for more advanced reviews.

selection issues, and allows inferences to be drawn about the effect of firm-level or macroeconomic age-varying covariates[8]. However, the usefulness of popular hazard regression models are somewhat limited by their strong assumptions (such as proportionality, additive hazards etc.) on the separation of the effects of age and other explanatory factors in determining conditional hazard rates.

Third, and probably the most important reason why these models are useful for studying firm dynamics is that they admit a huge range of flexible regression structures. It is well known that the Cox PH model (Cox, 1972; Kalbfleisch and Prentice, 1980) allows the shape of the hazard function, after conditioning on explanatory factors other than age, to be completely flexible; in other words, the nature of duration dependence can be left completely unrestricted. Similarly, recent research shows that with some modifications, the Cox regression model can also handle very flexible types of covariate dependence, including interaction between the duration and other covariates in the form of age-varying covariate effects (Bhattacharjee and Das, 2002; Scheike, 2002; Bhattacharjee, 2004). Hence, these models can also have completely unrestricted pattern of covariate dependence. This flexibility allows suitably constructed hazard regression models not only to accommodate quite arbitrary patterns of duration dependence and covariate dependence, but also render inference under various kinds of order restrictions on the nature of duration dependence and covariate dependence very convenient.

The models considered here will accommodate effects of initial size that are decreasing with age, as predicted by the active learning model. Further, the models can accommodate age-varying effects of macroeconomic conditions (Bhattacharjee *et al.*, 2002, 2003) and hazard rates that are decreasing with age even after controlling for observed heterogeneity in initial and current firm-level measures of efficiency (Agarwal and Audretsch, 2001; Bhattacharjee *et al.*, 2002, 2003). Thus, hazard regression models of firm exit incorporating age-varying covariate effects with possible order restrictions in both the nature of covariate dependence and duration dependence can be conveniently used for studying the nature of firm-specific differences in outcome paths.

In the following sub-section, we review the literature on age-varying covariate effects and order restrictions on the nature of covariate dependence

[8] See Bhattacharjee *et al.* (2002, 2003) for further discussion.

and duration dependence in hazard regression models. Finally, we present a flexible hazard regression model for the study of firm dynamics.

3.1 Flexible semi-parametric hazard regression models

The Cox PH model (or Cox regression model) is the most popular hazard regression model in empirical studies. In this model, the relationship between the hazard rate at duration t and the explanatory factors other than age (the covariates), denoted by $\underline{z}$, is expressed as

$$\lambda\left(t|s,\underline{z}\right) = \lambda_0(t).\exp\left[\underline{\beta}'.\underline{z}\right], \tag{1}$$

where $\lambda_0(t)$ is the so-called baseline hazard function. Under the model, changes in the covariates induce only a multiplicative shift in the hazard function. In other words, the hazard functions for different values of the covariates are proportional to each other and have the same shape as the baseline hazard function. The model provides a very convenient representation of the relationship between the hazard rate at a given age conditional on covariates and the values of the covariates, and is very useful for empirical work on duration data. However, it suffers from a major limitation. The PH specification in the model substantially restricts interdependence between the explanatory variables and the duration in determining the hazard. In particular, the PH model restricts hazard functions conditional on different values of the covariates to be proportional to each other; in other words, the model makes the strong assumption that the coefficients of the regressors in the logarithm of the hazard function are constant over age. This restriction may not hold in many situations, or may even be unreasonable from the point of view of relevant economic theory. For example, the active learning model would imply that the effect of initial size should monotonically decrease with age of the firm. This constitutes a violation of the proportionality assumption in the Cox PH model. Testing the PH model, either against the omnibus alternative[9] or against ordered alternatives, has therefore been an area of active research. The tests against ordered alternatives,

[9]The null hypothesis of proportionality is often tested against an omnibus (arbitrary non-PH) alternative hypothesis. The test proposed by Grambsch and Therneau (1994) is quite popular; Martinussen *et al.* (2002) also propose a test. However, rejection of the null hypothesis of PH in such tests do not provide any insight into the nature of the departure from proportionality.

such as the alternative that the covariate effect monotonically decreases with age, are often more useful. Bhattacharjee and Das (2002) propose tests of the proportional hazards assumption against alternatives of monotonic (and simple non-monotonic) covariate effects[10].

Empirical evidence of ordered departures from proportionality, such as age-varying covariate effect that is increasing or decreasing with age, are abundant in the literature on economic duration models, and such violations of the PH model can be conveniently interpreted in terms of age-varying covariate effects (Scheike, 2002; Bhattacharjee, 2004). Several authors have suggested validation of the PH assumption by testing for age-varying covariate effects (see, for example, Grambsch and Therneau, 1994; Martinussen *et al.*, 2002), and several methods for estimation of these age-varying covariate effects have been proposed (see, for example, Murphy and Sen, 1991; Martinussen *et al.*, 2002)[11]. While these estimators allow the covariate effects to vary with age, they do not provide estimates under maintained order restrictions suggested by theory, or observed to hold in a given application (by applying the test proposed in Bhattacharjee and Das (2002), for example).

Bhattacharjee (2004) proposes kernel-based biased bootstrap methods such as data tilting (Hall and Presnell, 1999) or local adaptive bandwidths (Brockmann *et al.*, 1993; Chaudhuri and Marron, 1999)[12] to estimate the covariate effects smoothly under order restrictions given by the monotonicity constraints. In particular, the method using local adaptive bandwidths is found to have good performance in moderate sample sizes, are simple to construct and useful in applications. The estimator is designed to modify the underlying kernel estimates only in certain limited regions; these modifications are typically aimed at adapting to the density of design points and/ or the structure of the regression function (Brockmann *et al.*, 1993). The estimators smooth away "spurious wiggles" by increasing the local band-

[10]For further details and review of literature, refer to Bhattacharjee and Das (2002).

[11]In particular, the histogram-sieve estimator (Murphy and Sen, 1991) is easy to implement and intuitively appealing. This estimator divides the duration scale into several intervals and estimates the (age-varying) covariate effects as a step function.

[12]Biased bootstrap methods are powerful techniques for order restricted inference, based on reweighting the original data, that are gaining increasing popularity. In data tilting, the frequencies of the original sample points are modified minimally to achieve the maintained or hypothesized order restriction. In adaptive bandwidth methods, the bandwidth used for kernel estimation is locally modified to account for sampling fluctuations. See Bhattacharjee (2004) for further details.

width at the middle of the wiggles, and reducing the bandwidth towards the boundaries. While in large samples, the monotone nature of the data would dominate, and adaptive bandwidth estimators may not be necessary, these methods usually produce statistically superior and more visually appealing curve estimates in small samples. This method will be illustrated with an empirical application later in this paper.

In summary, recent literature on duration models provides a wide range of inferential tools for situations where the PH model assumption may not hold. Of particular interest are applications in which there are order restrictions on the nature of covariate dependence that are either suggested by theory, or observed in particular data. In addition, one may have order restrictions on the nature of duration dependence. For example, both data and learning models are consistent with the view that unconditional hazard rates may decrease with age, at least at higher ages. Some work on hazard regression models with order restrictions on both the nature of covariate dependence and duration dependence have been reported in the literature (Bhattacharjee and Bhattacharjee, 2004).

3.2 A hazard regression model of firm dynamics

In this paper, we argue that hazard regression models with order restrictions on the nature of covariate dependence, and possibly also duration dependence, are a very convenient empirical framework for studying firm dynamics. We consider the following flexible hazard regression model that allows for age-varying (both monotonically and non-monotonically age-varying) covariate effects (Bhattacharjee, 2004)

$$\lambda\left(t|s,\underline{z}\right) = \lambda_0(t).\exp\left[\beta_{s_0}(t).s_0 + \beta_s(t).s + \underline{\beta}_z^{\prime}(t).\underline{z}\right], \tag{2}$$

where t denotes the age of the firm, s_0 ($s_0 \geq 0$) is the age-constant covariate "initial size", s ($s \geq 0$) is the age-varying covariate "current size" and $\underline{z}$ represents other possibly-age-varying covariates representing firm-level, industry-level and macroeconomic factors.

Equation (2) presents one of the most general hazard regression model one can think about, with a great degree of flexibility in the specification of both the baseline hazard rate and the age-varying covariate effects. It accommodates non-proportional hazards situations where there is interaction between the duration t and a covariate as reflected in age-varying nature of $\beta(t)$; the Cox regression model is included as a special case when the β parameters are constant over age. In fact, in many economic situations, one not only expects the covariate effect to be age-varying, but often to either increase or decrease with age; such situations can be incorporated in the model through order restrictions on the shape of $\beta(t)$. As we shall see, in models of firm exits, the effect of initial size (proxy for efficiency) is expected to be negative and increase with age.

As in the Cox PH model, the baseline hazard function $\lambda_0(t)$ is allowed to take any shape (nonparametrically specified). However, some economic applications may suggest order restrictions on the nature of duration dependence, which would then imply restrictions on $\lambda_0(t)$. In empirical models of firm dynamics, for example, conditional (or unconditional) hazard functions decreasing with age at higher ages may constitute such an order restriction.

Proposition 1: Within the class of regression models considered above,

a. Relationship 1 ($R1$) is satisfied if and only if $\beta_s(t) \leq 0$ for all t.

b. Also, Relationship 2 ($R2$) is satisfied if $\beta_s(t) \leq 0$ for all t. $R2$ is satisfied whenever $R1$ is satistied ($R1 \Rightarrow R2$).

c. Relationship 4a. ($R4a$) holds if $\beta_{s_0}(t) \leq 0$ for all t. However, Relationship 4b. ($R4b$) does not hold if $\beta_{s_0}(t) \uparrow 0$ as $t \uparrow \infty$. On the other hand, $R4b$ holds if $\beta_{s_0}(t) \leq 0$ and is bounded away from zero for large t.

d. Relationship 3a. ($R3a$) is satisfied if and only if $\lambda_0^{/}(t) \leq 0$ at all continuity points of $\lambda_0(t)$. Equivalently, the baseline cumulative hazard function $\Lambda_0(t) = \int_0^t \lambda_0(s).ds$ is convex. Similarly, the relationship holds only at higher ages if, there exists a $t_0 \geq 0$ such that $\lambda_0^{/}(t) \leq 0$ at all continuity points of $\lambda_0(t)$ higher than t_0; equivalently $\Lambda_{0,t_0}(t - t_0) = \int_{t_0}^t \lambda_0(s).ds$ is convex.

e. Assume that $\lambda_0(t)$, $\beta_{s_0}(t)$ and $\beta_s(t)$ are continuous almost everywhere. Then Relationship 3b. ($R3b$) is satisfied if and only if $\lambda_0^{/}(t) \leq \lambda_0(t).\beta_s^{/}(t).s$ for all s and at every continuity point of $\lambda_0(t)$ and $\beta_s(t)$. Similarly for initial size.

Corollary 1: In particular, the conditions of Proposition 1e. are satisfied if the following conditions hold for current size. Similarly, for initial size.

a. $\beta_s(t) \leq 0$, $\beta_s^{/}(t) \geq 0$.
b. $\lambda_0^{/}(t) \leq 0$, and
c. $s \geq 0$ almost everywhere.

Corollary 2: Relationships 1, 2 and 3 are satisfied if:
a. $\beta_s^{/}(t) \geq 0$,
b. $\lambda_0^{/}(t) \leq 0$ (that is, conditions of Proposition 1d. hold), and
c. $s \geq 0$ almost everywhere.

While all the results (in the Proposition and the Corollaries) are valid only within the context of the model in Equation (2), this age-varying covariate effects hazard regression model is about the most general hazard regression model one can think about; hence, the results are quite general. These results chracterize the effect of age, initial size and current size on the hazard rate, holding other factors fixed. The nature of firm dynamics given by the model and contained in these results are based on the shape of the baseline hazard function, and the sign and slope of the age-varying covariate effects of initial and current size. The proofs of the Proposition and the Corollaries are quite simple and omitted here, but we provide some intuition of the results and explain their implications for models of firm exits.

Proposition 1b. gives only a sufficient condition for Relationship 2, that the size distribution of firms increases (in a stochastic dominance sense) with age. The intuitive reasoning here is that, if $\beta_s(t) \leq 0$, lower size firms would have a higher hazard rate and more of these firms will exit. As a result, the size distribution will be stochastically increasing in age. Similar reasoning gives Propositions 1a. and 1c.

The results have several implications for models of firm dynamics. First, all the theoretical models discussed here predict that initial size (and by implication, current size) has a negative effect on the hazard rate at every age. Further, the active learning model allows firms to change their level of efficiency through investment and to that extent, predicts that the effect of initial size may not be persistent. In other words, the active learning model will predict a negative, but age-varying and increasing effect of initial size (Figure 1). By contrast, the passive learning model predicts a significant effect of initial size even at higher ages. Here too, the effect will be negative, and may also be age-varying and increasing, but the effect will not fall to zero

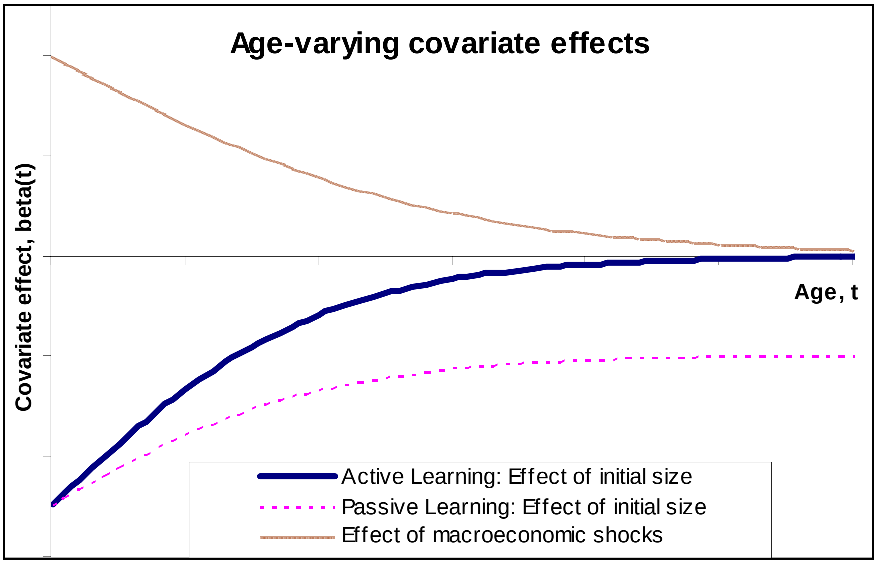

Figure 1:

even for older firms (Figure 1). Second, empirical studies find that hazard rates are decreasing with age (particularly at higher ages), even when initial size is controlled for[13]. This observation is consistent with learning models. Unconditional hazard rates should decline with age because of heterogeneity in efficiency; firms with an unfavourable efficiency level exit earlier than more efficient firms. To the extent that there is residual unobserved heterogeneity after controlling for initial (and current) size, even the baseline (conditional) hazard function may decrease with age at higher ages[14].

Like Pakes and Ericson (1998), we use Relationship $4b$ (and Proposition 1c.) to develop a test for the active and passive learning model in three steps. In the first step, we test the null hypothesis of no covariate effect against the alternative $\beta_{s_0}(t) < 0$ for some t. This can be done in the usual way using the Murphy-Sen histogram sieve estimates to construct a Wald test. If the null hypothesis is rejected, we go to step two and test the hypothesis that $\beta_{s_0}(t) = 0$ for some large enough t; if the null is accepted for some t, we go

[13]See Agarwal and Audretsch (2001), for example.
[14]See also Abbring and Campbell (2004).

to step three. At this step, we can use the test proposed by Bhattacharjee and Das (2002) to test the null hypothesis of proportionality $\beta_{s_0}(t) = c < 0$ against the monotone alternative $\beta_{s_0}(t) \uparrow t$. Because the active learning model allows $\beta_{s_0}(t)$ to start increasing to zero only after some threshold duration, we may have to conduct the test several times, with left-censoring at different fixed durations. The passive learning model can be said to hold if we either accept the null hypothesis at the third step, or cannot proceed from the second to the third step. The active learning model holds if the null hypothesis is rejected at the third step. The steps in this sequence of tests are shown in Table 1. The crucial idea here is to use the nature of age-variation in the covariate effect to distinguish between the two models.

TABLE 1: **Steps of the Test (Passive versus Active Learning)**

Step 1.	**Test**: $\mathbb{H}_0 : \beta_{s_0}(t) = 0$ for all t vs. $\mathbb{H}_1 : \beta_{s_0}(t) < 0$ for some t. $\mathbb{H}_1$ is consistent with passive and active learning. If $\mathbb{H}_0$ is rejected, go to **Step 2**.
Step 2.	**Test**: $\mathbb{H}_0 : \beta_{s_0}(t) = 0$ for large t vs. $\mathbb{H}_1 : \beta_{s_0}(t) < 0$ for large t. $\mathbb{H}_1$ is consistent with passive learning, and *vice versa*. If $\mathbb{H}_0$ is rejected (not consistent with passive learning), go to **Step 3**. If $\mathbb{H}_0$ is accepted (not consistent with active learning, but consistent with passive learning), it may still be interesting to understand the age-varying nature of covariate effects. Go to **Step 3**.
Step 3.	**Test**: $\mathbb{H}_0 : \beta_{s_0}(t) = c < 0$ for all t (PH) vs. $\mathbb{H}_1 : \beta_{s_0}(t) \uparrow t$. $\mathbb{H}_1$ is consistent with active learning; both $\mathbb{H}_0$ and $\mathbb{H}_1$ are consistent with passive learning. If $\mathbb{H}_0$ is rejected, evidence consistent with active learning.

Corollaries 1 and 2 give sufficient conditions for the unconditional hazard rate to decrease with age, and the conditional hazard rate to decrease for each initial value of the state variable. If (a) current size has a negative effect on the hazard rate, and if (b) this age-varying effect is increasing with age of the firm, and further if (c) the baseline hazard rate has negative duration dependence, then the hazard rate conditional on size decreases with age.

As explained earlier, both the active and the passive learning models imply negative effects of initial size. We can test the proportionality assumption (effect of size is constant over the age of the firm) against an omnibus alternative using any of several tests for proportionality available in the literature. In particular, one may use the test proposed by Grambsch and Therneau (1994). If this test suggests proportionality, negative duration dependence can be tested by testing that the estimated cumulative baseline hazard function is convex. This is equivalent to testing for an exponential duration distribution against an increasing hazard rate alternative. There are several tests reported in the literature[15]; the tests based on convexity/ concavity of the plot of estimated baseline hazard functions (Lee and Pirie, 1987) are very popular. If, on the other hand, proportionality is rejected, one can first test for monotone age-varying covariate effects using the test proposed by Bhattacharjee and Das (2002), followed by a test of convexity of the cumulative baseline hazard. However, since the conditions in Corollary 2 are only sufficient, we may take the Pakes and Ericson (1998) view that more appropriate tests for alternative models of firm dynamics may be based on Relationship 4*b*. (using Proposition 1c.).

As in the case of size, allowing for age-varying covariate effects in the other covariates included in the hazard regression model (such as measures of macroeconomic instability) also enriches the econometric framework for understanding firm dynamics. Bhattacharjee *et al.* (2002, 2003) develop a model where macroeconomic shocks and instability have a detrimental effect on the survival of firms. Further, an older firm is more capable of withstanding these negative shocks than a younger firm. This situation can be modelled by incorporating an age-varying macroeconomic effect that decreases with the age of the firm (Figure 1).

In summary, allowing the effect of size (and other explanatory factors) to be age-varying facilitates better understanding of the nature of firm dynamics using data on firm exits. We can use the tests available in the literature (Grambsch and Therneau, 1994; Bhattacharjee and Das, 2002) to understand the nature of covariate dependence more explicitly. Estimation under order restrictions on the nature of covariate dependence implied by the tests can be conducted using biased bootstrap methods proposed in Bhattacharjee (2004). In Section 4, we shall illustrate the use of these techniques with an application.

[15] The literature is reviewed in Gill and Schumacher (1987).

4 An application

Bhattacharjee *et al.* (2002) have analysed business failure due to bankruptcy in the quoted segment of the UK corporate sector over the period 1965 to 1998. The focus of the analysis was to understand the impact of macroeconomic instability on firm exits, after controlling for firm- and industry-specific factors. The data are right censored (by the competing risks of acquisitions, delisting etc.), left truncated in 1965, and contain delayed entries[16]. Bhattacharjee *et al.* (2002) give estimates of a hazard regression model of age since listing, including histogram sieve estimates of age-varying covariate effects wherever the proportionality assumption does not hold[17]. Further, Bhattacharjee (2004) estimate age-varying covariate effects for measures of macroeconomic instability under order restrictions implied by the tests of proportionality.

The data used here are for the period 1965 to 2002 and pertain to 4,105 listed nonfinancial companies covering approximately 48,000 company years, and include 203 exits due to bankruptcy. We take the squared natural logarithm of real fixed assets in the year of listing as the measure of initial size[18]. Several other measures of firm-level performance (measures of current size, profitability, debt sustainability and leverage), industry dummies, and several macroeconomic variables (domestic output, US output, interest rate, exchange rate, and instability in inflation, interest rates and exchange rate) are also included in the model[19]. The notion of age used here is age since listing.

The tests of proportionality with respect to the different covariates included in our model indicate age-varying covariate effects for initial size, current size, profitability and instability in the exchange rate. Using the test proposed by Bhattacharjee and Das (2002), the null hypothesis of proportional hazards was overwhelmingly rejected against an increasing covariate

[16]This is because the panel is unbalanced, and there is left truncation of the sample period in 1965.

[17]Tests proposed by Grambsch and Therneau (1994) and Bhattacharjee and Das (2002) were used.

[18]The earliest year for which we have data on assets is 1949. Around 900 of the 4,105 companies were listed in 1949 or earlier. For these companies, we consider assets in 1949 as a proxy for initial size.

[19]See Bhattacharjee *et al.* (2002) for discussion on data and construction of variables.

effect hypothesis for initial size (p-value 0.00) and a decreasing covariate effect hypothesis for exchange rate instability (p-value 4.09e-7).

We estimated a hazard regression model allowing for age-varying covariate effects for these two covariates, using the histogram sieve method (Murphy and Sen, 1991). These estimates are presented in Table 2; for economy of presentation, we do not report individual industry dummies. The reported z-scores are based on robust standard error estimates proposed by Lin and Wei (1989). In addition to tests of proportionality conducted separately with respect to each covariate, the overall validity of the proportionality assumption is also assessed using the test proposed by Grambsch and Therneau (1994). The overall fit of the model is judged using a Wald chi-square test.

The histogram sieve estimates (Table 2) show evidence of a negative and age-increasing effect of initial size[20], and a positive and age-decreasing effect of exchange rate instability. In particular, the histogram sieve estimates of the effect of initial size on the hazard rate of exits was negative for firms less than eight years old since listing (and significant at 1 per cent level); the effect was insignificant for older firms. Similarly, exchange rate instability had a positive effect (significant at 5 per cent level) on these firms while the effect on older firms was insignificant.

However, these histogram sieve estimates are based on an ad-hoc partitioning of the duration scale, and do not take into account the fact that the covariate effects have been tested and found to have monotone age-varying effects. Estimates of age-varying covariate effects under appropriate order restrictions would do much better in this situation (Bhattacharjee, 2004). Kernel estimates of age-varying covariate effects of initial size for three different bandwidths are plotted in Figure 2 and Figure 3 shows estimates (and pointwise 95 per cent confidence bands) based on locally adaptive bandwidths proposed by Bhattacharjee (2004). The estimates indicate that the effect of initial size is negative and increasing to zero; there is a significant negative effect at least till the age of 16 years after listing. Similar plots for instability in exchange rates (Figures 4 and 5) show a positive covariate effect sharply decreasing to zero with age post-listing; exchange rate instability is significantly detrimental to survival of quoted firms till about the age of 14 years post-listing.

[20] We also estimated models without initial size as a covariate. Here, the effect of current size shows a similar pattern.

TABLE 1: Estimated Model

Variable	Coefficient (z-score)
INDUSTRY DUMMIES (estimates omitted)	
TIME-CONSTANT FIRM LEVEL	
Initial size: ln(initial real fixed capl.)-sq. $= s_0^2$	
– $s_0^2\times$ I(age 0-7 yrs.)	-0.0557 (-3.2)**
– $s_0^2\times$ I(age 8-25 yrs.)	-0.0275 (-1.3)
– $s_0^2\times$ I(age $>$ 25 yrs.)	0.0021 (0.1)
FIRM $\times$ YEAR LEVEL	
Current size: ln(real fixed capital +1)	0.1193 (0.6)
Size-squared $= s^2$	
– $s^2\times$ I(age 0-7 yrs.)	0.0146 (0.6)
– $s^2\times$ I(age 8-25 yrs.)	-0.0279 (-1.1)
– $s^2\times$ I(age $>$ 25 yrs.)	-0.0352 (-1.4)
Cash flow to Capital $= \pi$	
– $\pi\times$ I(age 0-7 yrs.)	-0.0117 (-1.3)
– $\pi\times$ I(age 8-25 yrs.)	0.1301 (0.4)
– $\pi\times$ I(age $>$ 25 yrs.)	0.2237 (1.1)
Interest cover	-2.77e-4 (-4.1)**
Gearing ratio	0.0117 (3.8)**
MACROECONOMIC ACTIVITY	
UK business cycle	-6.0782 (-1.0)
US business cycle	-6.1877 (-1.9)$^+$
Long-term real interest rate	-0.0322 (-1.2)
Real effective exchange rate	1.8253 (1.2)
MACROECONOMIC INSTABILITY	
Instability – £ – \$ exchange rate $= v$	
– $v\times$ I(age 0-7 yrs.)	1.8262 (2.3)*
– $v\times$ I(age 8-25 yrs.)	0.9672 (0.8)
– $v\times$ I(age $>$ 25 yrs.)	0.6648 (0.6)
Instability – RPI inflation	-0.6061 (-1.5)
Instability – Long term int. rate	-0.2223 (-0.6)
No. of firms / No. of exits	4,105 / 203
Total time at risk (in firm-yrs.)	48,101
Log-likelihood	-1356.284
Wald χ^2 goodness-of-fit test (d.f. / p-value)	134.20 (32 / 0.00)
χ^2 test (PH assumption) (d.f. / p-value)	11.63 (32 / 0.9996)

** , *and $^+$– Significant at 1%, 5% and 10% level respectively.

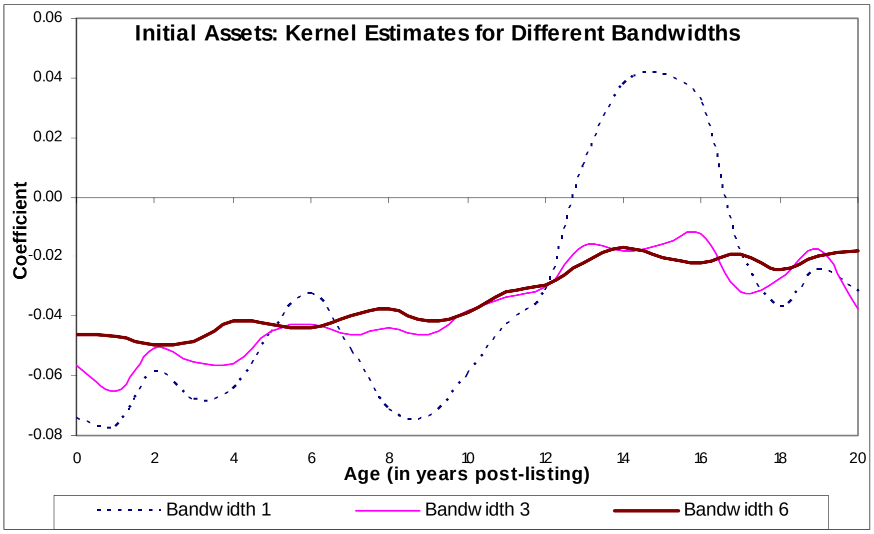

Figure 2:

The local adaptive bandwidth estimates provide substantial insight into the nature of firm dynamics in the UK quoted population. The effect of initial size is negative and falls to zero for older firms, providing evidence in favour of active learning rather than passive learning. The formal tests of hypotheses also favour the active learning model. The detrimental effect of exchange rate instability also decreases with age, and is insignificant for older firms. These effects are, of course, in addition to other firm level, industry level and macroeconomic effects.

After assessing the nature of covariate dependence, we next examine duration dependence in conditional hazard rates. As discussed in Sections 2 and 3, there is considerable debate in the literature on the expected variation of hazard rates with age, under different alternative models of firm dynamics. Empirically, hazard rates are observed to decrease with age, at least at higher ages, even after conditioning on initial size and other proxies for the state variable; this feature of residual negative duration dependence may be interpreted as the effect of unobserved heterogeneity. Learning models predict negative ageing at higher ages, but it is not necessary that this ageing will be in the nature of a decreasing hazard rate (Pakes and Ericson, 1998).

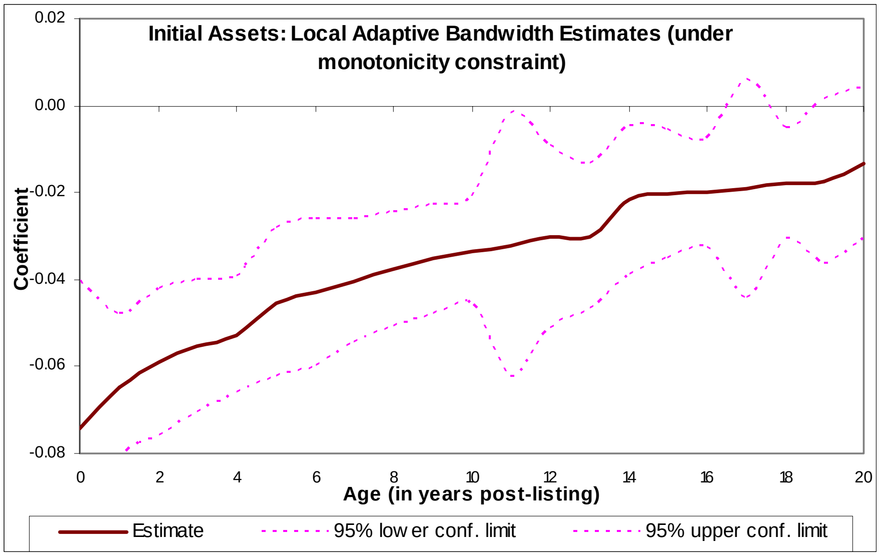

Figure 3:

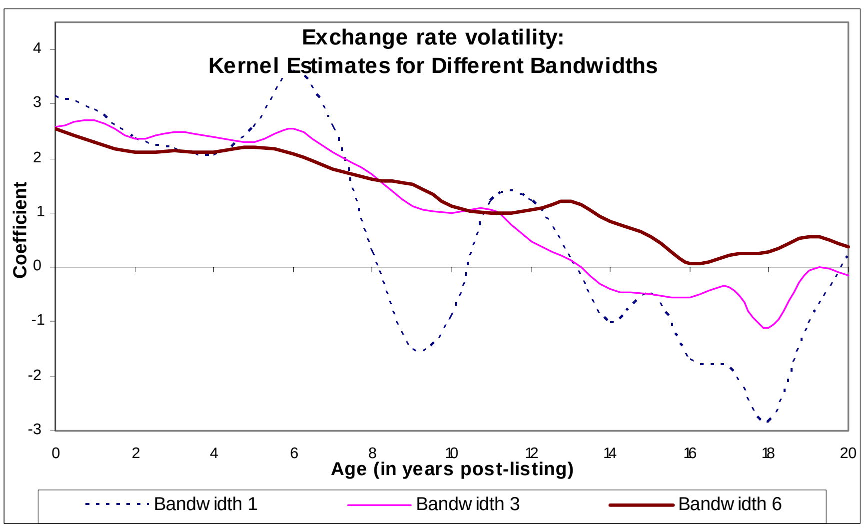

Figure 4:

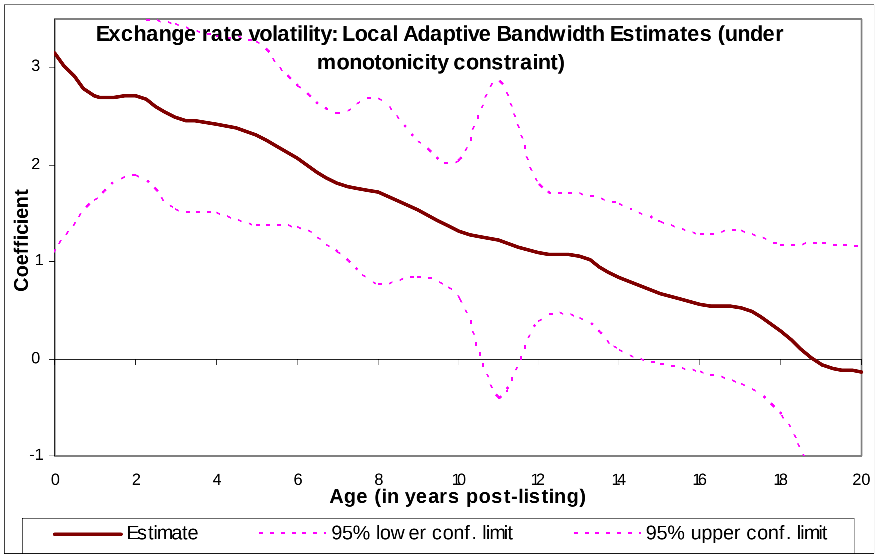

Figure 5:

The estimates of the cumulative baseline hazard function after controlling for initial size, current size and other covariates is shown in Figure 6. The plot indicates that the baseline hazard function may be decreasing with age, but only after the age of about three years, post-listing. This is confirmed by a test for convexity of the cumulative baseline hazard function (see also Gill and Schumacher, 1987).

Further, even though our data does not support strictly negative duration dependence, we observe a weaker notion of ageing from age zero. The estimated baseline hazard rate has the property of Increasing Mean Residual Life (IMRL). The plot of the estimated mean residual life function (Figure 7) visually illustrates this property; this is confirmed by statistical tests.

The above application demonstrates the usefulness of the proposed econometric framework in empirical studies of firm dynamics. The observed evidence appears consistent with the predictions of models of firm dynamics in the presence of macroeconomic instability. Consistent with the active learning model, the impact of initial size on the hazard rate of exits seems to be

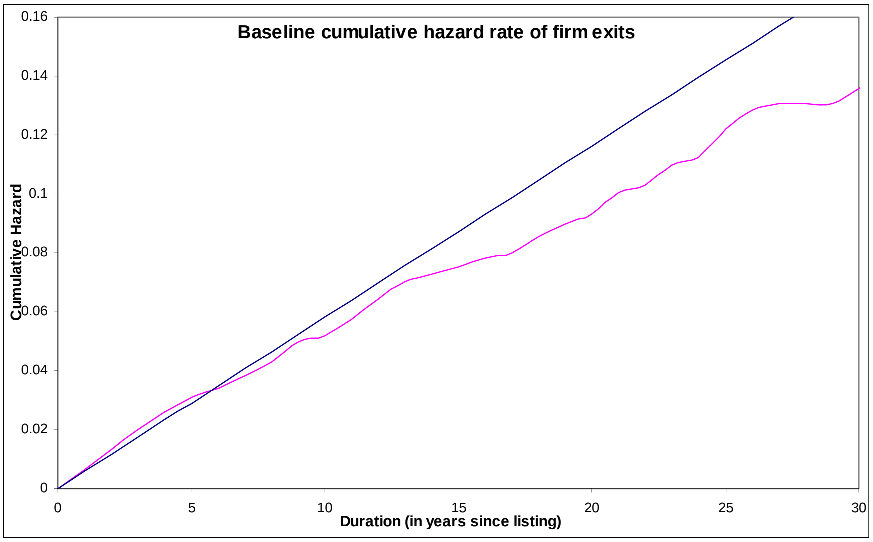

Figure 6:

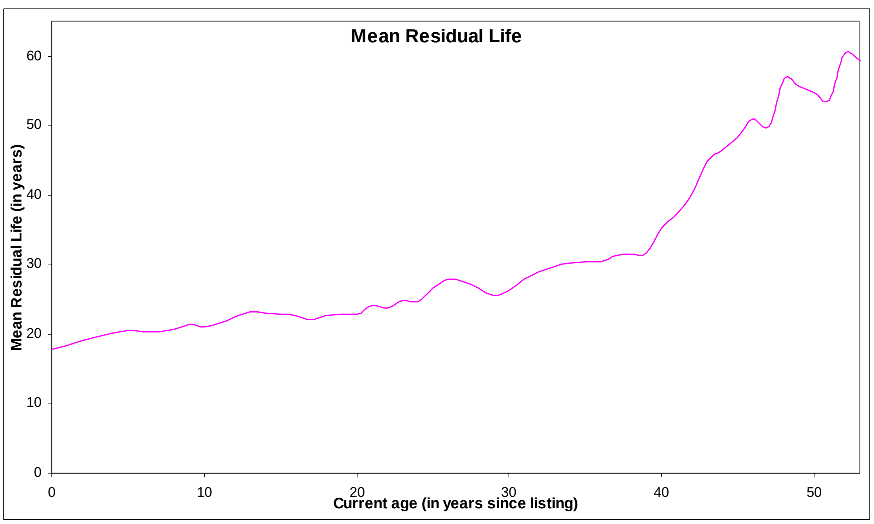

Figure 7:

strongly negative for a very young firm and increase to zero with age. The impact of macroeconomic conditions is also consistent with predictions in the literarture (see Bhattacharjee *et al.*, 2002). The negative ageing pattern in the baseline hazard rate is consistent with learning models, in the presence of unobserved heterogeneity.

5 Conclusions

This paper proposes an econometric model that facilitates empirical study of many common features of models of firm dynamics. The flexible hazard regression model makes a simple adjustment to the Cox regression model by allowing the effects of regressors to vary with age of the firm. In particular, we allow the effect of initial size and other covariates to be age-varying, and illustrate that many of the important issues of interest in studies of firm dynamics can be studied within this framework through inference on the signs, magnitudes and age-varying nature of covariate effects.

Using the model, we develop a framework for testing the implications of alternative theoretical models, and in particular testing whether data on firm exits conform to the active or the passive learning model. Using data on exits due to bankruptcy for quoted UK firms, we illustrate how our flexible hazard regression framework facilitates exploration of the nature of firm dynamics.

Our empirical results suggest that for quoted firms in the UK, effect of initial size is negative and increases with age to zero at higher ages. This supports the active learning model. We find evidence of detrimental effect of macroeconomic instability on the survival probabilities of young firms, and that this effect is negligible for older firms. Further, even after taking into account the effect of a wide range of explanatory factors, the conditional hazard rate shows negative ageing, and this ageing in the nature of increasing mean residual life.

References

1. Abbring, J.H. and Campbell, J.R. (2004). A firm's first year. Free University, Amsterdam. *Mimeo.*
2. Agarwal, R. and Audretsch, D. (2001). Does start-up size matter? The impact of technology and product life-cycle on firm survival. *Journal of Industrial Economics* **49**, 21–44.

3. Asplund, M. and Nocke, V. (2003). Firm turnover in imperfectly competitive markets. *PIER Working Paper* **03-010**, University of Pennsylvania.
4. Bergin, J. and Bernhardt, D. (1999). Industry dynamics. Queens University and University of Illinois. *Mimeo.*
5. Bhattacharjee, A. (2004). Estimation in hazard regression models under monotone departures from proportionality. *Computational Statistics and Data Analysis* **47** (3), 517–536.
6. —— and Bhattacharjee, A. (2004). Bayesian inference in hazard regression models under order restrictions on the nature of covariate dependence and duration dependence. Paper presented at the Computational Management Science Conference and Workshop on Computational Econometrics and Statistics, Neuchatel (April 2004).
7. —— and Das, S. (2002). Testing proportionality in duration models with respect to continuous covariates. *DAE Working Paper* **0220**, Department of Applied Economics, University of Cambridge.
8. ——, Higson, C., Holly, S. and Kattuman, P. (2002). Macroeconomic instability and business exit: Determinants of failures and acquisitions of large UK firms. *DAE Working Paper* **0206**, Department of Applied Economics, University of Cambridge.
9. ——, ——, —— and —— (2003). Business failure in US and UK quoted firms: Impact of macroeconomic instability and the role of legal institutions. *Cambridge Working Papers in Economics* No. **0420**, Department of Applied Economics, University of Cambridge.
10. Brockmann, M., Gasser, T. and Herrmann, E. (1993). Locally adaptive bandwidth choice for kernel regression estimators. *Journal of the American Statistical Association* **88**, 1302–1309.
11. Cabral, L.M.B. (1993). Experience advantages and entry dynamics. *Journal of Economic Theory* **59** (2), 403–416.
12. —— (1997). Competitive industry dynamics: A selective survey of facts and theories. London Business School. *Mimeo.*
13. Caves, R.E. (1998). Industrial organization and new findings on the turnover and mobility of firms. *Journal of Economic Literature* **36**, 1947–1982.

14. Chaudhuri, P. and Marron, J.S. (1999). SiZer for exploration of structures in curves. *Journal of the American Statistical Association* **94**, 807–823.
15. Cox, D.R. (1972). Regression models and life tables (with discussion). *Journal of the Royal Statistical Scociety* Series B **34**, 187–220.
16. Davis, S.J. and Haltiwanger, J.C. (1992). Gross job creation, gross job destruction and employment reallocation. *Quarterly Journal of Economics* **107** (3), 819–863.
17. —— and —— (1999). Gross job flows. In: Ashenfelter, O. and Card, D. (Eds.) *Handbook of Labor Economics* Vol. 3B. North Holland: Amsterdam.
18. Deshpande, J.V. and Sengupta, D. (1995). Testing for the hypothesis of proportional hazards in two populations. *Biometrika* **82**, 251–261.
19. Dunne, T., Roberts, M. and Samuelson, L. (1988). Patterns of firm entry and exit in US manufacturing industries. *Rand Journal of Economics* **19** (4), 495–515.
20. ——, —— and —— (1989). The growth and failure of US manufacturing plants. *Quarterly Journal of Economics* **104** (4), 671–698.
21. Ericson, R. and Pakes, A. (1995). Markov-perfect industry dynamics: A framework for empirical work. *Review of Economic Studies* **62**, 53–82.
22. Gill, R.D. and Schumacher, M. (1987). A simple test of the proportional hazards assumption. *Biometrika* **74**, 289–300.
23. Grambsch, P.M. and Therneau, T.M. (1994). Proportional hazards tests and diagnostics based on weighted residuals. *Biometrika* **81**, 515–526. Correction: 1995v82 p. 668.
24. Greene, W.H. (1993). *Econometric Analysis.* 2nd Edition. MacMillan: New York.
25. Hall, P. and Presnell, B. (1999). Intentionally biased bootstrap methods. *Journal of the Royal Statistical Society* Series B **61**, 143–158.
26. Hopenhayn, H.A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica* **60**, 1127–1150.
27. Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica* **50**. 649–670.

28. Kalbfleisch, J.D., and Prentice, R.L. (1980). *The Statistical Analysis of Failure Time Data*. John Wiley and Sons: New York.
29. Klepper, S. (1996). Entry, exit, growth, and innovation over the product life cycle. *American Economic Review* **86** (3), 562–583.
30. Lambson, V. (1991). Industry evolution with sunk costs and uncertain market conditions. *International Journal of Industrial Organization* **9** (2), 171–196.
31. —— (1992). Competitive profits in the long run. *Review of Economic Studies* **59**, 125–142.
32. Lancaster, T. (1990). *The Econometric Analysis of Transition Data*. Econometric Society Monographs (17). Cambridge University Press: Cambridge.
33. Lee, L. and Pirie, W. R. (1981). A graphical method of comparing trends in series of events. *Communications in Statistics: Theory and Methods* **10**, 827–848.
34. Lin, D.Y. and Wei, L.J. (1989). The robust inference for the Cox proportional hazards model. *Journal of the American Statistical Association* **84**. 1074–1078.
35. Lippman, S. and Rumelt, R. (1982). Uncertain immitability: An analysis of interfirm differences in efficiency under competition. *Bell Journal of Economics* **13**, 418–438.
36. Martinussen, T., Scheike, T.H. and Skovgaard, Ib M. (2002). Efficient estimation of fixed and time-varying covariate effects in multiplicative intensity models. *Scandinavian Journal of Statistics* **29**, 57–74.
37. Murphy, S.A. and Sen, P.K. (1991). Time-dependent coefficients in a Cox-type regression model. *Stochastic Processes and their Applications* **39**, 153–180.
38. Neumann, G.R. (1997). Search models and duration data. In: Pesaran, M.H. (Ed.) *Handbook of Applied Econometrics: Microeconometrics*. Basil Blackwell: Oxford.
39. Olley, G.S. and Pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica* **64**, 1263–1298.
40. Pakes, A. and Ericson, R. (1998). Empirical implications of alternative models of firm dynamics. *Journal of Economic Theory* **79**, 1–46.

41. Scheike, T.H. (2002). Time varying effects in survival analysis. Department of Biostatistics, University of Copenhagen. *Mimeo.*
42. Siegfried, J.J. and Evans, L.B. (1994). Empirical studies of entry and exit: A survey of the evidence. *Review of Industrial Organization* **9**, 121–155.
43. Sutton, J. (1997). Gibrats legacy. *Journal of Economic Literature* **35** (1), 40–59.
44. van den Berg , G.J. (2001). Duration models: Specification, identification, and multiple durations. In: Heckman, J.J. and Leamer, E. (Eds.) *Handbook of Econometrics* Volume V. North Holland: Amsterdam.