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1 Coastal Engineering Journal  
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3 **Investigation of Long Waves Generated by Bottom-Tilting Wave Maker**

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18 Motivated by recent field observations of tsunamis, a new wave maker, namely bottom-tilting wave  
19 maker, has been designed and investigated in order to generate very long waves in the laboratory.  
20 Theoretical results from the linear wave theory and the numerical modelling based on the weakly  
21 nonlinear and weakly dispersive wave theory show good agreement with the measurements. Using  
22 both theoretical and experimental results, the relation between the bottom motion and the resulting  
23 waves have been investigated. Wave amplitude and period of the generated waves are the subject of  
24 the parametric analysis, which verifies that the wave maker is able to generate waves longer than the  
25 effective wavelength of the solitary wave with the same wave amplitude.

26 *Keywords:* Long waves; wave maker; Boussinesq equations; nonlinear shallow equations; linear wave  
27 theory.

28 **1. Introduction**

29 Tsunamis can be caused by undersea earthquakes, such as the recent tragic events includ-  
30 ing the 2004 Indian tsunami, the 2011 Tohoku tsunami and the 2015 Chile tsunami [Tsuji  
31 *et al.*, 2006; Hayashi *et al.*, 2011; Aránguiz *et al.*, 2016]. They can have extremely long  
32 wavelengths and very small amplitudes compared to ocean depths [Mei, 1989] . It is of  
33 great importance to build an appropriate physical wave model in tsunami research for more  
34 accurate theoretical and experimental investigations. Over the past few decades, solitary

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35 waves have been extensively used as model tsunami [e.g. Hall and Watts, 1953; Hammack,  
36 1973; Synolakis, 1987; Li and Raichlen, 2002]. A solitary wave propagates in constant  
37 depth with permanent form, whose surface elevation is described as

$$\eta'(x', t') = A'_s \operatorname{sech}^2 [K'_s(x' - c't')] , \quad K'_s = \frac{1}{h'_0} \sqrt{\frac{3A'_s}{4h'_0}} , \quad (1)$$

38 in the horizontal coordinate  $x'$  and time  $t'$ , where  $\eta'$ ,  $A'_s$ ,  $c'$  and  $h'_0$  denote free surface ele-  
39 vation, wave height, phase velocity and static water depth, respectively. Boussinesq [1872]  
40 first developed the wave profile which is also the exact solution of the Korteweg–de Vries  
41 (KdV) equation. Owing to the solid theoretical foundation, solitary waves have become  
42 popular in tsunami science. The fact that the generation of a solitary wave is relatively  
43 straightforward [Goring, 1978] might be another reason why they have attracted interests  
44 from many researchers .

45 However, recent research [e.g. Madsen *et al.*, 2008] has questioned the relevance of  
46 solitary wave in tsunami research as the link between its wavenumber  $K'_s$  and wave height  
47  $A'_s$  is not realistic for geophysical tsunamis. The real-time field records of the 2011 Japan  
48 Tohoku tsunami outlined in Fujii *et al.* [2011] clearly show that the effective wavelength  
49 of the solitary wave is an order-of-magnitude short and the wave front is too steep when  
50 compared to the observed leading tsunami. Note that we define the effective wave length  $L'_s$   
51 of a solitary wave as  $L'_s = 2\pi/K'_s$ .

52 There have been many types of wave makers that have been developed, among which  
53 piston-type wave makers are commonly used to generate long waves [e.g. Li and Raichlen,  
54 2002; Fujima *et al.*, 2009; Lo *et al.*, 2013; Schimmels *et al.*, 2016]. Piston-type wave makers  
55 can create approximately uniform flow field in the vertical direction, which is an important  
56 characteristic of long waves. However, as illustrated in figure 1, the wavelength of the gener-  
57 ated waves is limited by the stroke length  $L'_p$ .

58 On the other hand, Hammack [1973] developed a bottom-moving wave maker to create  
59 solitary waves excited by vertical bottom motions. He also found that solitary waves were  
60 generated by positive bed motion while negative waves were generated by negative bed  
61 motion. After some distance (e.g., 180 times the still water depth) , the positive waves de-  
62 veloped into a train of solitary waves which were ordered by amplitude, and negative waves  
63 developed into a dispersive train of waves. As Hammack [1973] pointed out, however, the  
64 wavelength is largely determined by the length of the moving bottom, and the resulting  
65 waves were still shorter than tsunamis.

66 Dam-break bores have also been used in laboratory experiments [e.g. O'Donoghue  
67 *et al.*, 2010; Kihara *et al.*, 2015]. O'Donoghue *et al.* [2010] used a dam-break rig to model  
68 a swash event of a turbulent bore by rapidly lifting a gate and releasing water stored behind  
69 the gate. A Large-scale Tsunami Physical Simulator was utilised by Kihara *et al.* [2015]  
70 to generate a tsunami bore traveling over a dry bed to investigate the pressure on a vertical  
71 tide wall by controlling a radial gate and the water height in the overhead tank.

72 Moreover, Rossetto *et al.* [2011] used a pneumatic-type wave maker to create long  
73 waves by releasing a volume of water into wave basin in a controlled manner. Goseberg

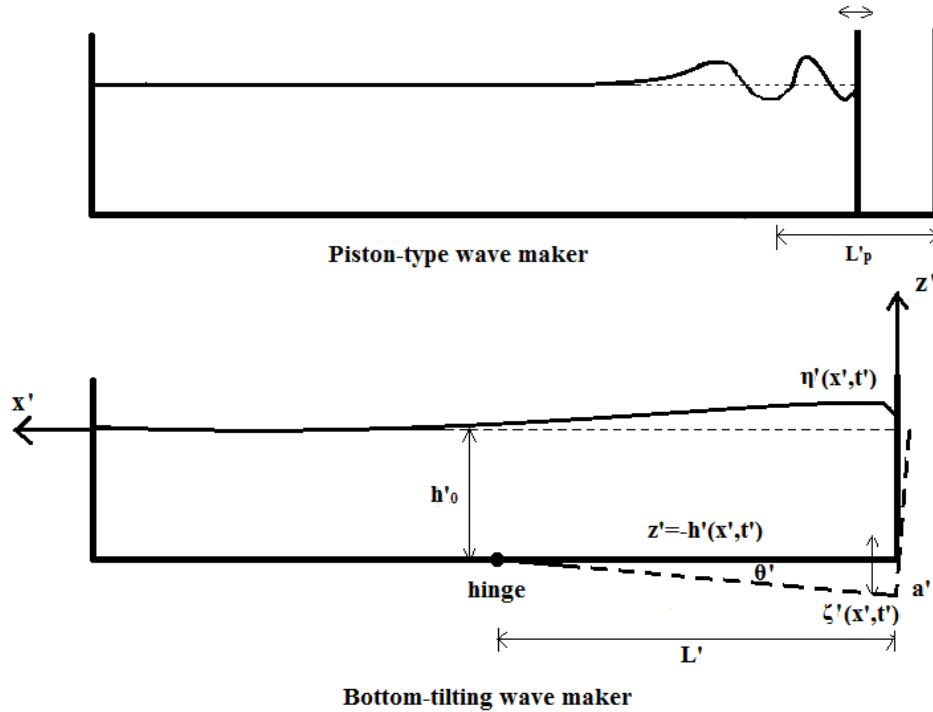


Fig. 1. Comparison between piston-type wave maker and the bottom-tilting wave maker.

74 *et al.* [2013] investigated pump-driven long wave generation in a closed-circuit wave flume,  
 75 which successfully generated long waves resembling tsunamis. While it is encouraging that  
 76 these novel wave makers are not limited by the stroke length, it is not easy to develop  
 77 theories for these new types of wave makers due to their unusual boundary conditions.

78 In order to make much longer waves in laboratory, perhaps a straightforward idea would  
 79 be moving the entire bottom and generating waves as long as the tank itself. In the present  
 80 research, a tilting bottom was used, as the simplicity of the design was attractive. Therefore,  
 81 the new wave maker used in this study was designed based on the concept depicted in  
 82 figure 1. In comparison with the typical piston-type wave maker, the bottom tilting wave  
 83 maker has a much longer moving part with length  $L'$ , which can thus produce longer waves.  
 84 Unlike the disintegration of the waves observed in Hammack [1973]'s study, the generated  
 85 wave will be used immediately before it disintegrates into a train of shorter waves. The long  
 86 wave will soon run up the adjustable beach which is directly hinged to the moving bottom.

87 Recently, Lu *et al.* [2017] provided the theoretical foundation needed in controlling the  
 88 new wave maker. In the present paper, using a combined methodology, including linear and  
 89 nonlinear solutions as well as experimental measurements, we aim to verify that the new  
 90 wave maker is able to create long waves through a comparison of the effective wavelengths  
 91 between the generated long waves and solitary waves and to explore its capacity of mod-

92 elling real tsunamis. Furthermore, we investigate the relation between the bottom motion  
 93 and the resulting wave, expecting to describe the generated waves in terms of its generation  
 94 parameters.

95 The rest of the article is organized as follows. For completeness, the theoretical back-  
 96 ground is briefly reviewed in Section 2. The preliminary design of a bottom-tilting wave  
 97 maker based on the linear wave theory is introduced in Section 3. Section 4 provides the  
 98 details of the wave maker and the corresponding experimental procedures. Theoretical and  
 99 experimental results are compared to each other in Section 5, and the relationship between  
 100 the bottom motion and the resulting waves is sought. Further discussion on the waves gener-  
 101 ated by bottom-tilting wave maker are presented in Section 6. Finally, concluding remarks  
 102 are given in Section 7.

## 103 2. Theoretical background

104 A schematic sketch of the two-dimensional wave tank is depicted in figure 1 with the cor-  
 105 responding coordinate system. Lu *et al.* [2017] provided the analytical solution of the free  
 106 surface elevation by solving the two-dimensional Laplace equation along with the specific  
 107 boundary conditions derived from the linear wave theory. The free surface elevation  $\eta'(x, t)$   
 108 is given by

$$\eta'(x' = L', t') = \frac{a' L'}{\pi} \int_0^{t'} du' \int_0^\infty dk' \frac{\sin^2(k' L'/2)}{(k' L'/2)^2} \frac{\cos k' L'}{\cosh k' h'_0} Q'(u') \cos \omega'(t' - u'), \quad (2)$$

109 with the bottom motion displacement  $\zeta'$  given by

$$\zeta'(x', t') = D'_0(x') T'(t'), \quad (3)$$

110 and

$$Q'(t') = \frac{d}{dt'} T'(t'), \quad (4)$$

111 where  $a'$  and  $h'_0$  denote the motion amplitude at the right end of the tilting bottom and  
 112 constant water depth, respectively.  $D'_0(x)$  describes the shape of the moving bottom, which  
 113 is given as for flat bottom of length  $L'$

$$D'_0(x') = \begin{cases} a'(1 - |x'|/L'), & |x'| \leq L', \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

114 The wavenumber  $k'$  and the frequency  $\omega'$  are related by the dispersion relation  $\omega'^2 =$   
 115  $g' k' \tanh k' h'_0$ . Note that the flow is assumed to be inviscid, incompressible and irrotational.

116 We remark here that the analytical solution Eq. (2) is obtained for an infinite domain.  
 117 Therefore comparison between Eq. (2) and the experimental results will be only valid in the  
 118 early stages of the experiments when the reflected wave from the other end has not affected  
 119 the incident wave.

120 The weakly dispersive and weakly nonlinear wave theory is also used to model the wave  
 121 maker. The Boussinesq equations (BE) derived by Wu [1987] are written in conservative  
 122 form as shown below:

$$\left. \begin{aligned} H'_t + [H'u']_{x'} &= 0, \\ u'_t + [\frac{1}{2}u'^2 + g'(H' - h')]_{x'} &= \frac{1}{2}h'h'_{x'tt'} + \frac{1}{2}h'(h'u')_{x'x't'} - \frac{1}{6}h'^2u'_{x'x't'}, \end{aligned} \right\} \quad (6)$$

123 where the subscript  $t$  and  $x$  indicate temporal and spatial derivatives, respectively, the total  
 124 water depth  $H' = \eta' + h'$ , and  $h'$  and  $u'$  denote the water depth under the static water  
 125 line and the horizontal velocity, respectively. According to Lu *et al.* [2017], Eq. (6) can be  
 126 rearranged as shown below:

$$\mathbf{V}_{t'} + [\mathbb{F}(\mathbf{V})]_{x'} = \mathbb{S}_b + \mathbb{M}(\mathbf{V}), \quad (7)$$

127 where the variable  $\mathbf{V}$ , the advective flux  $\mathbb{F}(\mathbf{V})$ , the source term  $\mathbb{S}_b$  and the dispersive  
 128 term  $\mathbb{M}(\mathbf{V})$  are denoted respectively by  $\mathbf{V} = \begin{pmatrix} H' \\ u' \end{pmatrix}$ ,  $\mathbb{F}(\mathbf{V}) = \begin{pmatrix} H'u' \\ \frac{1}{2}u'^2 + g'(H' - h') \end{pmatrix}$ ,  $\mathbb{S}_b =$   
 129  $\begin{pmatrix} 0 \\ \frac{1}{2}h'h'_{x'tt'} \end{pmatrix}$  and  $\mathbb{M}(\mathbf{V}) = \begin{pmatrix} 0 \\ \frac{1}{2}h'(h'u')_{x'x't'} - \frac{1}{6}h'^2u'_{x'x't'} \end{pmatrix}$ . They can be simplified by omit-  
 130 ting the dispersive terms and become the nonlinear shallow water equations (NSWE). The  
 131 equations are solved by a shock-capturing finite volume method with high-order recon-  
 132 struction schemes [Lu *et al.*, 2017], so that the discontinuities at cell interfaces in the dis-  
 133 crete solution are treated by higher-order approximations, such as second-order-accurate  
 134 UNO2 [Harten and Osher, 1987], third-order-accurate WENO3 and fifth-order-accurate  
 135 WENO5 [Shu, 1998]. In practice, UNO2 is used for the Boussinesq equations system, while  
 136 WENO schemes are used for the nonlinear shallow water equations system. The analysis in  
 137 section 5 is based on the BE system, while the NSWE system will be compared with the BE  
 138 system to demonstrate the effects of the dispersion. A semi-infinite computational is used  
 139 to obtain the wavelength and the back profile of the generated waves. In all the other cases,  
 140 the computational analysis is based on the confined domain, identical to the physical wave  
 141 tank with full reflective boundaries considered.

142 In this study, the bottom motion is of finite duration, which can excite the fluid and  
 143 produce transient waves. For simplicity, only the bottom motions with constant velocity for  
 144 given duration are considered as shown below:

$$Q'(t') = \begin{cases} 1/b', & 0 \leq t' \leq b' & \text{upward motion,} \\ -1/b', & 0 \leq t' \leq b' & \text{downward motion,} \end{cases} \quad (8)$$

145 where  $b'$  denotes the motion duration time. More complex bottom motions are also pos-  
 146 sible, for example the resulting wave shown by a dashed line in figure 12 which will be  
 147 discussed later. As the duration of the bottom motion increases, however, the effects of  
 148 reflected waves become dominant, especially in the wave tank with a limited length. There-  
 149 fore, more general motions are left as future studies when the challenges of dealing with  
 150 reflected waves are fully addressed.

151 In the following discussions, results are described in terms of dimensionless variables.  
 152 The constant water depth  $h'_0$  is used to normalise the length parameters, so the relevant  
 153 dimensionless variables are given by

$$L = \frac{L'}{h'_0}, x = \frac{x'}{h'_0}, h' = \frac{h'}{h'_0}, \eta = \frac{\eta'}{h'_0}, \zeta = \frac{\zeta'}{h'_0}, a = \frac{a'}{h'_0}, A = \frac{A'}{h'_0}, \quad (9)$$

154 while time is normalised by  $L' / \sqrt{g'h'_0}$

$$t = t' \sqrt{g'h'_0/L'}, b = b' \sqrt{g'h'_0/L'}, T = T' \sqrt{g'h'_0/L'}, \quad (10)$$

155 where  $L'$ ,  $A'$  and  $T'$  denote the length of the tilting bottom, the wave amplitude and the  
 156 wave period, respectively. Moreover, as the length of the moving bottom is fixed in reality,  
 157  $\alpha = h'_0/L'$  can be used to represent the constant water depth, in some cases discussed later.

### 158 3. Preliminary design

159 As can be seen in figure 1, the wave maker consists of a moving bottom hinged to a fixed  
 160 bottom, so that the bottom will move in a rotational motion with the vertical displacement  
 161 of the moving part  $\zeta(x, t) \approx (L - x)\theta(t)$ . The moving bottom part will generate long waves,  
 162 and the other part is for the generated waves to propagate in the constant water depth or  
 163 to run up the slope. The origin of the coordinate system is at the right end of the static  
 164 water surface with  $x$  axis pointing leftwards and  $z$  axis pointing upwards. The fluid domain  
 165 is bounded by the two end walls, the free surface  $z = \eta(x, t)$  and the solid impermeable  
 166 bottom boundary  $z = -h(x, t)$ . Note that  $h(x, t) = 1 - \zeta(x, t)$  with  $h_0 = 1$ .

167 Our purpose in this section is to demonstrate that the wave maker with the specific  
 168 geometry is capable of generating waves longer than solitary waves. Then, the ratio  $\alpha$  is  
 169 of interest for limiting the generation of long waves and is discussed by considering the  
 170 linear wave theory (2). It can be found that the wavelength increases with growing motion  
 171 duration, so that the minimum and observable duration time (0.5 s) the actuator can provide  
 172 is used to generate the shortest wave for the following cases discussed. It is also remarked  
 173 here that, as the analytical solution for downward motion is just opposite to that for upward  
 174 motion, it is efficient to use only upward motion for design purposes. Moreover, both  
 175 motions begin from the initial position where  $\theta \neq 0$  and stop at  $\theta = 0$  to ensure flat bottom  
 176 throughout the wave tank for wave propagation, and slope is not considered here.

177 Based on the typical relation of  $\alpha = O(0.1)$ , a range of water depths  $\alpha$  and the bottom  
 178 motion amplitudes  $a$  are tested as shown in figure 2. The analytical solution (2) was used to  
 179 compare the ratio  $L_w/L_s$  indicating the ability of the wave maker of generating waves longer  
 180 than the solitary waves, where  $L_s$  is the solitary wave length and  $L_w$  is the generated wave  
 181 length. It is observed in figure 2 that  $L_w/L_s$  grows with decreasing  $\alpha$  and with increasing  $a$ .  
 182 For the smallest  $a = 0.1$  tested here, it is expected that the generated wave will be longer  
 183 than the solitary wave for  $\alpha < 0.07$ .

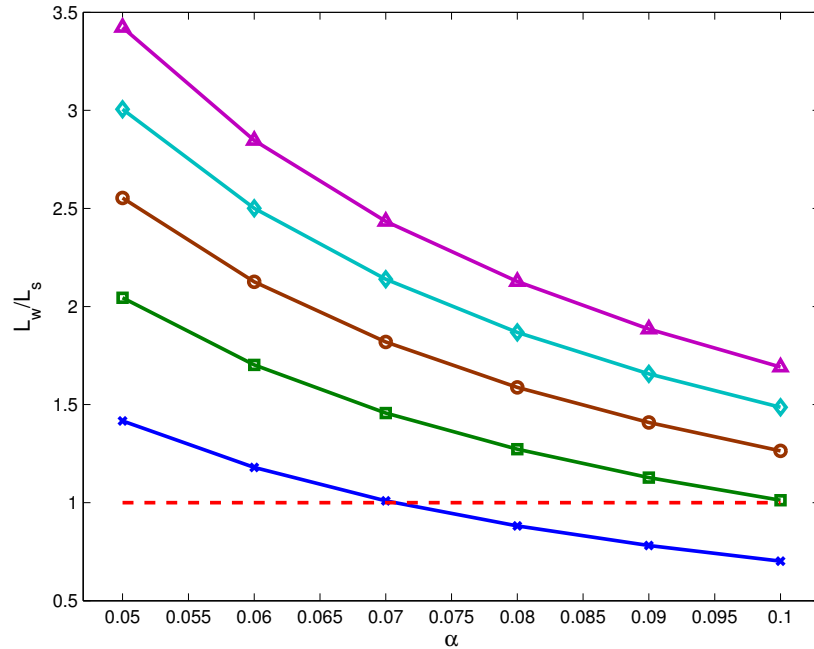


Fig. 2. The ratios of the generated wave length to the solitary wave length,  $L_w/L_s$ , plotted against varying  $\alpha$  and  $a$ : ---,  $L_w/L_s = 1$ ; +,  $a = 0.1$ ;  $\square$ ,  $a = 0.2$ ;  $\circ$ ,  $a = 0.3$ ;  $\diamond$ ,  $a = 0.4$ ;  $\triangle$ ,  $a = 0.5$ .

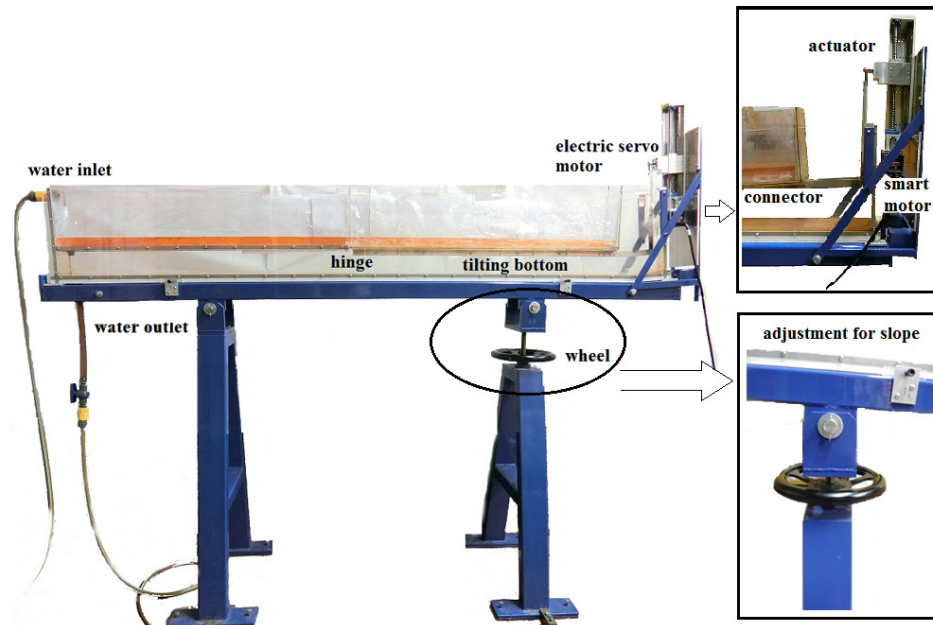
#### 184 4. Experimental equipment and procedures

185 A series of experiments was carried out in the wave tank shown in figure 3, which also  
 186 presents the set-up of the experiments. The tank is 2.185 m long, 0.11 m wide and 0.3 m  
 187 deep, and consists of an adjustable slope and the bottom-tilting wave maker. The wave tank  
 188 is regarded as being two-dimensional, so that the resulting waves are also two-dimensional.  
 189 The bottom-tilting wave generator has a 1 m long and 0.11 m wide moving bottom which is  
 190 hinged to the slope which has the same dimensions. The hinge is located at 0.2 m below the  
 191 top of the tank, which leaves enough space for varying bottom motions (upward, downward  
 192 or combination).

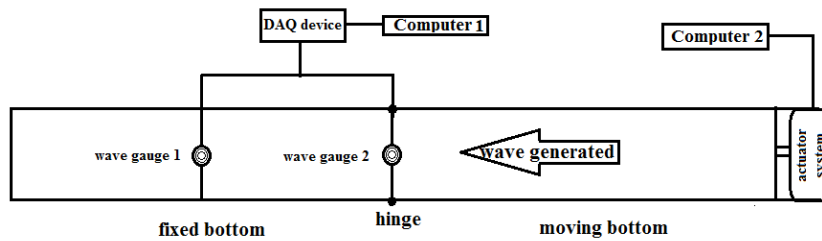
193 The moving bottom is controlled by an ANIMATICS<sup>®</sup> SM23165DT electrical servo  
 194 motor located at the right end of the tank which has a rotating rod to move the actuator.  
 195 The actuator moves vertically under the command of the programme called SmartMotor<sup>™</sup>  
 196 Interface, and it provides the moving bottom with the prescribed vertical velocity and dis-  
 197 placement directly through a steel connector attached to the back of the bottom.

198 Rubber seals were attached around the moving bottom. In addition, a 2.3 m long and  
 199 0.31 m wide PVC membrane covers the inner surface of the wave tank to ensure water-  
 200 proofness. The whole wave tank is supported by two legs, and there is a wheel on the right  
 201 leg used for adjusting the slope of the fixed bed. Then, the beach with desired slope was





(a) Photo of the bottom-tilting wave maker (components are shown as indicated)



(b) Plot of the set-up

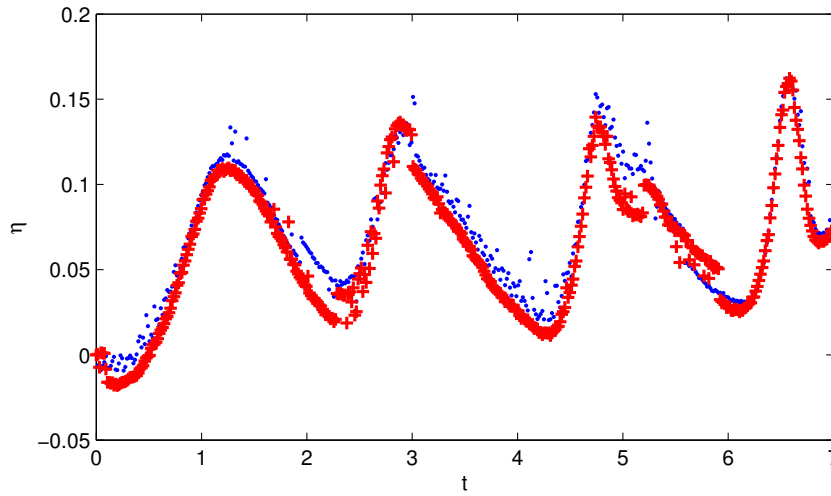
Fig. 3. Set-up of the bottom-tilting wave maker.

202 created by a specific height difference. In the present study, the fixed bottom was always  
 203 flat.

204 Two acoustic wave gauges (BANNER<sup>®</sup> U-STAGE<sup>™</sup>S18UUA) were used to measure  
 205 the free surface elevation at the hinge (1 m from the right end) and middle of the fixed bot-  
 206 tom (1.5 m from the right end) as shown in figure 3 (b), respectively. The two wave gauges  
 207 are ultrasonic sensors with analog output relying on time-varying voltage proportional to  
 208 the time history of the displacement. The measurement frequency was set to be 50 Hz and  
 209 measurement duration time 10 s which begins with the bottom motion, and is sufficient  
 210 to measure the time history of the free surface elevation. The accuracy of the gauges is  
 211  $\pm 0.5$  mm. Data is collected by a computer through a National Instruments<sup>™</sup> Low-Cost

Table 1. Normalised bottom motion parameters ( $a$  and  $b$ ) of the bottom motions for different  $\alpha$ .

| $\alpha$ | $a$                          | $b$                            |
|----------|------------------------------|--------------------------------|
| 0.04     | 0.125, 0.250 $\dots$ , 1.000 | 0.3132, 0.6264, 0.9396, 1.2528 |
| 0.05     | 0.100, 0.200 $\dots$ , 0.800 | 0.3502, 0.7004, 1.0505, 1.4007 |
| 0.06     | 0.083, 0.167 $\dots$ , 0.677 | 0.3836, 0.7672, 1.1508, 1.5344 |

Fig. 4. Comparison of two repetitions of the time history of  $\eta$  at the hinge.

212 USB Data Acquisition (DAQ) 6003.

213 Motion displacement  $a$  and duration time  $b$  are the two defining parameters of the basic  
 214 upward or downward bottom motion. Limited by the maximum value of the ratio  $\alpha$  being  
 215 0.07, three values 0.04, 0.05 and 0.06 were tested. After some trials, the maximum allow-  
 216 able bottom motion displacement without causing splash-up at the end wall was found to  
 217 be under  $a=1$ , 0.8 and 0.677 for the three different  $\alpha$ , respectively. On the other hand, the  
 218 corresponding rotating angle would be  $0.013\pi$ , which is small enough to ignore the hor-  
 219 izontal velocity induced by the moving bottom as long as the bottom motion is not very  
 220 strong. Thus, the bottom motion displacement within the range of 0.083 to 1 were chosen  
 221 in this study. Moreover, bottom motion duration time ranges from  $b=0.3$  to 1.5. Ranges of  
 222 the normalized parameters used in the experiments are summarized in Table 1. Experiments  
 223 are repeated a second time to ensure repeatability. For example, figure 4 shows two repe-  
 224 titions of the time history of the free surface elevation at the hinge. There are only small  
 225 differences between the two measurements, and the error is within the accuracy of the wave  
 226 gauge ( $\pm 0.01$ ).

## 227 **5. Experimental results**

228 The purpose of the present study is to relate the wave amplitude  $A$  and the wave period  
 229  $T$  of the waves generated in the new wave tank in terms of the two parameters ( $a$  and  $b$ )  
 230 of the simple upward and downward motions in Eq. (8) and (3). Firstly, the authors will  
 231 demonstrates that there is good agreement between experimental and theoretical results for  
 232 all the cases in Table 1, even with the multiple reflections on both ends of the tank. It is  
 233 straightforward to get  $A$  from experimental data, but  $T$  is more difficult to measure due to  
 234 the presence of the reflected wave. Instead, the authors use theoretical solutions in semi-  
 235 infinite domain to estimate  $T$ . It is noted here that the cases shown in this section are for  
 236  $\alpha = 0.05$  if not specified otherwise, but similar results were observed in the other cases as  
 237 well.

### 238 **5.1. Comparison of experimental and theoretical results**

239 The numerical results of the time-histories of the free surface elevation at the hinge are  
 240 compared to the experimental data, and examples are shown in figure 5 covering a wide  
 241 range of bottom motion amplitude  $a$  and duration  $b$ . For small  $a$ , the wave amplitude is  
 242 small and sometimes comparable to the uncertainty (0.5 mm) of the wave gauges, which  
 243 resulted in scattering of the experimental data for  $a = 0.2$  in figure 5. Nevertheless, both  
 244 the Boussinesq equations and the nonlinear shallow water equations show good agreement  
 245 with the experimental data. Eventually dispersion becomes no longer negligible, and the  
 246 Boussinesq equations capture it well. It is also observed that the resulting waves become  
 247 increasingly asymmetric with greater motion displacement because of the growing nonlin-  
 248 earity, in particular the negative waves. In addition, it can be found that the wave amplitudes  
 249 of the resulting waves are smaller than the corresponding motion displacement, roughly half  
 250 as much as the relevant motion amplitude for most cases.

### 251 **5.2. Wave amplitudes of the waves generated using the bottom-tilting wave maker**

252 The amplitudes of the waves generated in the new wave tank are plotted as a function of the  
 253 bottom motion displacement  $a$  and the duration  $b$  in figure 6. The amplitude was defined by  
 254 the elevation of the first peak for the positive waves or the first trough for the negative waves.  
 255 It is observed that greater motion displacement  $a$  and smaller bottom motion duration  $b$   
 256 leads to increasing wave amplitude  $A$  for both upward and downward motions. However,  
 257 effects of the motion duration on the wave amplitude become less important for downward  
 258 motions as the motion amplitude increases. This suggests that early disintegration of the  
 259 high-amplitude leading depression wave caused by dispersion plays a role in determining  
 260 the wave amplitude of negative waves.

261 In an effort to succinctly describe the wave amplitude in terms of  $a$  and  $b$ , it is instruc-  
 262 tive to consider scaling analysis. In their experimental study of tsunami generation due to  
 263 subaerial mass flow, Walder *et al.* [2003] argued that amplitude of tsunami is mainly a func-  
 264 tion of volume flux of displaced water. In the present research, the volume (per unit width)  
 265 of displaced water in the bottom-tilting wave maker is  $V_w = aL/2$ , which was in motion

Table 2. Parameters ( $m$  and  $n$ ) of the fitting functions for different bottom motion type.

| motion type | $m$     | $n$    |
|-------------|---------|--------|
| upward      | 0.06547 | 0.4994 |
| downward    | 0.09224 | 0.4380 |

for the duration of  $b$ . Following Walder *et al.* [2003], wave amplitudes measured from the experiments are plotted against the inverse of the volume flux, that is  $b/V_w$ , in figure 7. The two fitting functions are of form  $A = m(b/V_w)^{-n}$ , and the results are summarised in Table 2. Walder *et al.* [2003] reported  $m = 1.32$  and  $n = 0.68$ . On the other hand, the values of  $m$  for our experimental data are smaller, possibly because of the different time normalization, different generation mechanism and absence of acceleration for most of the bottom displacement.

### 5.3. Estimation of wave period

Due to the limited length of the wave tank, the reflected wave makes measuring  $T$  rather difficult. Two different methods were employed to estimate the wave periods. One is to measure time from the beginning of the wave to the peak of the wave, namely wave peak time  $T_a$ , in which the beginning is defined as the point where the water surface elevation is 1% of the wave amplitude. Then the wave period is estimated to be  $T = 2T_a$ . This method, however, works only for the waves that are more or less symmetric. Waves generated using the bottom-tilting wave maker becomes increasingly skewed as the  $ab^{-1}$  grows. In those cases, theoretical solutions in semi-infinite domain are used instead. Figure 8 presents the comparison between the theoretical results for  $a = 0.5$  and  $b = 0.70$  and the experimental data up to the arrival of the reflected wave. Both the Boussinesq equations and the nonlinear shallow water equations show good agreement with the experimental data before they are affected by the reflected wave. On the other hand, the linear analytic solution is quite different from the data, which means that nonlinear effects of the deformation of the wave profile are not negligible in wave generation. Using the numerical results from the Boussinesq equations in semi-infinite domain, the wave period  $T$  was estimated as the time difference between two points where the surface elevation is 1% of the amplitude of the wave.

### 5.4. Wave periods of the waves generated using the bottom-tilting wave maker

Figure 9 shows the wave peak time  $T_a$  plotted as a function of  $a$  and  $b$  for both upward and downward motions. It is observed that greater bottom motion duration  $b$  results in greater wave peak time  $T_a$ . However, dependence of  $T_a$  on  $a$  is different according to the direction of bottom movements. More specifically, while  $T_a$  decreases with  $a$  for upward motions, the opposite trend is found for downward motions. This interesting observation may be attributed to the nonlinear effects of the deformation of the wave profile. For upward motions, water surface elevation increases from the beginning of the wave to the peak, and the local wave celerity also increases with the surface elevation. Therefore the wave form

299 becomes squeezed and this tendency would be stronger for higher-amplitude waves. On the  
 300 other hand, water surface elevation decreases from the beginning to the (negative) peak of  
 301 the waves generated by the downward motions, and the wave form will be elongated at least  
 302 up to the peak. Of course, this nonlinear effect cannot be expected from the linear analytic  
 303 solution, which shows no functional dependence of  $T_a$  on  $a$ .

304 Wave periods estimated from the Boussinesq equations in semi-infinite domain are plot-  
 305 ted against  $a$  and  $b$  in figure 10. Small-amplitude waves ( $a = 0.1$ ) are more or less sym-  
 306 metric, and their wave periods can also be estimated using the peak time, that is  $T = 2T_a$   
 307 (marked with star in figure 10). As in the case for the wave peak time, the wave period  
 308 also increases with the bottom motion duration  $b$ . Dependence of the wave period on the  
 309 bottom motion amplitude is, however, much more complicated. Unlike the peak time, now  
 310 it is observed that the wave period increases monotonically with the motion amplitude for  
 311 the waves generated by the upward motions. This is only explained if the waves are skewed  
 312 with long tails so that  $T > 2T_a$ , which is also due to nonlinearity.

313 Wave periods of the waves generated by the downward motions no longer show mono-  
 314 tonic dependence on  $a$ . For small motion amplitudes, the increase of peak time with  $a$  is  
 315 almost cancelled due to the opposite trend of the tail. For larger amplitudes, dispersion man-  
 316 ifests itself as disintegration of the wave form, which effectively reduces the wave period  
 317 (see second and third panels of figure 5 (a) and (b)). After sudden decrease of the wave  
 318 period, the nonlinear effects come into play again, and the wave period starts to increase  
 319 with increasing  $a$  just like the waves generated by the upward motions.

320 Due to the rather complex response of the wave periods to the motion amplitudes, a  
 321 simple functional description of  $T$  with respect to  $a$  and  $b$  could not be found. More in-  
 322 vestigation is needed to elucidate the roles of competing mechanism of nonlinearity and  
 323 dispersion on wave periods of the long waves generated in the bottom-tilting wave maker.

## 324 6. Further discussion

325 So far, characteristics of the waves generated by the bottom-tilting wave maker, namely the  
 326 wave amplitude and the wave period, have been discussed in terms of the parameters ( $a$  and  
 327  $b$ ) of simple upward and downward bottom motions. In this section, we further investigate  
 328 the new wave maker. First of all, the waves generated in the new wave tank are compared  
 329 to the relevant solitary waves, demonstrating that the new wave maker can indeed generate  
 330 waves that are longer than the solitary waves. Then the bottom-tilting-generated wave is  
 331 compared to the field data of 2011 Japan Tohoku tsunami. Finally, the effects of the length  
 332 of the tilting bottom are also discussed.

333 Figure 11 illustrates the comparison of the ratio  $L_w/L_s$  of wave length between the  
 334 solitary waves and the bottom-tilting generated waves for  $\alpha = 0.05$ . The ratio  $L_w/L_s$  is  
 335 always greater than 1, demonstrating that the bottom-tilting wave maker can generate waves  
 336 longer than the solitary waves. Also notice that the  $L_w/L_s$  mostly grows with increasing  
 337 motion amplitude and with increasing motion duration, except some sudden drop due to  
 338 dispersion for downward motion.

339 The field data of the 2011 Japan Tohoku tsunami shown in figure 12 (a) by solid line was

340 obtained at a location with a water depth of 204 m. The amplitude of the leading wave was  
341 6.6 m and the wave period was 1500 s [Fujii *et al.*, 2011], resulting in the normalised wave  
342 amplitude ( $A_t$ ) to be  $A_t = 0.032$ . The upward bottom motion in the new wave maker with  
343  $a = 0.1$  and  $b = 1.40$  just satisfies  $A = 0.032$ , and the result (dash-dotted) is compared to the  
344 field data (solid) as shown in figure 12 (a). The corresponding wave period of the bottom-  
345 tilting-generated wave is still shorter than the field data, albeit much better than the solitary  
346 wave (dotted). However, closer inspection of the figure shows that the wave agrees well with  
347 the field data near the peak of the tsunami record. It is an encouraging result considering that  
348 we only used a simple upward motion and suggests that more sophisticated operation of the  
349 tilting bottom should be able to achieve better agreement with the field data. Theoretically,  
350 if the wave is allowed to propagate by applying the bottom motion displacement  $\zeta(x = 0, t)$   
351 (solid) as shown in figure 12 (b), the new wave maker is able to generate a wave (dashed)  
352 that is very similar to the field data as shown in figure 12 (a). Furthermore, the time histories  
353 of the motion displacement  $a$  and the relative motion speed  $Q$  determined by Eq. (4) are  
354 described in figure 12 (b) as well.

355 The wavelengths of the bottom-tilting-generated waves are mainly limited by the length  
356 of the tilting bottom and limited motion tested. Previously, in section 3, we showed that the  
357 length of the tilting bottom relative to the water depth plays an important role in determining  
358 the characteristics of the resulting waves. In figure 13, the dependence of wave amplitude  
359 and wave period on the tilting bottom length is plotted. It can be seen that increasing length  
360 of the moving bottom leads to growing wave amplitude but decreasing (dimensionless)  
361 period for both kinds of bottom motions.

## 362 7. Conclusions

363 In this paper, we have presented our new wave maker, namely the bottom-tilting wave  
364 maker, which can generate waves that are significantly longer than solitary waves. This was  
365 motivated by recent advancement in tsunami research [e.g. Madsen *et al.*, 2008], which  
366 points out that use of solitary waves to model tsunamis is not theoretically justified.

367 By changing the water depth, bottom motion displacement and speed, different waves  
368 have been created and investigated. The amplitudes and the periods of the generated waves  
369 were related to the parameters of the simple bottom motions. The wave amplitudes are well  
370 described in terms of the volume flux of the displaced water, but the wave periods show  
371 much more complicated trends due to combined effects of nonlinearity and dispersion.

372 With one moving bottom in simple monotonic motions, we were able to generate waves  
373 that are markedly similar to the field data of 2011 Japan Tohoku tsunami. We expect that  
374 more sophisticated operation would result in even better agreement. This wave maker ef-  
375 fectively changes the length of moving bottom and provides more degrees-of-freedom in  
376 operation.

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 382 20140437].

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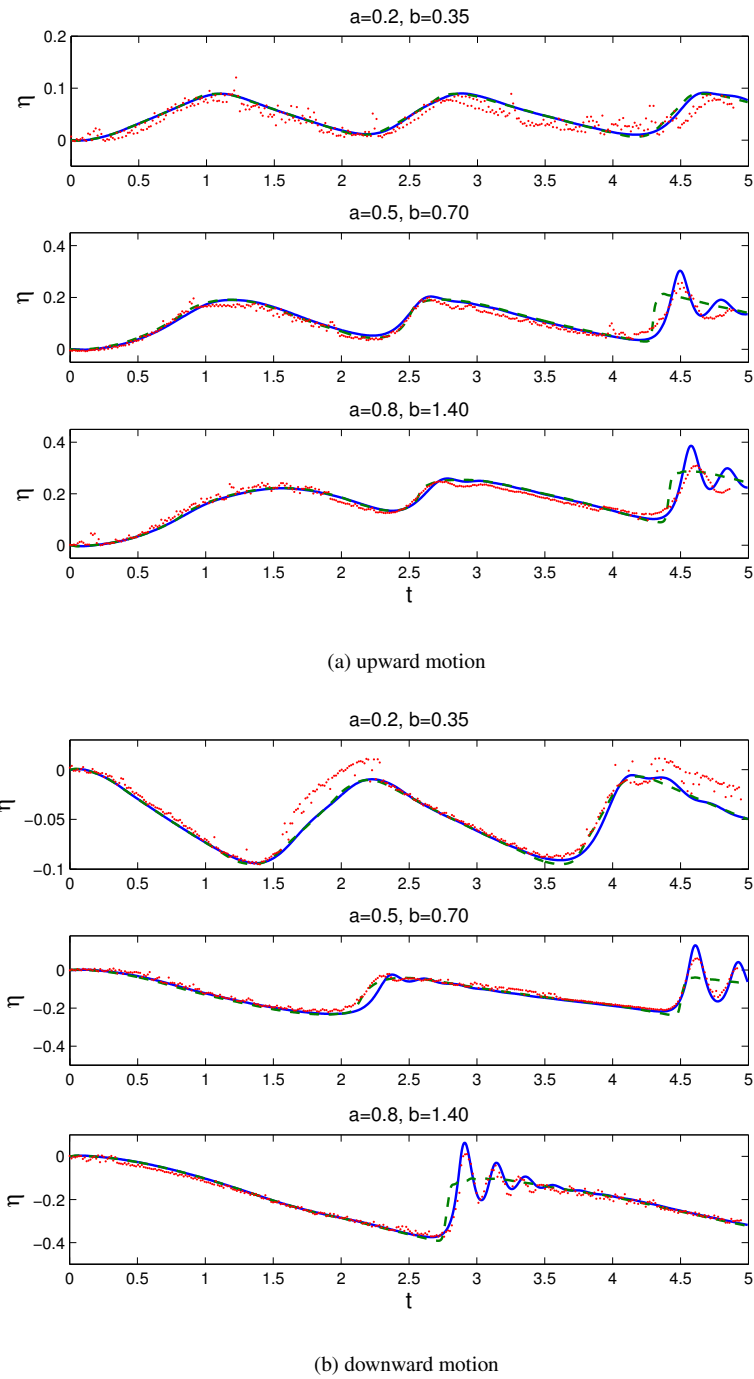
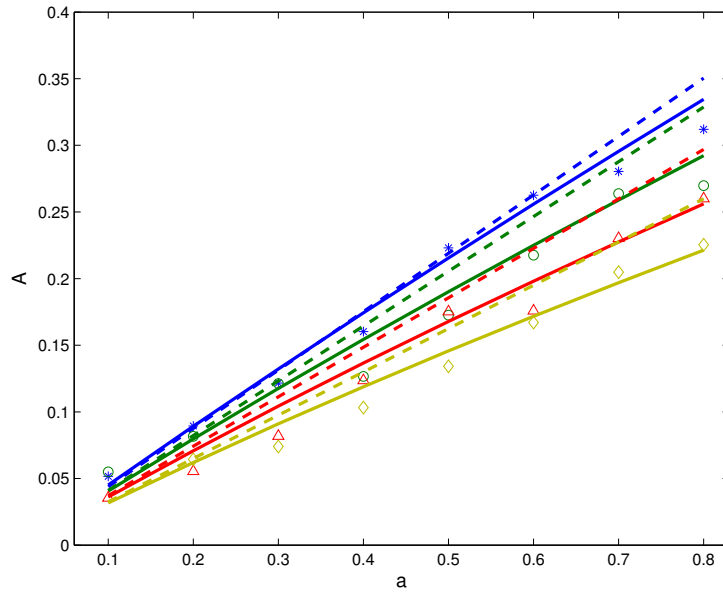
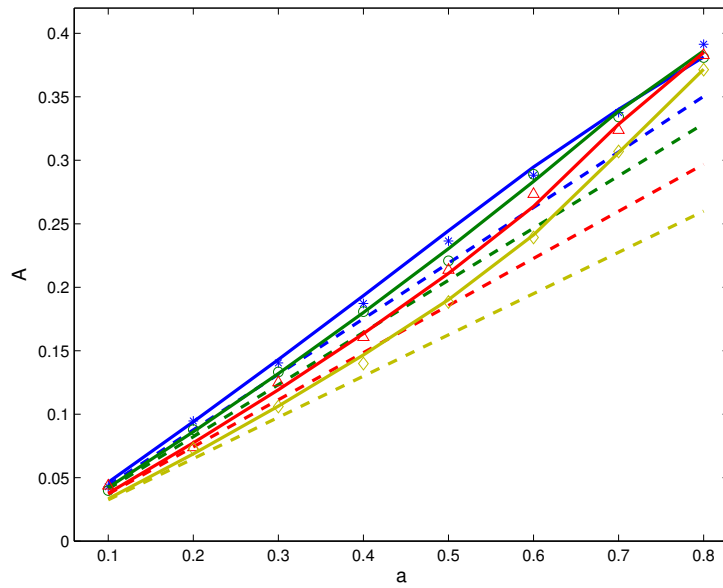


Fig. 5. Comparison of the free surface elevation at the hinge: solid line, numerical results by BE; dashed line, numerical results by NSWE; dotted line, experimental data.



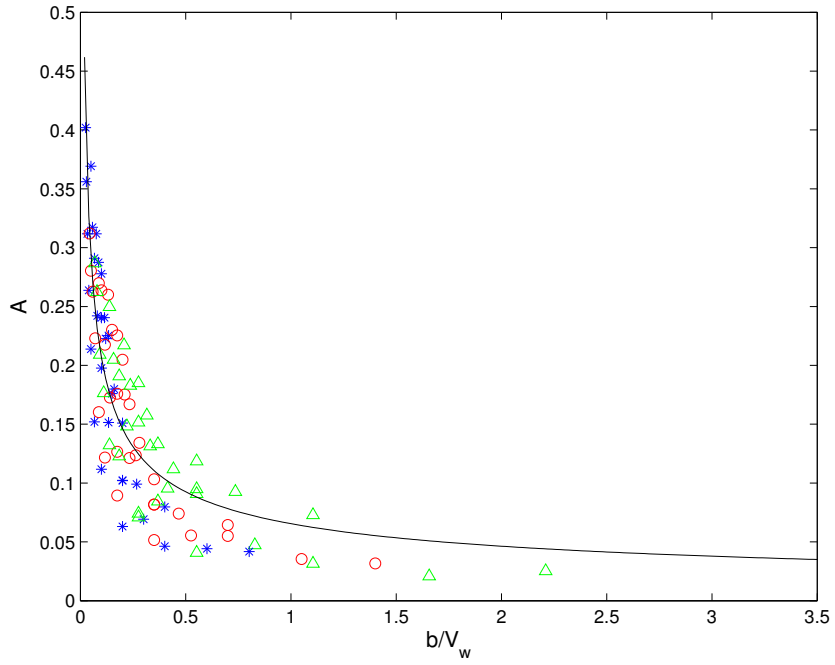
(a) Amplitude  $A$  for upward motions



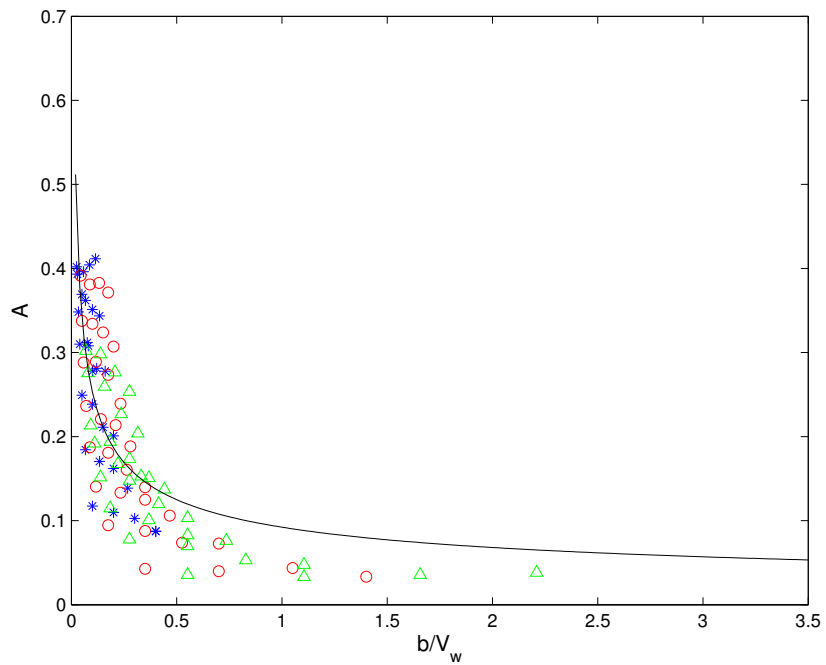
(b) Amplitude  $A$  for downward motions

Fig. 6. Effects of bottom motion on wave amplitude of the waves generated in the wave maker: \*,  $b = 0.35$ ;  $\circ$ ,  $b = 0.70$ ;  $\triangle$ ,  $b = 1.05$ ;  $\diamond$ ,  $b = 1.40$ ; solid line, numerical results by BE; dashed line, analytical results by linear wave theory (lines are distinguished following the symbols from top to bottom).

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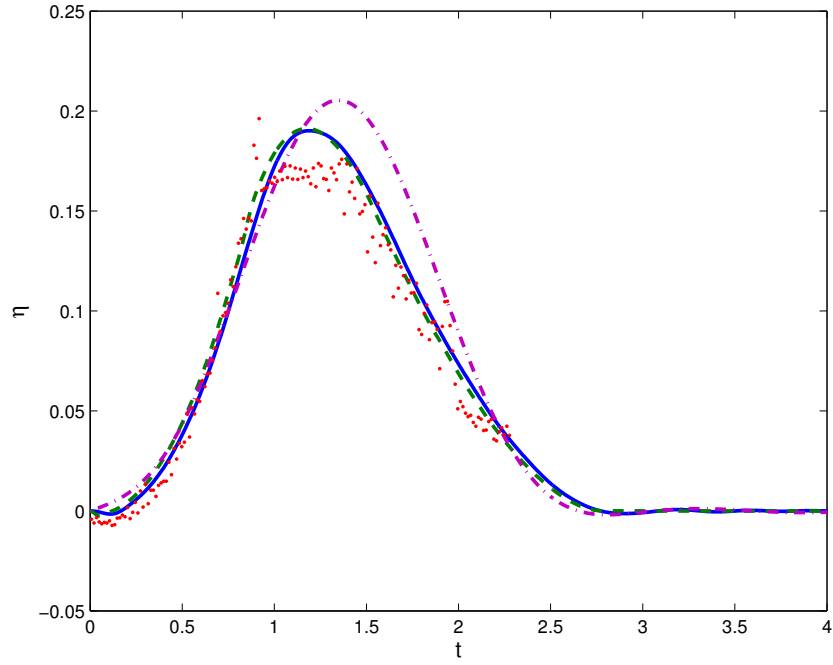


(a) upward motion

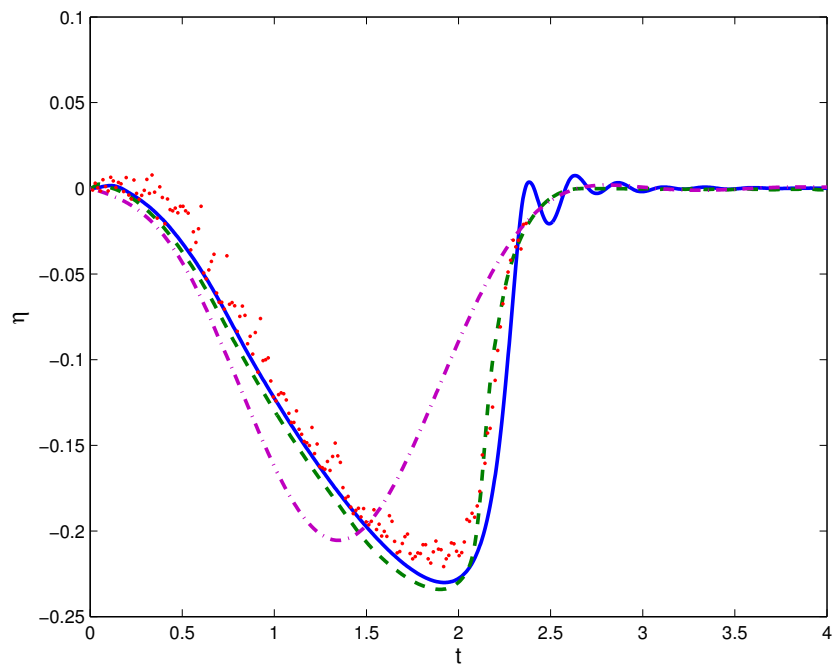


(b) downward motion

Fig. 7. Plots of wave amplitude  $A$  as functions of  $b/V_w$ : solid line, the fitting function; \*,  $\alpha = 0.04$ ;  $\circ$ ,  $\alpha = 0.05$ ;  $\triangle$ ,  $\alpha = 0.06$ .



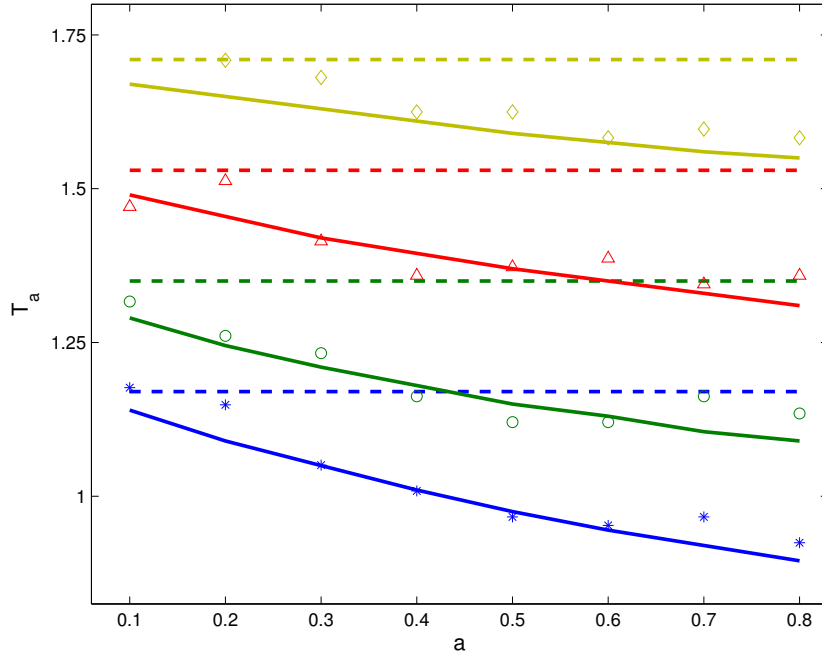
(a) upward motion



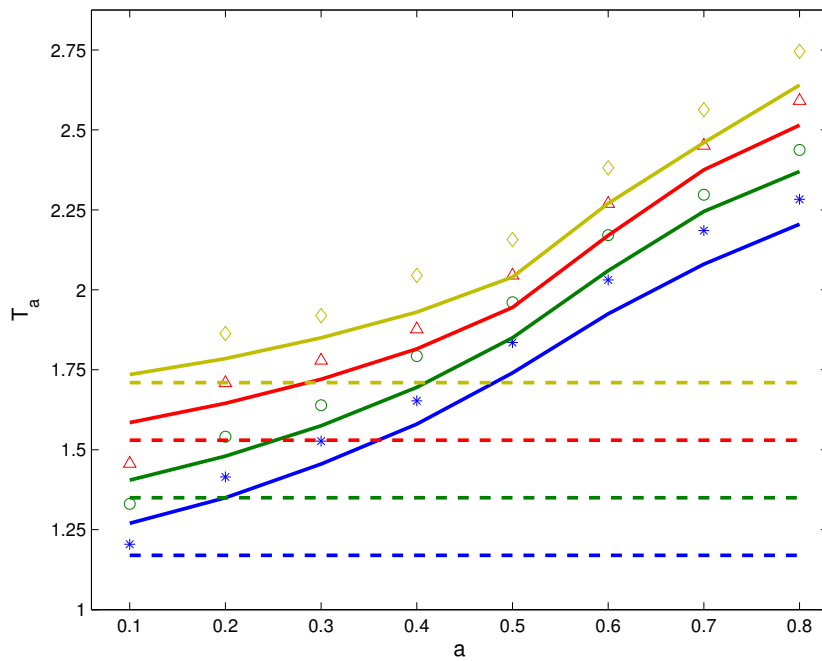
(b) downward motion

Fig. 8. Theoretical results for waves in semi-infinte domain compared to experimental data: solid line, results by BE; dashed line, results by NSW; dash dotted line, linear analytical results; dot, experimental data.

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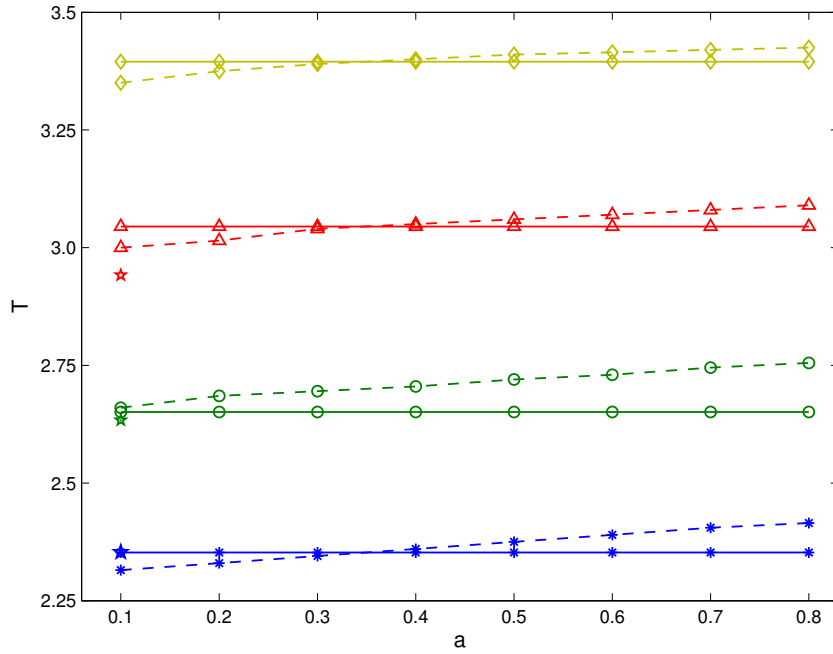


(b) Peak time  $T_a$  with varying bottom motions

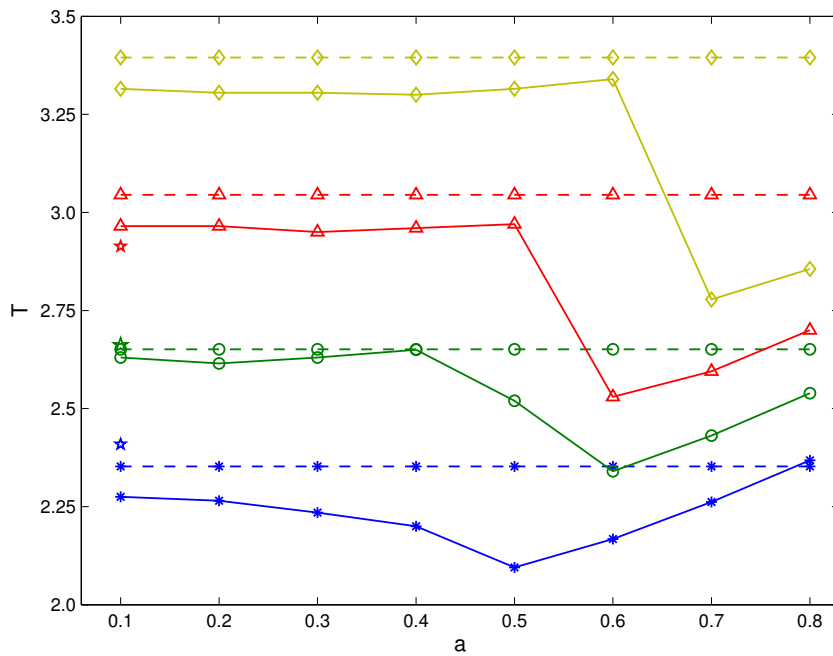


(b) Peak time  $T_a$  with varying bottom motions

Fig. 9. Effects of bottom motion on wave peak time  $T_a$  of the resulting waves: \*,  $b = 0.35$ ; o,  $b = 0.70$ ;  $\Delta$ ,  $b = 1.05$ ;  $\diamond$ ,  $b = 1.40$ ; solid line, numerical results by BE; dashed line, analytical results by linear wave theory (lines are distinguished following the symbols from top to bottom).



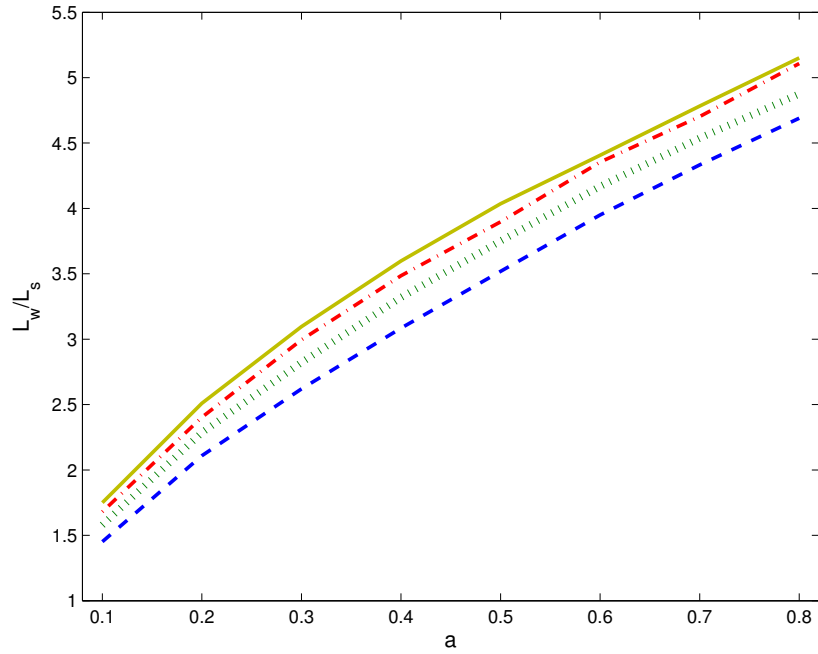
(a) Period  $T$  with varying  $a$  for upward motion



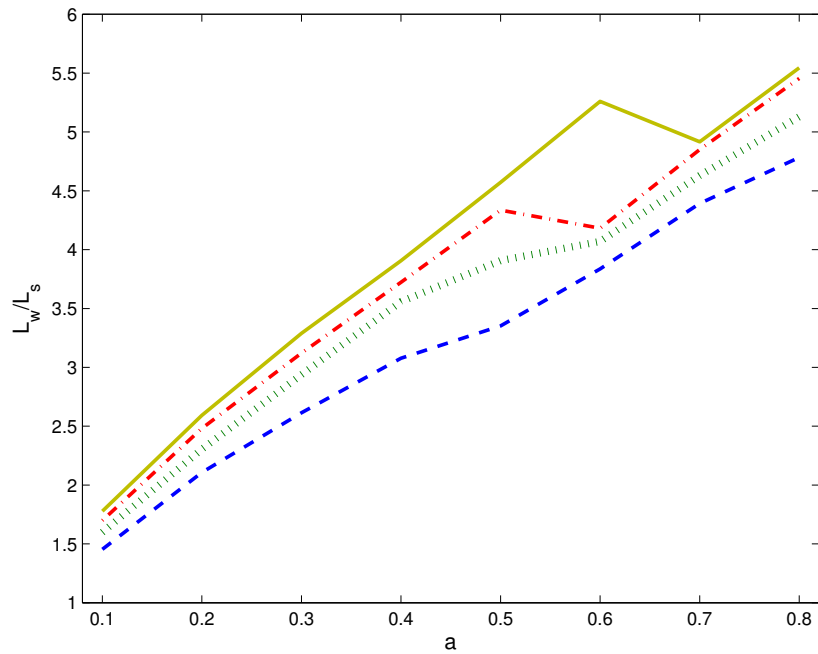
(b) Period  $T$  with varying  $a$  for downward motion

Fig. 10. Effects of bottom motion on wave period of the resulting waves: \*,  $b = 0.35$ ;  $\circ$ ,  $b = 0.70$ ;  $\triangle$ ,  $b = 1.05$ ;  $\diamond$ ,  $b = 1.40$ ; solid line, numerical results by BE; dashed line, analytical results by linear wave theory;  $\star$ , experimental data (adjacent to its corresponding  $b$ ).

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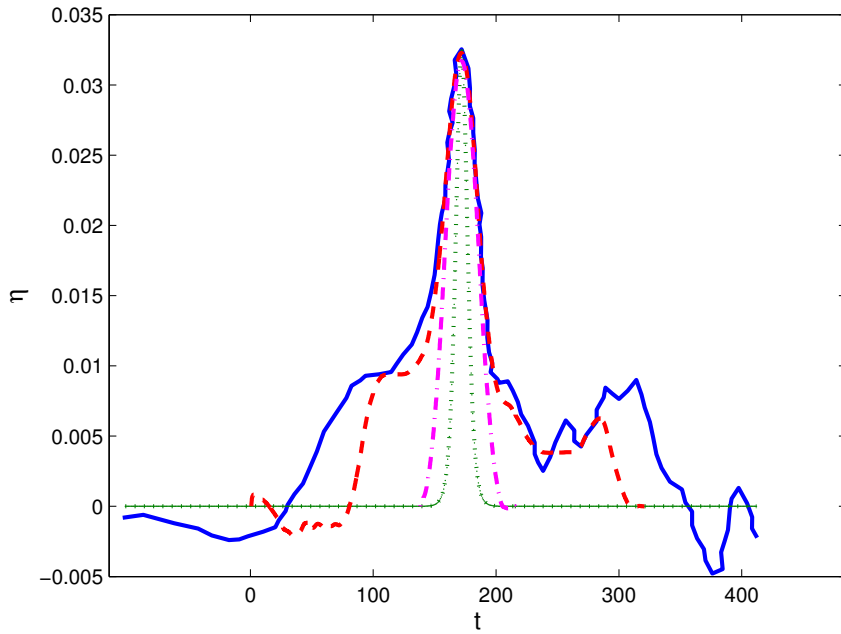


(a) upward motion

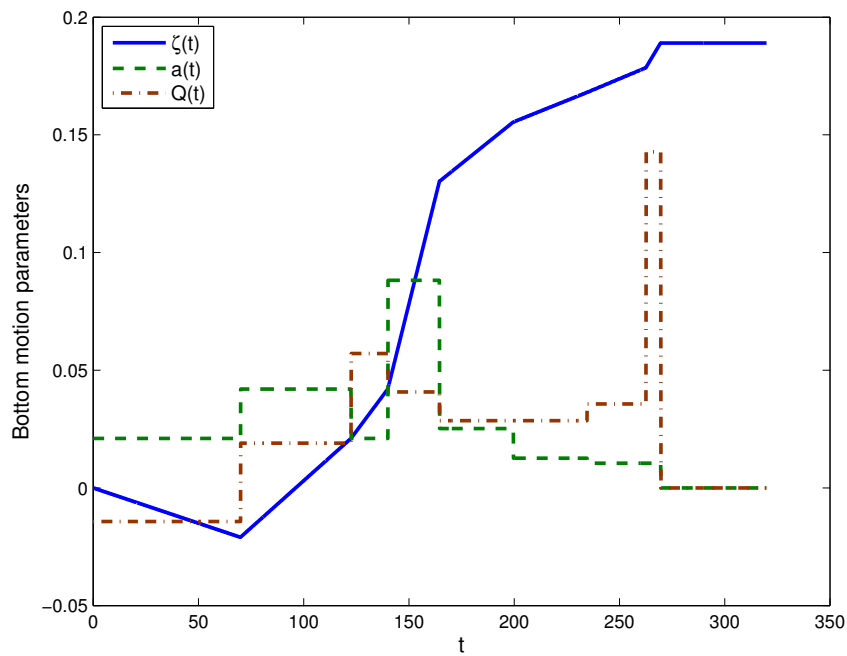


(b) downward motion

Fig. 11. Comparison of the ratio of  $L_w/L_s$  with varying  $a$  and  $b$  for upward motion and downward motion: dashed line,  $b = 0.35$ ; dotted line,  $b = 0.70$ ; dash dotted line,  $b = 1.05$ ; solid line,  $b = 1.40$ .



(a) Comparison between field data and fitted waves: solid line, the observed field data; dashed line, the wave generated by a sophisticated bottom motion; dash dotted line, the wave generated by a simple upward motion; dotted line, the fitted solitary wave .

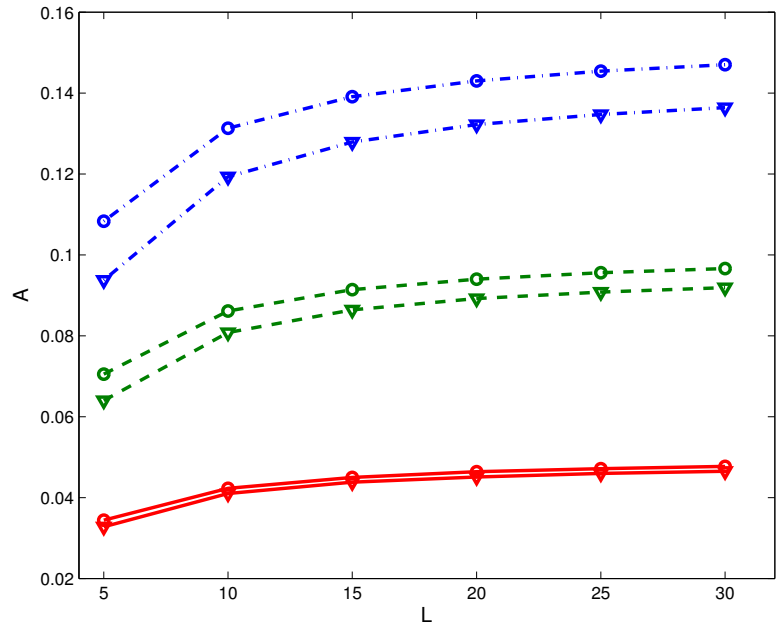


(b) Bottom motion parameters  $\zeta(x = 0, t)$  (solid line),  $Q(t)$  (dash dotted line) and  $a(t)$  (dashed line) of the fitted wave by a sophisticated bottom motion

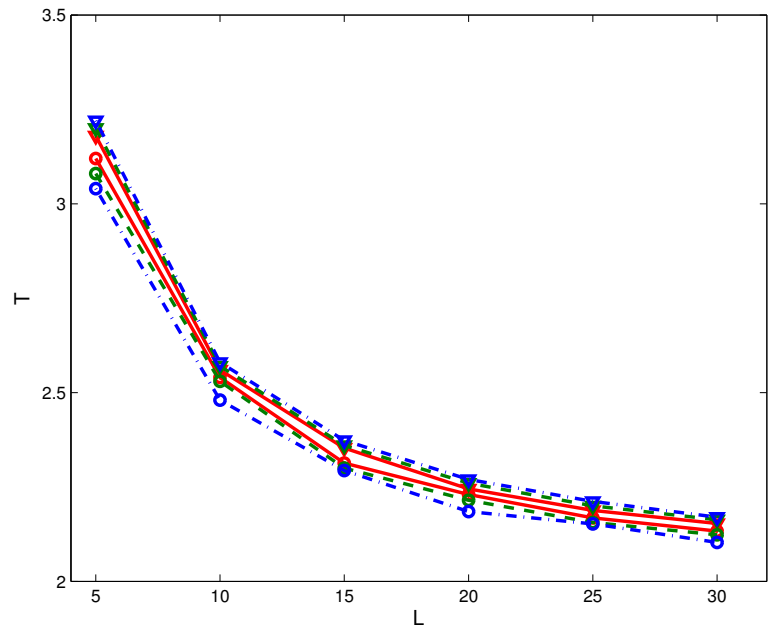
Fig. 12. Comparison between field data at Iwate South from Japan Tohoku tsunami in 2011 (Fujii *et al.* [2011]), fitted bottom-tilting-generated waves and a fitted solitary wave.



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(a) wave amplitude



(b) wave period

Fig. 13. Comparison of wave characteristics with varying moving bottom length:  $\nabla$ , upward motion;  $\circ$ , downward motion; solid line,  $a = 0.1$ ; dashed line,  $a = 0.2$ ; dash dotted line,  $a = 0.3$ .