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Determination of Urban Runoff Coefficient using Time Series Inverse Modeling

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ABSTRCT: Runoff coefficient is an important parameter for the decision support of urban stormwater management. However, factors like comprehensive land-use type, variable spatial elevation, dynamic rainfall and groundwater elevation, make the direct estimation of runoff coefficient difficult. This paper presented a novel method to estimate the urban runoff coefficient using the inverse method, where observed time-series catchment outfall flow volume was employed as input for the water balance model and runoff coefficients of different catchments were treated as unknown parameters. A developed constrained minimization objective function was combined to solve the model and minimized error between observed and modeled outfall flow is satisfactory for the presenting of a set of runoff coefficients. Estimated runoff coefficients for the urban catchments in Shanghai downtown area demonstrated that practice of low impact design could play an important role in reducing the urban runoff.

KEY WORDS: runoff coefficient, urban stormwater management, inverse modeling, low impact design.

Event runoff coefficient is defined as the portion of rainfall that becomes direct runoff during an event[1]. This coefficient is difficult to determine due to the rapid changes of urban landscape in developing countries. In addition, due to the complexity of land-use zoning and surface-groundwater exchange, traditional estimation of runoff coefficients is time-consuming and inaccurate. Here, we propose a novel solution to determine the catchment runoff coefficient using an inverse model [2-5]. Such inverse method is shown to be convenient to use without the uncertainty associated with the factors mentioned above. This study demonstrates that runoff coefficient of urban catchments can be estimated using a time series inverse model. In such a model, the recorded catchment outflow rate and observed precipitation were treated

as known variables in the system of water balance equation.

The present study focuses on 22 downtown catchments in Shanghai, China, which has an area of 41.95 km^2 and 22 pumping stations of combined sewer system. Note that Shanghai is geologically flat, so urban drainage is being driven by forced pumping, instead of gravity flow as in many other cities. The locations of precipitation monitoring sites and pumping stations of the combined sewer system are shown in Fig.1.

The pumping stations and the precipitation sites are not collocated, so we interpolated the precipitation data to the pumping stations to create a correlated precipitation-pumping rate dataset. Specifically, the precipitation data of the monitoring sites on 54 raining days in 2013 and 2014 as well as the pumping flow rate data were collected from the website of Shanghai Urban Drainage Company (URL?). The precipitation data were interpolated to the pumping station locations using the Inverse Distance Weighted (IDW) interpolation method developed in ArcGIS.
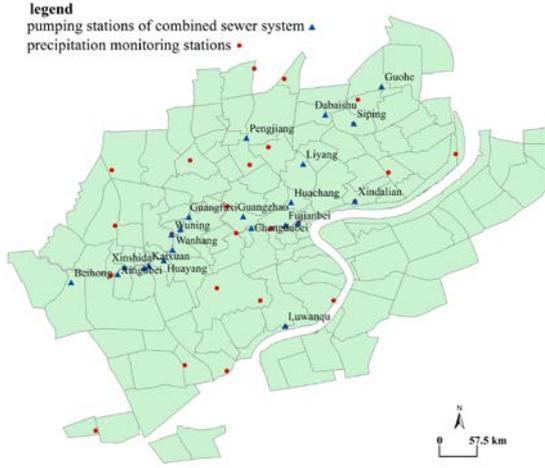
Fig. 1 The distribution of the precipitation monitoring stations and urbanized catchment outfalls in Shanghai.

We employed a water balance equation as follows:

$$\{ \text{EMBED Equation.DSMT4} \} \quad (1)$$

where $Q_{(p)i,j}$ is the j-th pumping flow rate of the i-th pumping stations, Q_{si} is the daily wastewater discharge flow rate per square kilometer of the i-th urban catchment, which was estimated based on the geographic information system of water pollution sources survey in Shanghai [6]. The average daily wastewater discharge flow rate per square kilometer of these catchments is 16751 m³/(km²·d). N_1 is recorded time of wet-weather period (=54 days), Q_{gi} is the daily groundwater seepage rate in each catchment (10.5% of Q_{si} [7]), $H_{i,j}$ is the j-th precipitation at the i-th pumping stations of combined sewer system, S_i is the catchment area corresponding to the i-th combined pumping station, x_i is the runoff coefficient of the i-th catchment.

To determine the unknown x_i, a minimization problem was solved with an objective function of

$$\{ \text{EMBED Equation.DSMT4} \} \quad (2)$$

Where, n is the number of catchments (i.e., number of combined sewer systems), $Q_{(p)}(X)_{i,j}$ is the predicted pumping flow rate data at the i-th catchment during the j-th precipitation event, $Q_{(data)i,j}$ is the recorded pumping flow rate data at the i-th catchment during the j-th precipitation event. The minimization problem of Eq. (2) is solved numerically by Fmincon, a nonlinear programming solver. Given the initial vector of the runoff coefficients $X_0(=x_{01}, x_{02}, x_{03},..., x_{22})$ in the feasible range, interior-point algorithm [8-9] was applied to obtain the solution, which is illustrated in the technical route of Fig.2.
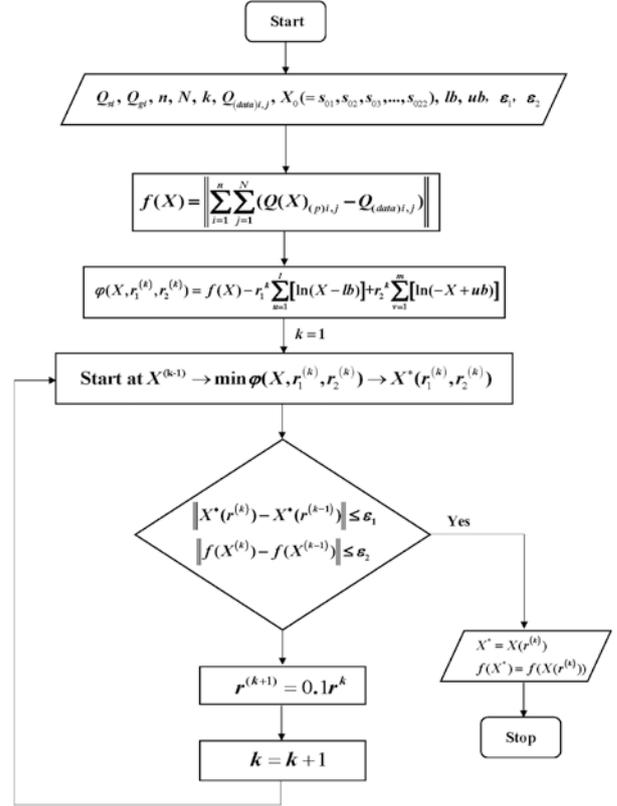

Fig. 2 Technical route to determine unknown parameter x_i using the time series inverse model.

In this technical route, lb and ub represent the lower and upper limits of x_i respectively that were determined using China's urban drainage design handbook [10]. $\phi(X, r^{(k)})$ is the logarithmic penalty function, which combined the original objective function and the original constraints including inequality constraint and equality constraint, so it was used to punish the iteration step that attempts to exceed the feasible range. $r_1^{(k)}$ and $r_2^{(k)}$ are the penalty factors ($r_1^{(k)}, r_2^{(k)}>0$), which are descending sequences. ε_1 ($=1.0\times10^{-10}$) is the tolerance of the lower bound using the L2-norm of (X_i-X_{i+1}). If the solver attempts to take a step that results smaller error than ε_1, the iteration ends. ε_2 ($=1.0\times10^{-6}$) is the tolerance of the first-order optimization. If the optimization error is less than ε_2, the iteration ends.

The process to find the runoff coefficient with the number of iteration steps is shown in Fig.3, where $f(x)$ gradually decreased to the order of 10^{-4} (i.e., 5.48×10^{-4} in Fig.3(a)) when ε_1 approached 10^{-10} (see Fig.3(b)). The trend shows that our algorithm has a quick convergence and the numerical configuration is appropriate.
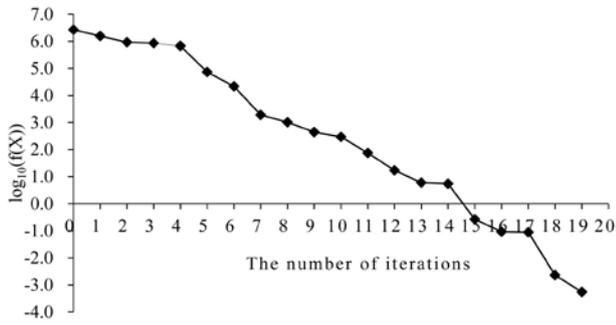
(a) log$_{10}$(f(x)) with the iteration number
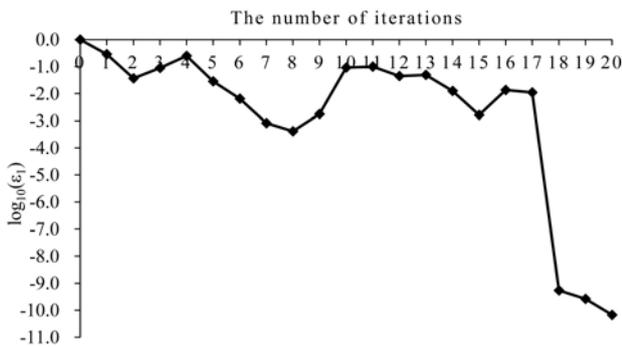
(b) log$_{10}$(ε_I) with the iteration number

Fig.3 The numerical iteration process to fine the runoff coefficient with f(x) and norm of step as the judgement criteria.

For each urban catchment, the estimated runoff coefficient using the time-series inverse model is shown in Fig.4. The maximum, minimum and average runoff coefficients are 0.55, 0.79 and 0.70, respectively. The urban runoff coefficients between the minimum and maximum values have a difference of 0.24. Focusing on a specific example, the Dabaishu catchment area is highly urbanized with a dense population, but the runoff coefficient is not necessarily high (see Fig. 5).
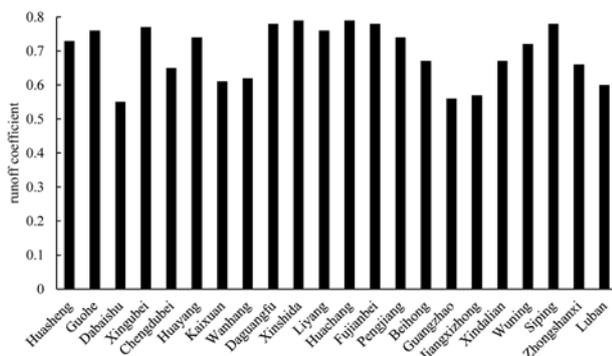

Fig.4 The estimated runoff coefficient of the urban catchments using inverse model.

Fig.5 Land use map of Dabaishi catchment area.

The estimated low runoff coefficient for Dabaishu catchment may be attributed to the implementation of low impact design (LID), including low elevation greenbelts, rain gardens and permeable pavements. Assuming that the downtown area of Shanghai is highly developed with similar city landscape, the major difference is the LID facilities. So our study shows that the difference of 0.24 between the high end value of 0.79 versus 0.55 at the lower end is the influence of LID development. LID is effective in this study.

The future work of this research will further our expedition to recognize the correlation between the runoff coefficient and the landscape, which aims at providing references to city planners.

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