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Breaks and the Statistical Process of Inflation: The Case of Estimating the 'Modern' Long-Run Phillips Curve*

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ABSTRACT

'Modern' theories of the Phillips curve inadvertently imply that inflation is an integrated or near integrated process but this implication is strongly rejected using United States data. Alternatively, if we assume that inflation is a stationary process around a shifting mean (due to changes in monetary policy) then any estimate of long-run relationships in the data will suffer from a 'small-sample' problem as there are too few stationary inflation 'regimes'. Using the extensive literature on identification of structural breaks we identify inflation regimes which are used in turn to estimate with panel data techniques the United States long-run Phillips curve.

Keywords: Phillips curve, inflation, structural breaks, non-stationary data
JEL Classification: C23, E31

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1. INTRODUCTION

There is a long literature on the identification and modelling of breaks in time series of data.[1] Early work focused on testing if the break occurred at a particular point in time where the break date is chosen from secondary information concerning economic priors about the data. This literature suffers from the number of breaks not being freely estimated and the date of the breaks are pre-determined. More recently the focus has turned towards identifying an unspecified number of break dates in a series as in Sen and Srivastava (1975), James *et al.* (1987), Banerjee *et al.* (1992), Andrews (1993, 2003), Bai (1994, 1997), Hawkins (2001), Sullivan (2002), Bai and Perron (1998, 2003), Starica and Granger (2005), and Fryzlewicz *et al.* (2006).[2]

These techniques for identifying the breaks in series suffer from the 'single-technique' problem. While there may be a 'true' underlying data generating process driving both the data and the associated breaks, the methodology used to identify these breaks in practice relies on the choice of a single technique and on a range of ancillary assumptions. These ancillary assumptions include the choice of the estimated structural break model, criteria to test for the number and position of the breaks, idiosyncratic parameter settings of the technique, and a range of further assumptions including those concerning the statistical process of the data. Each technique and ancillary assumptions will therefore produce a set of estimated breaks in the data. Given the unlimited range of possible techniques and assumptions this suggests that the maximum number of possible sets of estimated breaks in finite sets of data is large and limited only by the number of observations in the data. Faced with a large number of possible sets of breaks the standard approach is to focus on a 'single-technique' and ancillary assumptions in the hope that the estimated breaks mirror the 'true' breaks in the data.[3]

The 'single-technique' problem is important as the number of breaks identified in the data may vary widely depending on the approach taken. Even when different techniques identify the

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1 For example, see among many others, Page (1955, 1957), Quant (1958, 1960), Chow (1960), Gardner (1969) and MacNeill (1978). See also the historical survey of break analysis in Perron (2006).

2 Estimating breaks in time series data is closely related to the literature on testing whether data are stationary or persistent. This leads Perron (1989, 1990) and Rappoport and Reichlin (1989) to argue that highly persistent data may after allowing for breaks be stationary.

3 The robustness of the outcomes from a single technique may be subjected to a small and limited number of changes to the assumptions of the estimating technique.

same number of breaks in the data it is common for the break dates not to be the same. It is reasonable therefore for broad empirical findings based on any particular set of breaks from one break identification technique and set of assumptions to be met with some scepticism and the audience left wondering if different results would be forthcoming if a different technique or set of assumptions were employed in the analysis.

This scepticism is no more relevant than in work identifying long-run relationships in stationary data. Conceptually the long-run we refer to here is between the means of two or more variables in stationary data with shifting means.[4] For example, if the mean of variable A changes following a break, is it associated with a stable and significant change in the mean of variable B. So as to estimate a long-run relationship we must therefore identify breaks in the mean of the stationary series and this leads to the practical difficulties and scepticism discussed above.

An example of the 'single-technique' problem in relation to estimating long-run relationships in data is Russell (2011) who argues that United States quarterly inflation over the last 5 decades is a stationary process around a shifting mean and proceeds to identify 7 breaks in mean inflation. This simultaneously implies there are 8 inflation 'regimes' where statistically inflation appears to have had a constant mean. Based on the identified 8 inflation regimes, Russell estimates a significant negative sloping non-linear long-run Phillips curve between inflation and the markup where the mean values of these variables are assumed to be their long-run values in each inflation regime. Estimating the long-run curve in this way suffers not only from the 'single-technique' problem but also a 'small-sample' problem where the long-run estimates are based on a sample of only 8 observations. Consequently, although the results appear strongly significant, some may argue they may not be 'robust' to changes in either the technique used to estimate the breaks or changes in the ancillary assumptions and therefore the results may be unconvincing.

The developments and advances in the techniques to estimate breaks in series is very welcome. However, the methodology of trying to find the 'best' technique and ancillary assumptions in an attempt to identify the 'true' breaks in the data is misplaced and is the root cause of the

[4] The type of stationarity being discussed here is covariance stationarity once shifts in means have been accounted for. See Russell (2014) who finds that once the shifts in mean are accounted for in United States inflation the data are homoscedastic.

‘single-technique’ problem outlined above. Instead we propose a methodology that could *conceptually* be based on the set of all techniques and ancillary assumptions. If we assume that each individual technique and ancillary assumptions provide an unbiased estimate of the true breaks in the data then by averaging the estimates from all the techniques we can obtain a more efficient estimate of the breaks. In the process we not only overcome the ‘single-technique’ problem but we simultaneously overcome the ‘small-sample’ problem when estimating long-run relationships in data that are stationary around a shifting mean. We therefore propose the following four stage solution:

Stage 1: Determine if the data can be confidently described as a stationary process around a shifting mean.[5] Given the well-known low power of unit root tests this stage is sometimes contentious. However, in some cases, such as the inflation example we use below, the empirical evidence is overwhelming and conforms to a logical understanding of the data.

Stage 2: Estimate multiple breaks in the mean of the data using a range of techniques and parameter settings. This allows the identification of a number of sets of breaks where each set of breaks is based on a single-technique. In our example below we apply three techniques to identify multiple breaks in the mean rate of United States inflation and make use of three sets of ancillary assumptions.[6] This results in the identification of nine sets of breaks in the data.

Stage 3: Construct an unbalanced ‘long-run-panel’ of data. Assuming the mean value of the variable in a regime is its long-run value we construct an unbalanced ‘long-run-panel’ containing the mean, or long-run, values of the variables from each of the sets of breaks. In this way the mean values obtained from the first set of breaks become the first cross section of data, the second set provides the second cross section and this is repeated for all the sets of

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[5] If the data are integrated then the analysis proceeds within a cointegration framework. Within this framework Banerjee *et al.* (2001) and Banerjee and Russell (2001, 2005) argue that while the ‘true’ statistical process for inflation is most likely stationary around a frequently shifting mean they proceed under the maintained assumption that this process can be approximated by an integrated process.

[6] We thank a referee for drawing our attention to the break-point estimation techniques of Frick *et al.* (2014), Fryzlewicz (2014) and James and Matteson (2015) from the statistics literature. These techniques generate similar break dates to those reported in this paper below. Further information and results from these techniques can be found at www.billrussell.info.

breaks identified in stage two. Again in our example below we identify 9 sets of breaks and therefore our panel of long-run values contains 9 unbalanced cross-sections of data.

Stage 4: Estimate the long-run relationship in the 'long-run-panel' using a fixed effects panel estimator. Assuming the techniques used to estimate the breaks in stage two are unbiased but not necessarily efficient then variation between the estimated long-run relationships from each of the single cross sections of data is simply due to random error. The fixed effects panel estimator is then a valid method for estimating the long-run relationship using all of the data in the panel. This solution simultaneously overcomes the 'single-technique' and the 'small-sample' problems as the number of observations in the unbalanced cross-section panel is only limited by the number of techniques and parameter settings employed when estimating the breaks in the mean value of the variable.

These four stages are explained and demonstrated below with reference to estimating a long-run United States Phillips curve. However, before turning to the inflation example, we first set out the 'modern' theories of the Phillips curve so as to provide context to the example.[7]

2. 'MODERN' THEORIES OF THE PHILLIPS CURVE

Consider the following single equation representation of the hybrid Phillips curve:

$$\pi_t = \delta_f E_t\left(\pi_{t+1}\right) + \delta_b \pi_{t-1} + \delta_x x_t + \varepsilon_t \tag{1}$$

where inflation, π_t, depends on expected inflation, $E_t\left(\pi_{t+1}\right)$, conditioned on information available at time t, lagged inflation, π_{t-1}, a 'forcing' variable, x_t, and an error term, ε_t, due to the random errors of agents and the shocks to inflation.[8] 'Modern' theories of the Phillips curve can be thought of in terms of restrictions to equation (1). Friedman (1968) and Phelps (1967)

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7 An alternative approach to the one employed here is the unobserved components model of Stock and Watson (2006) that assumes the change in the mean rate of inflation evolves smoothly rather than abruptly across inflation regimes. However, the focus of this paper is to estimate the long-run relationship between the means of variables across inflation regimes rather than the transition between the means in successive inflation regimes.

8 For a recent survey on the wide range of theoretical and empirical approaches to modelling inflation see Mavroeidis *et. al.* (2014).

expectations augmented Phillips curve assumes agents hold backward-looking adaptive expectations and $\delta_f = 0$ and $\delta_b = 1$. In the New Keynesian (NK) Phillips Curve of Clarida, Galí and Gertler (1999) and Svensson (2000) agents hold forward-looking rational expectations and $\delta_f = 1-d$ and $\delta_b = 0$ where d is the rate of time discount. In the hybrid models of Galí and Gertler (1999) and Galí *et al.* (2001) agents look both forward and backward and $\delta_f + \delta_b = 1-d$. Assuming risk neutral agents, a symmetric loss function around the profit maximising price and an annual real interest rate of around 4 per cent, then d is approximately 0.04 and 0.01 on an annual and quarterly basis respectively. The standard interpretation, and a central 'tenet', of all three 'modern' theories of the Phillips curve is that the long-run Phillips curve is 'vertical' if $d = 0$ and $\delta_f + \delta_b = 1$ implying there is no relationship between inflation and the forcing variable in the long-run. Finally, equation (1) also nests the 'post-modern' Russell and Chowdhury (2013) statistical process consistent (SPC) Phillips curve where $\delta_f = 0$ and $0 \leq \delta_b < 1$.[9]

2.1 *What the 'Modern' Phillips Curve implies for the statistical process of inflation*[10]

On a theoretical level, equation (1) appears to be consistent with any statistical process for inflation depending on the assumed distribution of the shocks to inflation. However, if we assume the theory of the Phillips curve is a valid description of inflation data then specifying the magnitude of δ_f and δ_b simultaneously determines the statistical process of inflation. For example, consider the following proof by contradiction. Estimate equation (1) with ordinary least squares assuming the 'forcing variable', x_t, is a stationary process. Now assume there are two mutually exclusive and exhaustive states of the world where inflation, π_t, is either an integrated I(1) or a stationary I(0) process. Consider the first case where inflation is an integrated process. The forcing variable x_t is I(0) and therefore it will not enter the asymptotics of the estimation and can be ignored. In this case, standard cointegration theory implies that $\delta_f + \delta_b = 1$. If this is not the case then $\Delta\pi_t$ would be I(1) implying that π_t is I(2) which

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9 The SPC Phillips curve is 'post-modern' in the sense that it eschews the empirically invalid assumptions of full information and no missing markets as well as the logical implications of models incorporating identical representative agents. Instead, the knowledge set of agents contains elements that they can be reasonably expected to know and agents behave in ways consistent with the knowledge that agents are not identical.

10 This argument is considered in more detail in Russell (2015).

contradicts the initial assumption that π_t is I(1). Alternatively, in the second state when inflation is I(0) then $|\delta_f + \delta_b| < 1$. Again if this is not the case and $\delta_f + \delta_b = 1$ then this would imply that inflation is I(1) which contradicts our initial assumption.

The logic of estimating equation (1) suggests that (i) if inflation is I(1) then $\delta_f + \delta_b = 1$ and (ii) if inflation is I(0) then $|\delta_f + \delta_b| < 1$. The converse is equally true. If $\delta_f + \delta_b = 1$ in the theory then inflation needs to be I(1) so that the theory is an empirically valid description of the data. And if $|\delta_f + \delta_b| < 1$ in the theory then inflation needs to be I(0) so that the theory is empirically valid. By implication if $\delta_f + \delta_b$ is very close but not equal to 1 in the theory then inflation needs to be a near integrated process.

2.2 Where does the unit root come from in 'modern' theories of the Phillips curve?

The expectations augmented Phillips curve of Friedman (1968) and Phelps (1967) builds on Cagan (1956) who is the first to combine adaptive expectations with the Koyck (1954) transformation in a theory of inflation.[11] Adaptive expectations can be written:

$$E_{t-1}\{\pi_t\} = E_{t-2}\{\pi_{t-1}\} + \eta\left(\pi_{t-1} - E_{t-2}\{\pi_{t-1}\}\right) \qquad (2)$$

where a fixed proportion, η, of the errors in the expectation of inflation are corrected in each period. Cagan refers to η as the 'coefficient of expectations' which represents how quickly expected rates of inflation converge on actual rates of inflation. Backward induction of adaptive expectations implies that expected inflation is a geometrically declining distributed lag of all past rates of inflation such that:

$$E_{t-1}\{\pi_t\} = \sum_{i=1}^{\infty} \eta (1-\eta)^i \ \pi_{t-i} \qquad (3)$$

[11] See also Nerlove (1956). Griliches (1967) provides an early survey of the econometrics of distributed lagged models.

where $\sum_{i=0}^{\infty} \eta (1-\eta)^i = 1$. On a practical level equation (3) cannot be estimated with an infinite number of lags and so Cagan uses the Koyck (1954) transformation to truncate the number of lags and re-write (3) as:[12]

$$\pi_t = (1-\eta)\,\pi_{t-1} + \eta\,\pi_{t-1} = \pi_{t-1} \tag{4}$$

Equation (4) implies that inflation contains a unit root but this implication is simply the inadvertent result of using adaptive expectations and the Koyck transformation in the Friedman (1968) and Phelps (1967) expectations augmented Phillips curve. Following Friedman and Phelps' work in the mid-1960s, empirical work on inflation began to include the period of rising inflation in the late 1960s and early 1970s and this data incorporated a number of shifts in the mean rate of inflation. These shifts in mean inflation were not accounted for in the estimation of Phillips curves leading the estimate of the lagged dependent inflation variable to be biased towards 1. This bias also leads unit root tests to erroneously accept there is a unit root in the inflation data.[13] Given the 'universal' nature of the breaks in mean appearing across time periods, countries, and measures of inflation a similarly universal 'stylised fact' that inflation contains a unit root became entrenched and the vertical long-run Phillips curve unassailable in a policy sense. More importantly, all 'modern' theories of the Phillips curve following Friedman (1968) and Phelps (1967) had to explain this erroneous 'stylised fact' if they were to be a 'credible' theory of inflation.

3. THE STATISTICAL PROCESS OF INFLATION – STAGE 1

Underpinning all 'modern' Phillips curve theories of inflation is the notion that the long-run rate of inflation is ultimately determined by the setting of monetary policy and that this setting may change discretely in response to shocks, changes in institutional structure or changes in the personal characteristics of those setting policy. Changes in monetary policy therefore will lead

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12 Other possible transformations leading to equation (4) include that of Almon (1965) and the rational distributed lag function of Lucas and Rapping (1969).

13 See Peron (1989).

to changes in the long-run rate of inflation and therefore breaks in the mean rate of inflation.[14] Therefore, assuming monetary policy may have changed at least once over the past 5 ½ decades, any analysis of the statistical process of inflation should allow for the possibility of breaks in the inflation data.

Quarterly United States inflation for the period March 1960 to June 2015 is shown in Graph 1.[15] The graph shows that (i) inflation appears to be bounded below at around zero and above at some moderate rate in a way similar to most developed economies over the same period, (ii) the 1970s was a turbulent period for inflation, and (iii) visually there appears to be a number of breaks in the mean rate of inflation.

As explained in Perron (1989), when breaks are not accounted for, unit root tests are prone not to reject the null hypothesis of a unit root in the data. Similarly, tests may spuriously reject the null of stationarity when breaks are present. We examine two cases below to illustrate the impact of not accounting for structural breaks in the model. In the first case, we conduct tests of I(1) and I(0) without accounting for breaks in the data. In the second case, we report unit root tests when breaks are identified endogenously using the methods of Lumsdaine and Papell (1997) and Lee and Strazicich (2003). Finally we report the Enders and Lee (2012a, b) flexible Fourier ADF test to account for sharp and smooth breaks in the data.

We illustrate the first case in Table 1. In Panel A we see that when we do not allow for breaks in the data there is not enough evidence at the 5 per cent level to reject the null hypothesis of a unit root in the inflation data. Stronger evidence that inflation is an integrated process is shown in Panel B of Table 1 where the null of stationarity is strongly rejected at the 1 per cent level. As expected, the series appear stationary when considered in first differences suggesting inflation is an I(1) process. If considered without due consideration of breaks in the data then

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[14] Changes in monetary policy is used in the sense that the monetary policy instrument has either changed, or not changed, in such a way so that the mean rate of inflation does not remain constant.

[15] Inflation is measured as the quarterly change in the natural logarithm of the seasonally adjusted United States gross domestic product implicit price deflator at factor cost. See the data appendix for further details concerning the data.

we might conclude there is compelling evidence that the data contains a unit root and is therefore integrated.

However, Table 2 illustrates the case where breaks are allowed to be determined endogenously and the number of breaks are determined prior to the test. Reported are the Lumsdaine and Papell (1997) and Lee and Strazicich (2003) tests. In the case of the Lumsdaine and Papell test, breaks are considered in the trend and the intercept term. However the critical values are computed based on no breaks under the null hypothesis that the data is I(1). Lee and Strazicich (2003) consider the critical values when there are breaks under the I(1) null hypothesis. Importantly, we find that if only one break is allowed for in the model we can reject the null of a unit root in the data.

Finally, the flexible Fourier approach of Enders and Lee (2012a, b) reported in Table 3 allows for an unknown number and dates of endogenous breaks while testing for unit roots in the data. This technique also allows for non-linear deterministic trends in the data and for breaks to evolve at different rates and referred to in the literature as 'sharp' (fast) and 'smooth' (slow) breaks.[16] The Enders-Lee test assumes that the cosine and sine terms are jointly different from zero under the null, which implies under the assumption that the data is stationary that there are nonlinearities in the deterministic term which logically are associated with breaks in the mean. The computed t-stat (-5.533) is significant at the 1 per cent level and we therefore reject the null hypothesis that the series contains a unit root with breaks.

The argument that there has been no change in monetary policy over the past 55 years in the United States is very hard to sustain given the difficulties and uncertainties in setting monetary policy in a world with incomplete information. At the very least, the widespread agreement in the existence of the 'Volker deflationary period' implies that there is a similar widespread agreement that the mean rate of inflation shifted at least once over this period in the early 1980s.[17] Therefore once we allow for a single break in the mean rate of United States inflation

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16 The Flexible Fourier approach is thought to deal more effectively with the spurious rejection problems surrounding the approaches of Lumsdaine and Papell (1997) and Lee and Strazicich (2003).

17 If the Volker deflation is a downward shift in the mean rate of inflation and the mean rates of inflation in the early 1960s and after 1990 are similar then there must have been a similar upward shift in the mean either prior to or after the 'Volker' break in the early 1980s. Consequently, the 'Volker' break cannot exist on its own and there must be a minimum of two breaks in the data.

we can confidently reject the hypothesis that inflation is either an integrated series with breaks or a near-integrated series with breaks and conclude that inflation is a stationary process around a shifting mean.

As we can confidently reject inflation is an integrated or near-integrated process with breaks we can therefore also reject with similar confidence that 'modern' theories of the Phillips curve are empirically valid description of the inflation data. In contrast, the 'post-modern' SPC Phillips curve is consistent with the statistical process of the data. Note that this is not a 'proof' of the SPC Phillips curve as other theories of inflation may also be consistent with inflation as a stationary process around a shifting mean.

Finding the 'modern' theories of the Phillips curve are inconsistent with the data does not invalidate the 'central tenet' of these theories that the long-run curve is vertical. It may be that once the analysis is undertaken within a framework allowing the data to be stationary around a shifting mean then the long-run Phillips curve may still be 'vertical' or possibly 'non-vertical' with a significant slope.[18] To examine the slope of the long-run Phillips curve requires the estimation of the relationship between the long-run values of inflation and the 'real' variable across inflation regimes. And it is this estimation of the long-run Phillips curve that we now turn to.

4. IDENTIFYING THE SHIFTS IN MEAN INFLATION – STAGE 2

Following the above, we proceed under the maintained assumption that inflation is a stationary process around a shifting mean. We employ in turn three techniques to estimate the breaks in mean inflation.

[18] If the long-run Phillips curve is not vertical and has a slope then the curve must be non-linear. If this is not the case then as inflation increases to an infinite rate the forcing variable will exceed its conceptual boundaries.

4.1 *Bai-Perron Technique*

The Bai and Perron (1998, 2003) algorithm identifies the dates of k breaks in the inflation series so as to minimise the sum of the squared residuals and thereby identify $k+1$ 'inflation regimes'. The estimated 'pure' shifting means model we estimate is:

$$\pi_t = \gamma_{k+1} + \varepsilon_t \tag{5}$$

where π_t is inflation and γ_{k+1} is a series of $k+1$ constants that estimate the mean rate of inflation in each of $k+1$ inflation regimes and ε_t is a random error.

Two issues need to be considered. First, to apply the Bai-Perron technique we need to impose by assumption the 'trim rate' which is the minimum distance between the breaks in mean. This is not simply an empirical issue but carries over into what the breaks represent in the data which in our case are changes in monetary policy. Bai and Perron (1998) recommend the minimum trim rate should be 5 per cent of the data but in Bai and Perron (2003) they argue that 15 per cent is more appropriate if the data is highly persistent. This implies that conceptually, changes in monetary policy are not fixed points in time but depend on the length of the data series. For example, 5 and 15 per cent trim rates correspond to a minimum distance between breaks of 12 and 33 quarters respectively for our data. As the data series lengthens then the minimum distance between breaks also increases and this is equivalent to the minimum distance between changes in monetary policy. Furthermore, when the minimum distance is 33 quarters this implies agents take a minimum of 8 years to identify a shift in mean inflation which is hard to justify if agents are in any way interested in the mean rate of inflation. The empirical arguments of Bai and Perron (1998, 2003) therefore conflict with the economic arguments of what the breaks represent conceptually.

We therefore consider three minimum distances between breaks, namely, 8 (trim of 3.6 per cent), 12 (5 per cent) and 33 quarters (15 per cent). The 8 quarters assumption is due to the data having very low persistence once a 'believable' number of breaks are allowed for in the data and 2 years is the forecasting horizon of most central banks.

The second issue is the choice of information criteria. The usual approach is to employ the sequential F test of the number of breaks. This approach outperforms the use of BIC and the modified BIC (LWZ test) only when the 'true' number of breaks is very small (i.e. 1, 2 or 3 breaks). When a larger number of breaks are in the data generating process then BIC performs better than the other two criteria.[19] In any case this issue is not particularly relevant as the same conclusions in an economic sense follow if we use any of these three criteria. Given that the BIC outperforms the other criteria in our supplementary Monte Carlo study of this issue we employ the BIC to determine the number of breaks in mean inflation.

4.2 OxMetrics Impulse Saturation Technique

The Impulse Saturation Technique identifies breaks in the data by using the impulse saturation approach of Hendry *et al.* (2008) and does not include a minimum distance between breaks. This technique makes use of the 'general-to-specific' approach to modelling data so as to identify the significance of the break in the impulse saturation dummies. The breaks model begins by dividing the total sample into 2 parts, and adding saturation dummy variables to the first half of the data. The saturation model is given as:

$$\pi_t = \mu + \sum_{j=1}^{T/2} \delta_j d_{j,t} + \varepsilon_t \tag{6}$$

where $d_{j,t}$ are the saturation dummies. The best fit to the above regression is determined such that the mis-specification tests are insignificant at the desired level where $\left|t_{1,\hat{\delta}}\right| < c_\alpha$ and α is the level of significance. This process is repeated until all significant saturation dummies are found in the estimated model. For our work we report three sets of breaks based on three significance levels of 1 per cent (small target), 0.1 per cent (tiny), and 0.01 per cent (minute).

4.3 Multiple Change Point Analysis

The Bai-Perron and Impulse Saturation techniques for identifying multiple breaks in time series were developed within the econometrics literature. However, within the mathematics and

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[19] Personal correspondence between Bill Russell and Pierre Perron between March and June 2015 considers the issue of minimum trim size and the number of breaks when applying the Bai-Perron technique to inflation data. In this correspondence Pierre Perron indicates that in his simulation work (based on 1 or 2 breaks in the data) he finds that BIC outperforms the other information criteria if persistence is low. He also acknowledges the importance of a practical approach to modelling breaks based on an understanding of the data when the number of breaks in the data may be large.

statistics literature there is large body of work on the detection of segmentation, breakpoints and change-points in the characteristics of time series data.[20] This literature plays an important role in the detection of breaks in important non-economic time series in a range disciplines.[21]

Within the statistics literature the binary segmentation method of Scott and Knott (1974) is one of the most used methods to estimate change points (or breaks) in time series of data. A range of algorithms, both exact and approximate, have been proposed to identify the multiple breaks in time series with the computational cost increasing as some function of the number of observations and the number of breaks in the series. For quarterly and monthly macroeconomic variables the computational cost is insignificant with modern computers but becomes significant if the data is over many years and recorded at high frequency such as every second, minute, day or week. Consequently accuracy and not computational cost is more relevant when identifying breaks in low frequency macroeconomic data of the type we use in our inflation example.

Recently Killick *et al.* (2012) proposes the Pruned Exact Linear Time (PELT) technique for estimating breaks that is based on, and extends, the algorithm of Jackson *et al.* (2005).[22] They show that PELT has an improved computational cost without losing its exactness under certain condition. More importantly they demonstrate that PELT is more accurate than a number of alternative popular algorithms.

Killick *et al.* (2012) computes breaks (or change-points) using a linear cost function. PELT minimises the following log likelihood linear cost function over a range of possible breaks:

$$L(\tau) = \sum_{i=1}^{m+1}\left[(\tau_i - \tau_{i-1})\left(\log(2\pi) + \log\left(\sum_{j=\tau_i+1}^{\tau_i}\left(\pi_j - \hat{\mu}\right)^2\right) + 1\right)\right] \quad (7)$$

where the number of breaks is given by m, τ_i represents the position of the break, and π_j is inflation measured in period j. This approach identifies breaks in terms of changes in the variance and means of the data where the relevant null hypothesis is that the mean and variances

[20] The statistics literature refers to change-points and segmentation in the same manner as the econometrics literature refers to breaks in time series data.

[21] For example see Braun and Muller (1998), Algama and Keith (2014) and Priyadarshana and Sofronov (2015).

[22] See also Killick *et al.* (2010).

are the same across different segments of the data that are identified recursively. We apply PELT to the 'pure' mean shift model of equation (5).

PELT provides different sets of breaks depending on the information criteria used to identify the preferred model of breaks and the minimum distance between breaks leading to different estimates of the number and dates of the breaks. We employ five information criteria, namely, BIC, modified BIC, Hannan-Quinn, SIC and the loss function above assuming no weighting to additional breaks and a minimum of 8 quarters between breaks. In our case three information criteria (BIC, Hannan-Quinn and SIC) deliver identical estimates and we therefore produce three and not five sets of breaks using PELT.

4.4 Estimates of the Breaks in Mean Inflation

Table 4 reports the estimates of breaks in mean inflation. We observe that while the different techniques identify a wide range in the number of breaks in the data, there are some similarities in the time pattern in the breaks. In particular, breaks in common appear around the time periods 1963/65, 1972/73, 1981/82, 1991, and 1999/01. The Bai-Perron estimated breaks in row 2 imply there are 9 inflation regimes and the mean rates of inflation in these 9 regimes are shown in Graph 1 as thin horizontal lines. Again from a purely visual perspective in the graph, the Bai-Perron estimated breaks in Row 2 appears to have identified all the important shifts in mean United States inflation over this period and might be thought of as our 'preferred' estimate of the breaks. However, note that a 'preferred' set of breaks is not necessary in the 4 stage analysis of inflation and is used here simply for illustrative purposes in Graph 1.

Models 1, 5 and 8 in Table 4 all report 4 breaks in mean inflation and jointly illustrate an important aspect of estimating breaks. In each model the first 3 breaks are similar and occur with the changes in monetary policy in the mid-1960s, the first OPEC price shock in 1972 and the 'Volker' deflation in the early 1980s. However, a reasonable case can be made to support the extra break in each model. In model 1 the extra break in March 1991 occurs at the time of the large recession in the United States and world-wide. In model 5 the extra break is June 2014 coincides with a slowdown in China and Europe. And finally, model 8 has the extra break in September 1998 at the time of the Asian financial crisis. With all three breaks a case can be made that they were associated with reductions in mean inflation in the United States and world-wide. These three sets of breaks demonstrate the variation in the results even when the number of breaks is the same using different techniques and ancillary assumptions to identify the breaks.

5. IDENTIFYING THE LONG-RUN PHILLIPS CURVE – STAGES 3 AND 4

Continuing with the maintained assumption that inflation is a stationary process around a shifting mean we estimate a non-linear long-run Phillips curve for the United States based on each set of breaks reported in Table 4. In keeping with the recent New Keynesian literature the forcing variable is defined as the markup of prices on unit labour costs at factor cost and the long-run Phillips curve is of the form:[23]

$$\bar{\pi}_m = \beta_0 \, exp\left(\beta_1 \, \overline{lmu}_m\right) \quad (7)$$

where $\bar{\pi}_m$ and $\overline{lmu}_m$ are the mean values of inflation and the markup respectively in regime m and where the number of regimes, m, is equal to the number identified by each technique to estimate the breaks.

The long-run relationship in equation (7) contains no lagged dependent variable because the m inflation regimes are independent of each other for each individual set of breaks based on one technique and ancillary assumptions. This can be explained with reference to a second proof by contradiction. Assume a stationary inflation regime of N periods where inflation has a constant mean. If agents can predict the break in mean inflation in period N-k then inflation will begin to adjust to the new mean rate of inflation k periods before the end of the regime. Therefore the mean rate of inflation in the last k periods will be different to the first N-k periods in the inflation regime which contradicts our initial assumption that inflation is stationary with a constant mean over all N periods of the inflation regime. Therefore, we can conclude that when inflation is a stationary process (i) agents cannot predict future shifts in mean inflation, (ii) mean rates of inflation are independent across inflation regimes, (iii) there is no ‘time’

[23] For sake of space the estimation of the short-run Phillips curves assuming inflation is a stationary process around a shifting mean is not pursued here and readers are directed to Russell (2011).

dimension to the regimes in equation (7), and (iv) there are no dynamics in the long-run relationship.[24]

The first 9 rows of Table 5 estimate with OLS and HAC standard errors the non-linear long-run United States Phillips curve of equation (7) for each of the 9 sets of inflation regimes identified and reported in Table 4. F and chi squared tests that $\beta_1 = 0$ are rejected at standard levels of significance indicating there is evidence of a significant negative non-linear long-run Phillips curve in the United States. What is clear from Table 5 is the similarity between all 9 long-run relationships identified in the data from the 9 sets of breaks identified in the inflation data.

However, note that the number of observations in each estimation of the long-run Phillips curve range from 3 to 24. The F and chi squared tests are asymptotic tests and may be unreliable with such small numbers of observations in each regression. Each of these individual estimates suffer from the 'single-technique' and 'small-sample' problems as described above and the identified long-run curve may simply be the result of the method and assumptions chosen to estimate the breaks in mean inflation.

To overcome these problems we construct in stage 3 the long-run-panel where each cross section of data contains the mean rates of inflation and mean markup based on the individual sets of breaks reported in rows 1 to 9 in Table 4. We can then estimate in Stage 4 the long-run Phillips curve:

$$\bar{\pi}_{m,n} = \beta_0 \, exp\left(\beta_1 \, \overline{lmu}_{m,n}\right) \qquad (8)$$

where m are the regimes identified by break technique n in stage 2. In our example there are 9 techniques used to estimate the breaks and n is 1, 2, , , 9 and m ranges between 3 and 24 depending on the break technique used to estimate the breaks.

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24 This proof by contradiction is another component of 'post-modern' theories of inflation in that agents cannot logically predict future breaks in mean inflation. Therefore this information must logically be 'missing' and all optimisation behaviour of agents based on agents holding unbiased predictions of future relative prices is logically moot. See Russell and Chowdhury (2013).

Stage 4 overcomes the 'single-technique' problem by estimating the *average* long-run Phillips curve from the 9 sets of long-run inflation data in the long-run-panel. Assuming the techniques for estimating the breaks in mean are unbiased but not necessarily efficient we estimate the long-run Phillips curve in equation (8) using panel estimation techniques. Simultaneously the 'small-sample' problem is overcome as the number of observations in the long-run panel has increased and we now estimate the long-run curve with 66 observations but could potentially be larger with the addition of further estimates of the breaks in mean inflation based on more techniques and more parameter settings.

The bottom two rows of Table 5 report estimates of the long-run Phillips curve using OLS panel estimation techniques. Row 10 pools the data from the 9 cross-sections and restricts the intercept, β_0, and the slope coefficient, β_1, to be the same across the cross-sections. Table 5 suggests that the assumption that the intercept is the same for all the cross-sections of data may be unrealistic. Therefore row 11 reports the model estimated with the fixed effects OLS panel estimator which allows β_0 to be unrestricted and we again find that there is a significant negative sloping long-run inflation-markup United States Phillips curve. Finally, Graph 2 reports the estimated long-run Phillips curve from row 11 in Table 5 as the thick red line and labelled LRPC. Also shown on the graph as crosses are all the combinations of mean inflation and the mean markup from all the regimes identified by all 9 break techniques. The thin black lines are the long-run Phillips curves estimated from the individual cross-sections of data in Rows 1 to 9 in Table 5. From the graph and the estimates reported in Table 5 we can conclude with some confidence that the estimated long-run Phillips curve is both significant and a valid description of the long-run relationship between inflation and the markup in United States data.

6. CONCLUSION

Standard estimates of the long-run Phillips curve assuming inflation is a stationary process around a shifting mean suffer from the problem that the inflation regimes are identified with one technique and the estimation is based on very few observations of inflation regimes. We argue the 4-stage solution outlined above overcomes both of these problems.

Applying the 4-stage solution to estimating the long-run inflation-markup Phillips curve for the United States we identify using 5 ½ decades of quarterly data a significant negative sloping non-linear long-run Phillips curve. Over the range in inflation from zero to an infinite rate the non-linear long-run Phillips curve is still ‘vertical’ to a first approximation. However, over the low to moderate rates of inflation experienced by the United States over the past 50 years there is a significant non-linear negative slope to the long-run inflation-markup Phillips curve which may be meaningful in an economic sense and consistent with Russell (1998), Chen and Russell (2002) and Russell *et al.* (2002).

This analysis argues that when modelling inflation it is best to proceed on the assumption that inflation is a non-stationary process. However, this still implies that there are two ways to proceed. The first is to assume inflation is a stationary process around a shift mean and the second is to assume inflation is an integrated process. This raises the issue of whether or not these two assumptions lead to estimates that are observationally equivalent. This issue is addressed in Russell (2011) where it is reported that a linear long-run Phillips curve estimated assuming the inflation data are stationary around shifting means are numerically very similar to estimates assuming that inflation is integrated. However, assuming the data are integrated means that we cannot estimate the short-run Phillips curves and these short-run estimates are important in a policy sense. Furthermore, the assumption that the data are integrated can only be an approximation of the true statistical process as argued above.

APPENDIX 1 DATA APPENDIX

The United States data are seasonally adjusted monthly and quarterly for the period March 1960 to June 2015. The United States national accounts data are from the National Income and Product Account (NIPA) tables from the United States of America, Bureau of Economic Analysis (BEA) and downloaded on 2 and 3 September 2015 except for Table 1.1.6 which was downloaded on 21 November. The database is available at www.BillRussell.info.

United States Data

Variable	*Details*
The price level	Defined as the gross domestic product implicit price deflator at factor cost (ipdfc) calculated from NIPA Table 1.10 as gross domestic income (line 1) less taxes on production and imports (line 7) plus subsidies (line 8) divided by real gross domestic product at constant prices (Billions of Chained 2009 Dollars) (NIPA Table 1.1.6 line 1). The price level is the natural logarithm of ipdfc (Database: lipdfc).
Inflation	Defined as the log change in the price level (Database dlipfc).
The Markup (National Accounts Basis)	Defined as gross domestic income at factor cost divided by total compensation paid to employees (Database: mufc). Calculated from NIPA Table 1.10 as gross domestic income (line 1) less taxes on production and imports (line 7) plus subsidies (line 8) divided by compensations of employees paid (line 2). The markup is the natural logarithm of the markup (mufc) (Database: lmufc).

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Table 1: Linear Unit Root Tests of United States Inflation Assuming No Breaks and No Trend

	Levels	First difference
Panel A: Null Hypothesis : Integrated of Order one		
Augmented Dickey-Fuller test	-3.240*	-11.223***
Phillips-Perron	10.45***	-50.07***
Elliott-Rothenberg GLS test	-2.737*	-9.845***
Panel B: Null Hypothesis : Integrated of Order zero		
KPSS	0.887***	0.112
Variance Scale Model test (I(0) against I(d))	0.516***	0.050
Harris-McCabe-Leybourne (I(0) against I(d))	4.472***	0.416
Robinson-Lobato (m=23) (I(0) against I(d))	6.851***	-3.069

Reported are the test statistics from the respective unit root tests. The alternative under V/S (Giraitis *et. al* (2003), and Harris et. al (2008), is that the order of integration is higher than zero. The Robinson and Lobato (1998) tests the alternative that it could be integrated greater than zero or less than zero. m illustrates the bandwidth level which is chosen to be 53. *** Rejection at the 1 % level of significance, **Rejection at the 5 % level, *Rejection at the 10 %.

Table 2: Test of unit root assuming breaks are determined endogenously:

Number of breaks	t-statistic	Lags	Break dates
Lumsdaine and Papell (1997) (null of unit root)			
0	-3.243	3	-
1	-7.193**	3	1982Q1
2	-7.517***	3	1973Q2, 1982Q1
3	-7.721***	2	1973Q2, 1982Q1, 2003Q4
Lee and Stazicich (2003) (null of a unit root)			
0	-3.415	3	
1	-6.064**	4	1980Q4
2	-8.574***	1	1981Q2, 2000Q1
3	-16.10***	0	1972Q3, 1981Q1, 2004Q3

Notes: *** Rejection at the 1 % level of significance, **Rejection at the 5 % level, *Rejection at the 10 %.

Table 3: Enders and Lee (2012a, b) test for unit roots with unspecified number and dates of structural breaks

Statistic	Estimate
$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$	5.134
Lags (l)	3
t-stat (τ)	-5.533***
ρ	-0.705

Notes: The number of lags have been selected using the BIC criterion. *** implies the null hypothesis of integration of order one is rejected at the 1 % level.

The Enders and Lee approach considers the following model:

$$y_t = d(t) + \rho\, y_{t-1} + \sum_{j=1}^{l} \Delta y_{t-j} + \varepsilon_t \quad \text{(A1)}$$

$$d(t) = \delta_0 + \delta_1 t + \sum_{j=1}^{k} \alpha_j \sin\left(\frac{2\pi k t}{T}\right) + \sum_{j=1}^{k} \beta_j \cos\left(\frac{2\pi k t}{T}\right) \quad \text{(A2)}$$

where $d(t)$ describes the Fourier approximation of the deterministic component the changing means in the data. The critical value (τ) for testing the null of a unit root ($\rho = 1$) depends on the regressors in (A2), k is the assumed frequency of the sin and cos terms, α_j and β_j are the parameters for the trigonometric terms, t is the trend and T is the total number of observations. If the coefficients on the trigonometric terms are equal to zero, then the Fourier ADF test is simply the ADF test. An F-test can be used to jointly determine the significance that the trigonometric terms are not equal to zero. To avoid problems of overfitting our data we consider k= 2.

Table 4: Estimated Breaks in Mean Inflation

	Model	*Number*	*Break Dates*
Bai-Perron Estimates			
1.	BIC, Trim 5.4% (12 quarters)	4	1965Q4, 1973Q2, 1982Q1, 1991Q1.
2.	BIC, Trim 3.6% (8 quarters)	8	1965Q4, 1973Q2, 1975Q3, 1977Q3, 1982Q1, 2001Q1, 2003Q4, 2006Q3.
3.	BIC, Trim 15% (33 quarters)	3	1972Q2, 1982Q1, 1991Q1.
OxMetrics Impulse Saturation Estimates			
4.	Minute (Prob < 0.0001)	3	1972Q2, 1981Q4, 2014Q2.
5.	Tiny (Prob < 0.001)	4	1963Q3, 1973Q2, 1982Q4, 2014Q2.
6.	Small (Prob < 0.01)	9	1966Q2, 1972Q1, 1972Q6, 1981Q4, 1999Q4, 2000Q1, 2000Q4, 2001Q1, 2015Q1.
Pruned Exact Linear Time Estimates			
7.	No information criteria.	23	1961Q4, 1964Q1, 1966Q1, 1968Q2, 1971Q2, 1973Q2, 1975Q3, 1977Q4, 1980Q1, 1982Q1, 1984Q4, 1986Q4, 1988Q4, 1991Q1, 1994Q1, 1996Q1, 1999Q4, 2001Q4, 2003Q4, 2006Q1, 2008Q1, 2010Q2, 2012Q4.
8.	BIC, SIC & Hannan-Quinn (all same breaks)	4	1965Q4, 1971Q3, 1982Q1, 1998Q3.
9.	Modified BIC	2	1968Q2, 1982Q3.

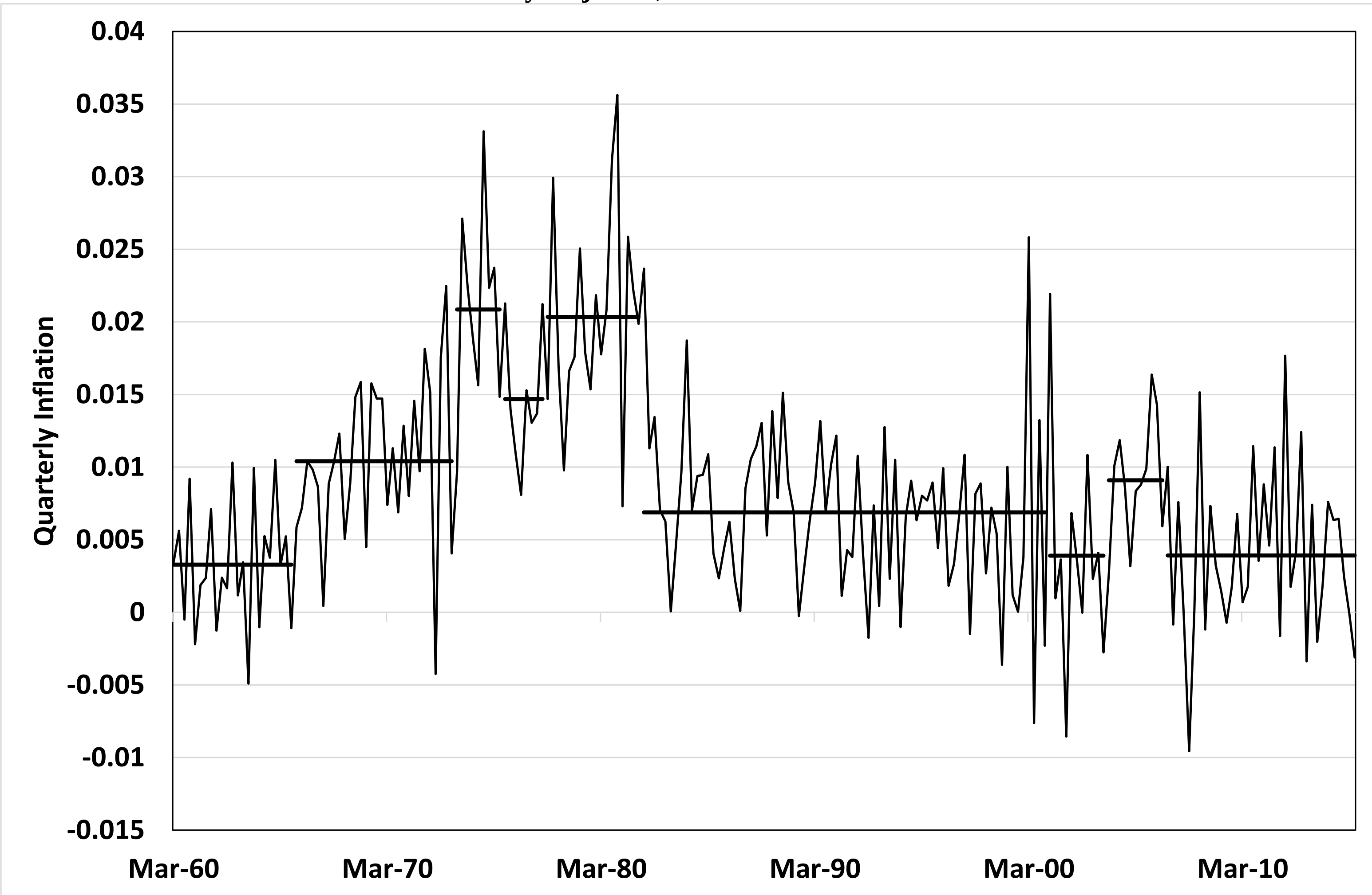
Notes: The Bai-Perron technique makes use of baiperron.src and multiplebreaks.src programmes written by Tom Doan and estimated with RATS 8.01 OxMetrics estimated with version 7.0.

Table 5: Estimates of the Non-Linear Long-Run United States Phillips Curve

Model	N	C	$\overline{lmu}^m$	Prob	$\bar{R}^2$
1. Bai-Perron: BIC, Trim 5.4% (12 quarters).	5	8.0000 (2.3)	-255230 (-3.4)	[0.0370] {0.0003}	0.46
2. Bai-Perron: BIC, Trim 3.6% (8 quarters).	9	4.4662 (2.1)	-18.2153 (-4.2)	[0.0040] {0.0000}	0.42
3. Bai-Perron: BIC, Trim 15% (33 quarters).	4	7.4595 (1.8)	-24.1820 (-3.0)	[0.0161] {0.0000}	0.63
4. OxMetrics: Minute	4	8.9297 (11.8)	-27.1387 (-21.0)	[0.0023] {0.0000}	0.88
5. OxMetrics: Tiny	5	8.2272 (5.6)	-26.1099 (-10.0)	[0.0021] {0.0000}	0.67
6. OxMetrics: Small*	7	4.2710 (2.0)	-17.9133 (-4.1)	[0.0093] {0.0000}	0.46
7. PELT: None	24	3.0633 (1.4)	-15.7392 (-3.6)	[0.0014] {0.0003}	0.37
8. PELT: BIC, SIC, & Hannan Quinn (all indicate the same number and dates of the breaks).	5	5.9685 (1.6)	-21.5220 (-2.8)	[0.0655] {0.0045}	0.47
9. PELT: Modified BIC	3	8.6215 (2.0)	-26.6880 (-3.1)	[0.2001] {0.0021}	0.62
10. Panel (or pooled): Restricted constant	66	5.3574 (4.3)	-20.1998 (-8.3)	[0.0000] {0.0000}	0.55
11. Panel: Fixed Effects	66	5.2234 (4.0)	-19.9368 (-7.8)	[0.0000] {0.0000}	0.50

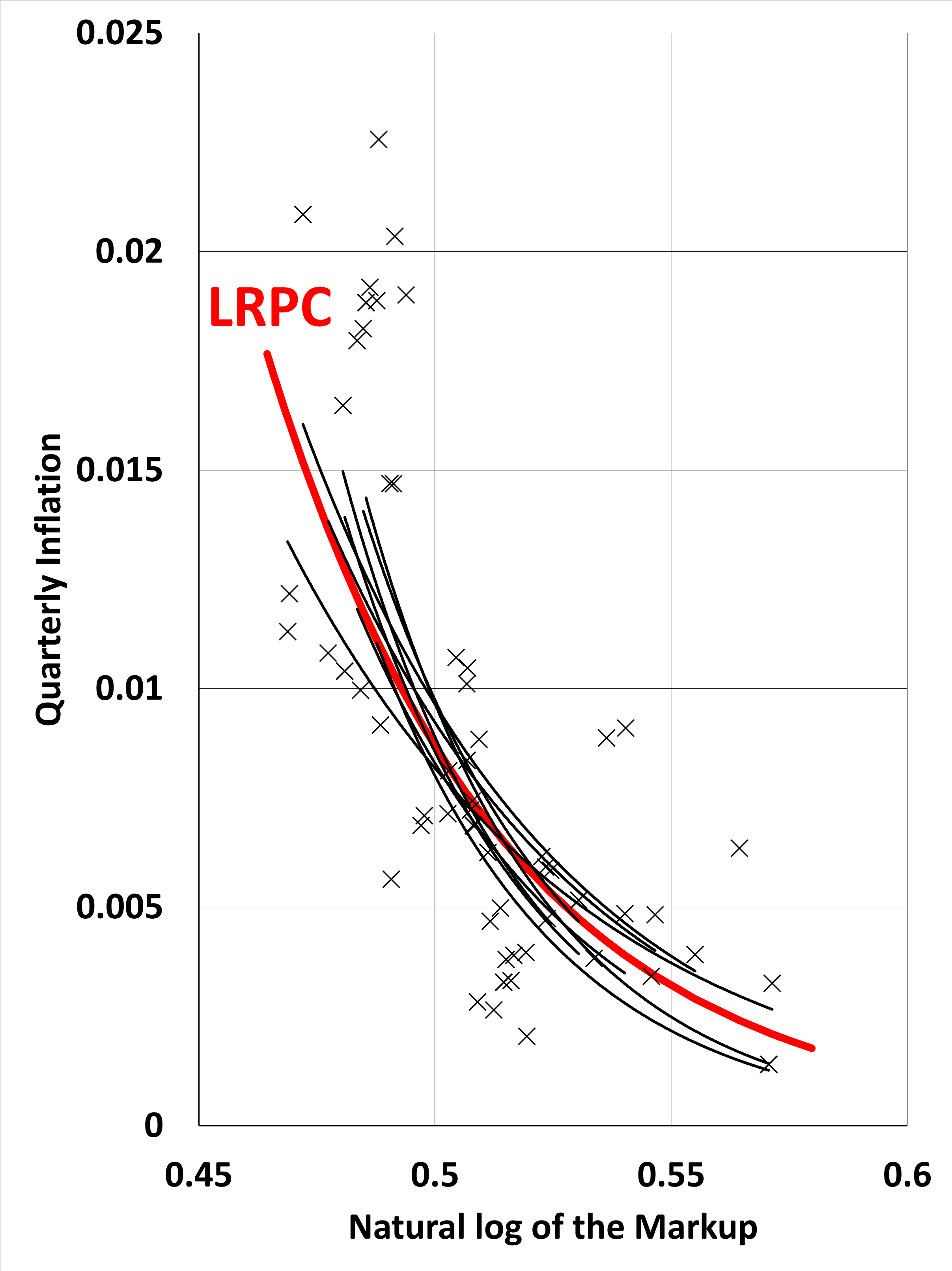
Notes: Rows 1 to 9 correspond to the breaks reported in Rows 1 to 9 in Table 4. Estimated non-linear exponential long-run model of inflation reported in the table: $Ln(\bar{\pi}^m) = C + \beta\,\overline{lmu}^m + \epsilon_m$ where $Ln(\bar{\pi}^m)$ is the natural logarthim of the mean rate of inflation in regime m, C is a constant, and $\overline{lmu}^m$ is the mean of the natural logarithm of the markup. See the data appendix for more details concerning the calculation of inflation and the markup. N is the number of observations (i.e. the same as the number of regimes) in the regression. *Note that in model 6 three single quarter inflation regimes are excluded from the analysis on the basis that a single quarter is a 'shock' and not a 'regime'. The single quarter break can occur when using the OxMetrics approach as no minimum distance between breaks is stipulated. Prob is the probability value of the F test in [ ] and chi squared test in { } that the estimated coefficient β is zero. Numbers in ( ) are *t* statistics. The models are estimated using ordinary least squares in Eviews 8 with Newey-West HAC standard errors. The data are the combinations of the mean (or long-run) rates of inflation and the markup in each of the N inflation regimes in the model. Bai-Perron is the Bai and Perron (1998, 2003) test of multiple structural breaks in time series of stationary data. OxMetrics is the Impulse Saturation Technique of Hendry *et al.* (2008). PELT is the Pruned Exact Linear Time test of Killick *et al.* (2012).

Graph 1: United States Quarterly Inflation
Seasonally Adjusted, March 1960 – June 2015



Notes: Horizontal dashed lines indicate the average inflation in the nine inflation regimes identified by the Bai-Perron technique (BIC, 8 quarters minimum distance model, see Table 4 row 2 for details). Quarterly inflation is measured as the change in the natural logarithm of the gross domestic product at factor cost implicit price deflator.

Graph 2: Non-Linear United States Long-Run Inflation-Markup Phillips Curves
Quarterly March 1960 to June 2015



Note: Long-run Phillips curve indicated by LRPC.