Productivity Growth in Network Models: An Application to Banking

During the Financial Crisis

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Abstract

We construct Malmquist Productivity indices for two-stage processes. A two-stage data envelopment analysis model with an additive efficiency decomposition is used for the modeling of the two-stage process. We incorporate prior information into the analysis using the Weight Assurance Region model. This model offers advantages such as the weights representing the contribution of each stage to the overall process are always positive and we also can restrict them into a region given the available prior information. We extend this model from efficiency analysis to productivity analysis and we calculate Malmquist Productivity indices using four alternative decomposition approaches. The model is applied to a panel of banks in Central and Eastern European countries and productivity change is evaluated for three periods of the financial crisis. The alternative decompositions allow us to examine the various sources of productivity change during the financial crisis. Convergence patterns are also examined.

Keywords: Data envelopment analysis, two-stage, productivity growth, transition economies, banking efficiency.
1. Introduction

One of the key questions regarding two-stage DEA models is the level by which each stage contributes to the whole process. The additive two-stage DEA model of Chen et al. (2009) calculates the contribution of each stage inside the model, in order to avoid any bias. Halkos et al. (2015) notified an extreme case where the contribution of one stage is zero. They proposed the Weight Assurance Region (WAR) model to overcome this problem. In addition, the WAR model allows to incorporate a priori value judgements into the model, such as known information and/or widely accepted beliefs or preferences, and other types of information as described by Thanassoulis et al. (2004). The WAR model is an advancement of the original additive two-stage DEA model which can be considered as a special case of the WAR model with no additional information.

This study investigates the productivity change of commercial banks in Central and Eastern European (CEE) countries during three sub-periods of the recent Economic Crisis. The Economic Crisis hit hard the Western economies on both sides of the Atlantic Ocean. Negative effects of the crisis have also been transmitted to other countries as well through global banks (Correa and Sapriza, 2014). The crisis put pressure on both the funding side (Iyer et al., 2014) and the lending side (De Haas and Van Horen, 2013) of the banks. During the pre-crisis period, the economies of CEE countries were catching up the growth rate and the income growth of western European economies due to foreign direct investment (FDI) and cross-border capital flows. In spite of the former remain stable during crisis, the later suffered a severe drop. The result was a sharp drop of investment and the reduction in the availability of loans (EBRD, 2015). In turn, small and medium-
sized firms were seriously affected by this credit crisis. To make matters worse, the debt to GDP ratio was significantly increased during the crisis period (ECB, 2011). However, the impact of the crisis varied significantly across the CEE countries (ECB, 2010). The banking system had a key role in the transmission of the financial crisis to the CEE countries and the investigation of the productivity growth of banks could provide valuable insights.

An advancement of the WAR model is constructed in order to accommodate the appealing features of the model from efficiency analysis into productivity analysis. The dual of the WAR model is presented for the first time and it is used for the approximation of the distance functions that are needed for the productivity indices. The new model is employed for the evaluation of the overall banking system which is composed by a value added activity index in the first stage and a profitability index in the second stage. The two-stage model serves as a solution for the deposits dilemma by ensuring their dual role as intermediate variables. DEA-based MPI index is used for the evaluation of productivity changes. Furthermore, four alternative decomposition approaches are applied which identify the sources of productivity change. To the best of our knowledge this is the first time a network DEA-based MPI approach uses the four decompositions. Furthermore, this is the first time that the dual of the WAR model is presented. This formulation allow us to define an approximation of the distance function for the network process. Last but not least, this paper checks whether economic crisis hampered the convergence process of the banking system in CEE countries.

The paper is structured as follows. Section 2 present the most recent review of the literature and Section 3 demonstrates the framework and the methodology used
throughout the paper. Section 4 is about the empirical application on banks of CEE
countries during the Financial Crisis and Section 5 concludes.

2. Review of the recent literature

The most commonly used measure for productivity is the Malmquist Productivity
Index (MPI) originated from the distance functions of Shephard (1953, 1970) and
Malmquist (1953). The theoretical framework for the MPI was introduced by Caves et al.
(1982) who examined productivity indices related to Shephard’s distance function and
Törnqvist index. Färe and Grosskopf (1992a) constructed an MPI directly from input and
output data using data envelopment analysis (DEA). Färe et al. (1994a) decomposed MPI
in order to identify its sources. There is a debate across the literature about whether
efficiency change or technical change is the primary source for productivity growth.

Malmquist Productivity indices with various decomposition approaches have been
used extensively in conjunction with DEA models. Conventional DEA models are single
stage models which treat the decision making unit (DMU) as a “black box” using inputs to
produce outputs without considering any internal procedure inside the DMU. Real life
applications require more complex models which may consists of two or more stages
linked with intermediate variables; variables which are treated as inputs in one stage and
outputs in another stage. Network DEA models allow for more than one stage, inputs may
enter in any stage and final outputs may also exist in any stage. Two-stage DEA models
are a special case of network models with only two stages.
Based on the seminal work of Färe and Grosskopf’s (1996) network DEA, Wang et al. (1997) were the first to develop a two-stage network DEA model. Two-stage network DEA models can be classified into four categories: independent, connected, relational and game theoretic (Halkos et al., 2014). The present study uses a relational network DEA model. Relational models consider the interactions between the two stages and assume an additive or multiplicative relationship between the overall and the stage efficiencies.

Berg et al. (1992) were the first to investigate the Malmquist productivity of banking institutions and sparked the beginning of a fast growing literature. There is an academic argument regarding the assessment of banking efficiency about whether to consider deposits as inputs or outputs in the process. We can summarize the conflicting approaches into three categories depending on the use of deposits and other liabilities as inputs or outputs: the intermediation approach, the production approach and the user cost approach (Berger and Humphrey, 1992). Berger and Humphrey (1992) pointed out that deposits have both input and output characteristics. There is an alternative approach if we consider the bank process as a two-stage process. In the first stage the bank uses inputs such as employees, capital and assets in order to attract deposits and other loanable funds, while in the second stage the bank uses its deposits to convert them into earning assets (Fukuyama and Weber, 2010; Fukuyama and Matousek, 2011; Holod and Lewis, 2011). This network approach is in accordance with Berger and Humphrey’s (1992) view about the dual role of deposits.

Network DEA studies are becoming very popular for analyzing the efficiency levels of banking institutions. Seiford and Zhu (1999) and Luo (2003) applied an independent
network DEA model in order to measure the profitability and marketability of 55 US commercial bank and 245 large banks respectively. Mukherjee et al. (2003) examined 27 Indian public banks, Liu and Lu (2012) 27 firms in the banking industry and Akther et al. (2013) 21 banks in Bangladesh using connected network DEA models. Degl’Innocenti et al. (2016, 2017) applied relational network DEA models in order to study banks in Eastern Europe and EU-28 respectively. Du et al. (2010) and Zha and Liand (2010) applied game theoretic network DEA models in order to investigated the top 30 US commercial banks. There is also a strand in the literature which applies a network DEA model in order to study the efficiency of bank branches (Cook et al., 2000; Yang et al., 2011).

Following Fukuyama and Matousek (2011) and Holod and Lewis (2011) we specify the input-output framework for our two-stage model. Specifically, number of employees and total assets are used as inputs while deposits are the only output in the first stage and they also serve as intermediate variable. In the second stage deposits are treated as input and loans and securities are the final outputs. Non-performing loans should also be considered when modelling the banking system. Fukuyama and Weber (2010) incorporated non-performing loans into their directional network slacks-based using weak disposability. We do not include non-performing loans here, however the incorporation of non-performing loans to the WAR model is an open research question that needs to be addressed. Specifically, it should be investigated how the WAR model can be modified in order to adapt non-performing loans.
3. Methodology

3.1 Weight Assurance Region (WAR) model

This section discusses the WAR model of Halkos et al. (2015) which follows Thompson et al.’s (1990) assurance region concept. The WAR model is a modification of the relational two-stage DEA model of Chen et al. (2009) in order to incorporate assurance region-based weights regarding the contribution of each stage to the overall process. WAR model has the ability to utilize prior information and solves a possible infeasibility problem of the original additive model.

The use of additional constraints and restrictions in single-stage DEA models has been studied extensively (see Thanassoulis et al., 2008 for a comprehensive review of the literature). Alternative approaches have been proposed across the literature such as the use of regression analysis to restrict weight flexibility (Dyson and Thanassoulis, 1988), restricting multiplier flexibility with inequalities (Beasley, 1990, 1995; Wong and Beasley, 1990), absolute weight restrictions (Podinovski and Athanassopoulos, 1998) and unobserved DMUs (Thanassoulis and Allen, 1998; Allen and Thanassoulis, 2004; Thanassoulis et al., 2012).

The additive model of Chen et al. (2009) assumes $n$ DMUs and $x_{ij}$ ($i = 1, \ldots, m$), $z_{dj}$ ($d = 1, \ldots, D$) and $y_{rj}$ ($r = 1, \ldots, s$) are the $i$th input, the $d$th intermediate variable and the $r$th output respectively of the $j$th DMU ($j = 1, \ldots, n$) and $v_i$, $w_d$ and $y_r$ are the multipliers of the model. The overall efficiency for DMU $k$ is defined as the weighted average of the stage efficiencies:
The relative contribution of each stage to the whole process is represented as $\xi_1$ and $\xi_2$ and they are proxied by the size of each stage. Chen et al. (2009) uses total inputs as a proper measure for the size of each stage. Therefore, the relative contribution of each stage to the whole process is defined as:

$$E_k = \xi_1 \frac{\sum_{i=1}^{m} v_i x_{ik}}{\sum_{i=1}^{m} v_i x_{ik} + \sum_{d=1}^{D} w_d z_{dk}} + \xi_2 \frac{\sum_{r=1}^{S} u_r y_{rk}}{\sum_{d=1}^{D} w_d z_{dk}},$$

(1)

where $0 \leq \xi_1, \xi_2 \leq 1$ and $\xi_1 + \xi_2 = 1$. Ang and Chen (2016) found that the weights of Chen et al. (2009) are non-increasing which means that the weights of the first stage are larger than the weights of the second stage. However, this is only true for the CRS version of the model. The VRS version of the model allows the weights of the second stage to be larger. Evidently, the present paper contains such cases. Furthermore, the additive model assigns weights to each stage which are greater than or equal to zero. The equality leads to infeasibility problems as described in the following paragraph. On the contrary, the WAR model assigns strictly positive weights to each stage.

A zero value for a weight means that the corresponding stage does not contribute to the overall process at all and a unity value means that the overall process is entirely based on this stage. Assigning zero values to one of the stages results both in an infeasibility and a conceptual problem (Halkos et al., 2015). On the one hand it is not possible to calculate both the overall and the stage’s efficiencies and on the other hand it is not reasonable to use a two-stage network model when one of the stages does not contribute to the whole process at all. The WAR model restricts the ratio of weights $\xi_1$
and $\xi_2$ to be inside a region defined by two positive scalars, $\beta$ and $\delta$:

$$
\beta \leq \frac{\xi_1}{\xi_2} \leq \delta
$$

(3)

Note that $\beta$ and $\delta$ represent the prior information and they cannot become zero. This ensures that neither $\xi_1$ nor $\xi_2$ are zero. By replacing (2) into (3) two new constraints are formed which are the advancement of the WAR model relative to the original additive model.

$$
- \sum_{i=1}^{m} \omega_i x_{ik} + \beta \sum_{d=1}^{D} \mu_d z_{dk} \leq 0
$$

(4)

$$
\sum_{i=1}^{m} \omega_i x_{ik} - \delta \sum_{d=1}^{D} \mu_d z_{dk} \leq 0
$$

Then, the WAR model for the overall efficiency of DMU $k$ satisfying variable returns to scale is the following (Halkos et al., 2015):

$$
E_k = \max \sum_{d=1}^{D} \mu_d z_{dk} + \sum_{r=1}^{s} y_{rk} + u^1 + u^2
$$

(5)

s.t.

$$
\sum_{d=1}^{D} \mu_d z_{dk} = 1
$$

$$
\sum_{i=1}^{m} \omega_i x_{ik} + \sum_{d=1}^{D} \mu_d z_{dk} \leq 0
$$

$$
\sum_{d=1}^{D} \mu_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} + u^1 \leq 0,
$$

$$
\sum_{r=1}^{s} y_{rk} - \sum_{d=1}^{D} \mu_d z_{dj} + u^2 \leq 0,
$$

$$
- \sum_{i=1}^{m} \omega_i x_{ik} + \beta \sum_{d=1}^{D} \mu_d z_{dk} \leq 0
$$

$$
\sum_{i=1}^{m} \omega_i x_{ik} - \delta \sum_{d=1}^{D} \mu_d z_{dk} \leq 0
$$

$$
\gamma_r, \mu_d, \omega_i \geq 0
$$

$$
j = 1, ..., n; i = 1, ..., m; d = 1, ..., D; r = 1, ..., s$$
β and δ are user specified and (0<β≤δ); u^1 and u^2 are free in sign

Note that multipliers γ_r, µ_d, ω_i are greater than or equal zero. A non-Archimedean infinitesimal ε can be included in order for the multipliers of all variables to be positive.

Next, we choose to give priority to the first stage as we will explain in a later section. The first stage efficiency is calculated in model (6). Note that here we omit the two additional constraints (4) for assurance region. These constraints are used only in the overall model (5) in order to constraint the weights of each stage. Then model (5) yields the optimal weights ξ^*_1 and ξ^*_2 along with the optimal overall efficiency E^*_k. If we include these constraints in model (6) then we constraint the first stage efficiency and the results will be biased in favor of the second stage. This is in line with previous studies which use weight constraints such as (Ho et al., 2013). Then, the first stage efficiency by omitting the assurance region constraints is the same as in Chen et al., 2009):

\[
E^*_k = \max \sum_{d=1}^{D} \mu_d z_{dk} + u^1
\]

s.t.
\[
\sum_{i=1}^{m} \omega_i x_{ik} = 1
\]
\[
(1 - E^*_k) \sum_{d=1}^{D} \mu_d z_{dk} + \sum_{r=1}^{s} y_r y_{rk} + u^1 + u^2 = E^*_k
\]
\[
\sum_{d=1}^{D} \mu_d z_{dj} - \sum_{i=1}^{m} \omega_i x_{ij} + u^1 \leq 0,
\]
\[
\sum_{r=1}^{s} y_r y_{rj} - \sum_{d=1}^{D} \mu_d z_{dj} + u^2 \leq 0
\]
\[
y_r, \mu_d, \omega_i \geq 0
\]
\[
j = 1, ..., n; i = 1, ..., m; d = 1, ..., D; r = 1, ..., s
\]
\[
u^1 and u^2 are free in sign
\]

Last, the efficiency for the second stage based on (5) and (6) is calculated as:
\[ E_k^2 = \frac{E_k^* - \xi_1 E_1^*}{\xi_2^*} \] (7)

3.2. Malmquist Productivity Index and decompositions

Alternative decomposition approaches are based on the difference among the benchmark technology which satisfies constant returns to scale and the best practice technology which satisfies variable returns to scale. Färe et al. (1992b) defined the Malmquist Productivity Index (MPI) among two periods (t and t+1) on a benchmark technology, as the geometric mean of the ratios of their respective distance functions from one period to the other. This paper adopts the input oriented version of the Malmquist productivity index (Färe et al., 1992b). Input distance functions measure the largest possible contraction of inputs relative to a reference technology. “D” stands for distance function, “c” for constant returns to scale and “v” for variable returns to scale. For example, \( D_c^c(t, y, x) \) is the input distance function in period t+1 using period t as a benchmark technology. This distance function measures the largest possible contraction of \( x_{t+1} \) relative to the benchmark technology of period t.

Färe et al. (1994a) decomposed the index into an efficiency change term and a productivity change term.

\[ MPI_c(y^{t+1}, x^{t+1}, y^t, x^t) = Eff_{ch} \cdot Tech_{ch} = \]

\[ = \frac{D_{c}^{t+1}(y^{t+1}, x^{t+1})}{D_{c}^{t}(y^{t}, x^{t})} \cdot \left[ \frac{D_{c}^{t}(y^{t+1}, x^{t+1})}{D_{c}^{t+1}(y^{t+1}, x^{t+1})} \cdot \frac{D_{c}^{t}(y^{t}, x^{t})}{D_{c}^{t+1}(y^{t}, x^{t})} \right]^{\frac{1}{2}} \] (8)

Färe et al. (1994b) restructured the efficiency change term into two new terms, an efficiency change term relative to best practice technology and a scale change term.
\[ MPI_c(y^{t+1}, x^{t+1}, y^t, x^t) = E_{ff\_ch} \cdot T_{ech\_ch} \cdot S_{cale\_ch} = \frac{D^{t+1}_b(y^{t+1}, x^{t+1})}{D^t_b(y^t, x^t)} \cdot \left[ \frac{D^t_c(y^{t+1}, x^{t+1})}{D^{t+1}_c(y^{t+1}, x^{t+1})} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \]  

Same as (5), this decomposition approach estimates the technical change term relative to a benchmark technology. Subsequently, a debate emerged regarding the economic interpretation and internal consistency of the decomposition (Lovell, 2003). In order to tackle the aforementioned issues, Ray and Desli (1997) proposed a decomposition which estimates both the efficiency change term and the technical change term relative to a best practice technology. However, this decomposition approach yields a number infeasible efficiency scores due to variable returns to scale in mixed periods (Grosskopf, 2003).

\[ MPI_c(y^{t+1}, x^{t+1}, y^t, x^t) = E_{ff\_ch} \cdot T_{ech\_ch} \cdot S_{cale\_ch} = \frac{D^{t+1}_b(y^{t+1}, x^{t+1})}{D^t_b(y^t, x^t)} \cdot \left[ \frac{D^t_c(y^{t+1}, x^{t+1})}{D^{t+1}_c(y^{t+1}, x^{t+1})} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \cdot \left[ \frac{D^{t+1}_c(y^{t+1}, x^{t+1})}{D^t_c(y^t, x^t)} \right]^{1/2} \]  

Wheelock and Wilson (1999) criticized the above decomposition approaches and proposed a four way decomposition. The efficiency change and the technical change terms are calculated relative to a best practice technology similar to Ray and Desli (1997). The scale change term is defined as in Färe et al. (1994b) and the last component is the scale bias of technical change. This new term is the geometric mean of two scale efficiency ratios, one of the \((x^t, y^t)\) on the two technologies (benchmark and best practice) and the
other of the \((x^{t+1}, y^{t+1})\). If there is a difference relative to the two technologies, then there is evidence for scale bias in the technical change term.

\[
MPI_c(y^{t+1}, x^{t+1}, y^t, x^t) = Eff\cdot Tech\cdot Scale\cdot Scale\_bias =
\]

\[
\frac{D_{v}^{t+1}(y^{t+1}, x^{t+1})}{D_{v}^{t}(y^{t}, x^{t})} \cdot \left[ \frac{D_{v}^{t}(y^{t}, x^{t})}{D_{v}^{t+1}(y^{t+1}, x^{t+1})} \right]^{1/2} \cdot \left[ \frac{D_{c}^{t+1}(y^{t+1}, x^{t+1})/D_{v}^{t+1}(y^{t+1}, x^{t+1})}{D_{c}^{t}(y^{t}, x^{t})/D_{v}^{t}(y^{t}, x^{t})} \right]^{1/2}
\]

3.3 Modified WAR model for MPI calculation

This section modifies the Weight Assurance region model in order to calculate the input distance functions for the Malmquist Index. The production technology is defined at each period \(p = t, t + 1\) to be the set of all feasible input, output and intermediate variables vectors. We denote \(x^p \epsilon R^m_+\) as the input vector at period \(p\), \(y^p \epsilon R^n_+\) as the output vector at period \(p\) and \(z^p \epsilon R^d_+\) as the intermediate variables vector at period \(p\). Then, the production technology can be expressed as:

\[
T^p = \{x^p, y^p, z^p\}: x^p \text{ can produce } z^p, z^p \text{ can produce } y^p
\]

According to production theory, \(T^p\) is assumed to be closed and bounded set. Based on the production technology and following Galagedera et al. (2016) we can calculate an approximation of the input distance function as the reciprocal of the Farrell technical efficiency (Färe et al., 1992b). It must be noted that our proposed distance function and the distance function of Galagedera et al. (2016) are approximations of the input distance function proposed by Färe et al. (1992b). In Galagedera et al. (2016) model (B.2), the component \((1 - \theta_0)z_{d0}\) at the third constraint does not allow for proportional reductions of all intermediate variables. Similarly, in our model (12) the components \(\sigma\)
and φ which are the assurance region components do not allow for proportional reductions of all inputs and intermediate variables. Therefore, we clarify that this is an approximation and not an exact calculation of the input distance function.

Models (5) and (6) are in multiplier form. We need their dual models in order to calculate the input distance functions. Here we present the dual of the WAR model for the first time. Model (12) is the VRS version of the model for DMU \( k \) where the reference technology is in period \( t \) and the observed values are also in period \( t \). By replacing period \( t \) with \( t+1 \), we can calculate the model for the next period where the reference technology is in period \( t+1 \) and the observed values are also in period \( t+1 \). The CRS version can be obtained by omitting the fourth and the fifth constraints.

\[
[D_v^t(y_k^t, z_k^t, x_k^t)]^{-1} = \min \ \theta_k \tag{12}
\]

\[
\sum_{j=1}^{n} \lambda_j x_{ij}^t \leq (\theta_k - \sigma + \varphi) x_{ik}^t,
\]

\[
\sum_{j=1}^{n} \mu_j z_{dj}^t - \sum_{j=1}^{n} \lambda_j z_{dj}^t \leq (\theta_k - 1 + \sigma\beta - \varphi\delta) z_{dk}^t,
\]

\[
\sum_{j=1}^{n} \mu_j y_{rj}^t \geq y_{rk}^t,
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\sum_{j=1}^{n} \mu_j = 1
\]

\[
\sigma, \varphi, \lambda_j, \mu_j \geq 0
\]

\[ j = 1, ..., n; \ i = 1, ..., m; \ d = 1, ..., D; \ r = 1, ..., s \]

\( \beta \) and \( \delta \) are user specified and \( 0 < \delta \leq \delta \).

In the solution of model (12), \( \theta_k \) is less than or equal to 1. \( \theta_k = 1 \) means that the observed DMU \( k \) is efficient. Furthermore, \( \lambda_j \) and \( \mu_j \) can be utilized to assess whether DMU \( j \) is a peer.
of the observed DMU $k$ in the first stage or second stage respectively. If $\lambda_j$ is zero, then DMU $j$ is not a peer for observed DMU $k$ in the first stage and if $\mu_j$ is zero, then DMU $j$ is not a peer for the observed DMU $k$ in the second stage. If either of them takes a positive value, then DMU $j$ is related with the observed DMU $k$ either in the first stage (for a positive $\lambda_j$) or in the second stage (for a positive $\mu_j$). A larger positive value for $\lambda_j$ and $\mu_j$ indicates a stronger relationship between DMU $j$ and the observed DMU $k$.

Similarly, we can calculate the input distance function for the first stage as the reciprocal of model (13).

\[
[D_{v,1}^t(y_k^t, z_k^t, x_k^t)]^{-1} = \min \tau \theta_k^* + \pi
\]

s.t.

\[
\sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \pi x_{ik}^t,
\]

\[
\sum_{j=1}^{n} \mu_j z_{aj}^t - \sum_{j=1}^{n} \lambda_j z_{aj}^t - \tau (1 - \theta_k^*) z_{ak}^t \leq -z_{dk}^t,
\]

\[
\sum_{j=1}^{n} \mu_j y_{rj}^t \leq \tau y_{rk}^t,
\]

\[
\sum_{j=1}^{n} \lambda_j + \tau = 1
\]

\[
\sum_{j=1}^{n} \mu_j + \tau = 0
\]

\[
\lambda_j, \mu_j \geq 0
\]

\[
j = 1, \ldots, n; \quad i = 1, \ldots, m; \quad d = 1, \ldots, D; \quad r = 1, \ldots, s
\]

$\tau$ and $\pi$ are free in sign

Model (12) which estimates the overall efficiency is the dual of model (5) and model (13) which estimates the first stage efficiency is the dual of model (6). Since the second stage efficiency in (7) is calculated residually and not from a linear model we
cannot calculate the distance function directly. As an alternative we will use the reciprocal of (7) as an estimation of the input distance function.

The most challenging part is the calculation of the problem in mixed periods. Model (14) is the VRS version of the model for DMU $k$ where the reference technology is in period $t+1$ and the observed values are in period $t$. The opposite can easily be obtained by setting the reference technology in period $t$ and the observed values in period $t+1$. The CRS version can be calculated by omitting the fourth and the fifth constraints.

$$[D_{v}^{t+1}(y_{k}^{t}, z_{k}^{t}, x_{k}^{t})]^{-1} = \min \theta_k$$

s.t.
$$\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq (\theta_k - \sigma + \varphi) x_{ik}^{t},$$
$$\sum_{j=1}^{n} \mu_j z_{dj}^{t+1} - \sum_{j=1}^{n} \lambda_j z_{dj}^{t+1} \leq (\theta_k - 1 + \sigma \beta - \varphi \delta) z_{dk}^{t},$$
$$\sum_{j=1}^{n} \mu_j y_{rj}^{t+1} \geq y_{rk}^{t},$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\sum_{j=1}^{n} \mu_j = 1$$
$$\sigma, \varphi, \lambda_j, \mu_j \geq 0$$

$j = 1, ..., n; i = 1, ..., m; d = 1, ..., D; r = 1, ..., s$

$\beta$ and $\delta$ are user specified and ($0<\delta\leq\delta$)

The input distance function for the first stage in mixed periods is as follows.

$$[D_{v}^{t+1,1}(y_{k}^{t}, z_{k}^{t}, x_{k}^{t})]^{-1} = \min \tau \theta_k + \pi$$

s.t.
$$\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \pi x_{ik}^{t},$$
$$\sum_{j=1}^{n} \mu_j z_{dj}^{t+1} - \sum_{j=1}^{n} \lambda_j z_{dj}^{t+1} - \tau (1 - \theta_k^{*}) z_{dk}^{t} \leq -z_{dk}^{t},$$

$\theta_k^{*}$ is the optimal input distance function for the VRS version.
\[
\sum_{j=1}^{j} \mu_j y_{r_j}^{t+1} \leq \tau y_{r_k}^t,
\]
\[
\sum_{j=1}^{n} \lambda_j + \tau = 1
\]
\[
\sum_{j=1}^{n} \mu_j + \tau = 0
\]
\[
\lambda_j, \mu_j \geq 0
\]
\[
j = 1, \ldots, n; \ i = 1, \ldots, m; \ d = 1, \ldots, D; \ r = 1, \ldots, s
\]
\[
\tau \text{ and } \pi \text{ are free in sign}
\]

Again, we calculate the input distance function for the second stage using the reciprocal of (7).

3.4 Test of convergence

After the assessment of the two-stage MPI indices, we check for productivity convergence. Following Barro and Sala-i-Martin (1995), we perform a β-convergence test which uses a GLS regression of the productivity growth rate on the initial level of productivity (Kumar and Russell, 2002). A statistically significant coefficient reveals the existence of a directional relationship; convergence if the coefficient is negative and divergence if it is positive. The existence of β-convergence indicates that DMUs with lower initial productivity performance achieve faster growth. Equation (16) shows the regression for β-convergence of the MPI index.

\[
\ln MPI_{j,t} - \ln MPI_{j,t-1} = \alpha + \beta \ln MPI_{j,t-1} + \varepsilon_{j,t}
\]

where \( MPI_{j,t} \) is the Malmquist Productivity Index for DMU \( j \) in time \( t \), \( MPI_{j,t-1} \) is the Malmquist Productivity Index for DMU \( j \) in time \( t-1 \), \( \alpha \) and \( \beta \) are the parameters which will be estimated and \( \varepsilon_{j,t} \) is the error term.
4. Empirical Application

This paper focuses on the productivity assessment of banks in transition economies in Central and Eastern Europe (CEE). The banking sectors remained highly concentrated and dominated by large state-owned banks and later by the privatized state-owned banks. Newly established domestic small and medium-sized banks (SMBs) competed with these banks through aggressive lending strategies and price competition. This business strategy was rather questionable and ended up in the failure of SMBs. Therefore it is of interest to assess how the banking sector has changed over the analyzed period that is after twenty years of the transition period.

4.1 Data and model description

The dataset of our empirical application consists of 88 commercial banks in 11 economies in transition located at Central and Eastern Europe, known as the new EU countries (Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia and Slovenia), for the time period from 2007-2012. All data has been collected from Bankscope database. All variables are deflated and are in constant euros. Following Fukuyama and Matousek (2011) and Holod and Lewis (2011) we specify the input-output framework for our two-stage model. Specifically, number of employees and total assets are used as inputs while deposits are the only output in the first stage and they also serve as intermediate variable. In the second stage deposits are treated as input and loans and securities are the final outputs. Figure 1 presents the two-stage framework where the first stage measures the value added activity where the bank uses its inputs to
accumulate deposits and the second stage measures the profitability of the bank where the deposits are being used to finance loans and other securities which generate profit for the bank.

Figure 1 about here

During financial crisis periods, banks focus heavily on deposits in order to finance their activities. In addition, banks reallocate their portfolios away from loans (Demirguc-Kunt et al., 2006). Furthermore, banks with finance from a strong deposit base tend to cut their loans less than their competitors (Ivashina and Scharfstein, 2010; Cornett et al., 2011). Taking the above into account, we can say that during financial crisis banks rely more on their deposit side relative to their loans. Now we demonstrate how we can utilize such an information using the WAR model. We assign values to the WAR model in order to ensure that the first stage (where the bank accumulates deposits) will contribute more than the second stage (where the bank finances loans) to the whole process. Specifically, we set $\beta=(0.55/0.45)=1.222$ and $\delta=(0.9/0.1)=9$. In this way, the first stage contributes 55%-90% to the whole process, while the second stage contributes 10%-45%. We assume that it is not reasonable for the second stage to contribute less than 10%.

We have also performed a robustness check by setting two alternative assurance regions. The first one is a less restricted model $\beta=(0.10/0.90)=0.111$ and $\delta=(0.9/0.1)=9$ and the second one is a more restrictive model $\beta=(0.70/0.30)=2.333$ and $\delta=(0.9/0.1)=9$. We performed Kruskal-Wallis tests in order to check if there is a statistically significant difference among our different groups. The results revealed that there is not a statistically
significant difference regarding the Malmquist scores. However there are differences in the productivity components when infeasible scores are present.

Next, we employ the DEA-based Malmquist Productivity Index measures for the calculation of the productivity change over time. Since we use an input based productivity change, a number less than one corresponds to productivity progress while a number greater than one corresponds to productivity regress. Following Färe et al. (1992b) in Tables 2 and 3 we take the reciprocal number in order to conform to the standard productivity literature. Therefore, banks with productivity change over 1 experience productivity progress, while banks with productivity change under 1 experience productivity regress. Four decomposition approaches are applied as presented in (8-11). Following Fiordelisi et al. (2014) we consider three periods of the financial crisis to examine the productivity change: the U.S. subprime crisis (2007-2008), the global financial crisis (2008-2010) and the sovereign debt crisis (2010-2012). Table 1 presents the descriptive statistics of the dataset.

Table 1 about here

4.2 Results

In this section, we turn our focus on the results regarding the productivity change and the four decomposition approaches (Färe et al., 1994a; Färe et al., 1994b; Ray and Desli, 1997; Wheelock and Wilson, 1999) for the overall model, the first stage and the second stage. Table 2, presents the overall trend of productivity change for the whole sample. Since the results are in a bank level, we need an aggregation method in order to report the results for the whole sample. Here we choose the denominator rule of Färe
and Karagiannis (2017) according to which an input oriented index is aggregated using input side shares as weights. Furthermore, following Färe and Zelenyuk (2003), Zelenyuk (2006) and Fox (2012) we assume equal weights for the firm’s share of each input.

We must note that all four decomposition approaches should yield the same productivity estimate. However, there are a few missing values due to infeasibility problems which lead to small differences in productivity for some cases. In accordance with Grosskopf (2003,p.460) who found that mixed period problems yield infeasibilities, we find that decomposition approaches of Färe et al. (1994a) and Färe et al. (1994b) yielded fewer infeasibilities than the other two approaches.

An overall assessment of the results reveal that financial crisis did not severely hamper the productivity growth of CEE countries. On the contrary they experience a slightly positive growth. Specifically, during the first period of the analysis the banks of CEE countries achieve a minor productivity growth (1.018) with productivity decline for the value-added activity stage (0.991) and productivity growth for the profitability stage (1.107). The results of the four decompositions are mixed. Only the first approach attributes productivity growth to the efficiency change, the second approach to scale change, the third approach to technical change and the last approach to technical change and scale change. During the second period, banks of CEE countries experience productivity growth for the overall model (1.020), the first (1.043) and the second stage (1.021). Here, the four decomposition approaches reveal that the growth is attributed to efficiency growth. The third period yielded a minor productivity growth (1.016) for the overall banking model, a productivity decline for the value-added activity stage (0.995)
and productivity growth for the profitability stage (1.098). Here the results of the four decomposition approaches are mixed. The three approaches which include the scale term indicate that there is an efficiency growth. In addition, two approaches reveal growth in technical change while Wheelock and Wilson’s (1999) approach shows a growth in the scale bias of technical change term.

**Table 2 about here**

Next, we focus on the productivity growth and its components per country as presented in Table 3. Here we present results from the decomposition approach of Wheelock and Wilson (1999). Results from the other approaches are available upon request. We apply the denominator rule of Färe and Karagiannis (2017) and the aggregation method we described above. The first column presents the countries and the second column the number of banks from the aforementioned country. The next five columns are the MPI, the efficiency change, the technical change, the scale change and the scale bias of technical change for the first time period 2007-2008 (U.S. subprime crisis). The following five columns are the same indices for the second period 2008-2010 (Global Financial crisis) and the last five columns are about the third period 2010-2012 (Sovereign debt crisis). There is no clear pattern about how Financial Crisis affected banks of CEE countries.

**Table 3 about here**

Furthermore, we turn our focus on productivity convergence among banks of CEE countries (Table 4). The first line shows the convergence results for the first (2007-2008) and the second (2008-2010) period and the second line shows the convergence for the
second and the third (2010-2012) period. The second column demonstrates the β-convergence scores for the overall model, the third and the fourth columns the corresponding results for the first stage and the last two columns for the second stage. All the β-convergence coefficients for the overall model, the first stage and the second stage are negative and statistically significant for 0.001. Evidently, our results support the convergence hypothesis for the banks of CEE countries during the Financial Crisis. Furthermore, Figures 2-4 presents a visual evidence for the results of the overall model, the first stage and the second stage respectively. Subfigures 2a and 2c present scatterplots of productivity growth on the initial level of productivity change with a fitted GLS regression line. Subfigures 2b and 2d present the densities of productivity change. In addition, Figures 2a and 2b examine the convergence of the first and the second period and Figures 2c and 2d examine the convergence of the second and the third period. Correspondingly, Figures 3 and 4 present the same for the first and the second stage. The visual representation validates the results from Table 4 and it is clear that there is a strong pattern of productivity convergence.

5. Conclusions

This paper uses the Weight Assurance Region (WAR) model in order to construct an approximation of the distance function and calculate Malmquist Productivity Indices for two-stage processes. The WAR model has the ability to incorporate any available prior information into the analysis such as value judgements, known information and/or widely accepted beliefs or preferences. We extend the WAR model to productivity analysis and
we adopt four decomposition approaches (Färe et al., 1994a; Färe et al., 1994b; Ray and Desli, 1997; Wheelock and Wilson, 1999) to examine the components of productivity change. Following Demirguc-Kunt et al. (2006), Ivashina and Scharfstein (2010) and Cornett et al. (2011) we utilize the information that during financial crisis banks rely more on their deposit side relative to their loans.

The model is applied to a panel of banks in Central and Eastern European (CEE) countries and productivity change is evaluated for the financial crisis period. Specifically, we assess the productivity change of CEE banks during three sub-periods of recent Economic Crisis; the U.S. subprime crisis (2007-2008), the Global financial crisis (2008-2010) and the sovereign debt crisis (2010-2012). The results do not reveal that the Financial Crisis affected the banks of CEE countries. Furthermore, the paper checks for β-convergence and finds a strong pattern for convergence among the banks of CEE countries.

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References


Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2010</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of employees</strong></td>
<td>Mean</td>
<td>2530.4</td>
<td>2569.6</td>
<td>2498</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>4733</td>
<td>4496.3</td>
<td>4420.6</td>
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<tr>
<td><strong>Fixed assets</strong></td>
<td>Mean</td>
<td>7334837.4</td>
<td>7484225.2</td>
<td>7236668.3</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>11830780.5</td>
<td>11324890.8</td>
<td>11199511.3</td>
</tr>
<tr>
<td><strong>Deposits</strong></td>
<td>Mean</td>
<td>5739474</td>
<td>5820687.6</td>
<td>5821677.7</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>9368730.9</td>
<td>8878535.9</td>
<td>9062205.7</td>
</tr>
<tr>
<td><strong>Loans</strong></td>
<td>Mean</td>
<td>4304088.1</td>
<td>4649243.5</td>
<td>4388002.6</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>6799117.2</td>
<td>7055150.3</td>
<td>6564710.6</td>
</tr>
<tr>
<td><strong>Securities</strong></td>
<td>Mean</td>
<td>1423636</td>
<td>1505656</td>
<td>1620629.4</td>
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<tr>
<td></td>
<td>Std. dev</td>
<td>2901118.6</td>
<td>2971998.3</td>
<td>3053082.1</td>
</tr>
</tbody>
</table>

Note: The table presents descriptive statistics (mean values and standard deviation) for all banks in our sample. (*) Values are in thousands of Euros.

Table 2: Overall Malmquist Productivity Index

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Overall Process</td>
<td>Value Added</td>
<td>Profitability</td>
<td>Overall Process</td>
</tr>
<tr>
<td><strong>MPI</strong></td>
<td>1.018</td>
<td>0.991</td>
<td>1.107</td>
<td>1.018(3)</td>
</tr>
<tr>
<td><strong>eff</strong></td>
<td>1.045</td>
<td>0.998</td>
<td>1.176</td>
<td>0.983</td>
</tr>
<tr>
<td><strong>tech</strong></td>
<td>0.975</td>
<td>0.999</td>
<td>0.944</td>
<td>0.975</td>
</tr>
<tr>
<td><strong>scale</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>bias</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: The numbers inside the brackets indicate the number of infeasible scores.
Table 3: Malmquist Productivity Index per Country

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPI¹</td>
<td>eff</td>
<td>tech</td>
<td>scale</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
<td>-------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>7</td>
<td>1.001</td>
<td>0.998</td>
<td>1.009</td>
</tr>
<tr>
<td>Croatia</td>
<td>15</td>
<td>1.022</td>
<td>1.031</td>
<td>1.000</td>
</tr>
<tr>
<td>Czech Rep.</td>
<td>10</td>
<td>1.003</td>
<td>0.974</td>
<td>1.041</td>
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<tr>
<td>Estonia</td>
<td>2</td>
<td>1.006</td>
<td>0.954</td>
<td>1.066</td>
</tr>
<tr>
<td>Hungary</td>
<td>6</td>
<td>1.046</td>
<td>1.028</td>
<td>1.039</td>
</tr>
<tr>
<td>Latvia</td>
<td>7</td>
<td>1.021</td>
<td>1.008</td>
<td>1.011</td>
</tr>
<tr>
<td>Lithuania</td>
<td>4</td>
<td>1.001</td>
<td>0.984</td>
<td>1.018</td>
</tr>
<tr>
<td>Poland</td>
<td>8</td>
<td>1.074</td>
<td>0.995</td>
<td>1.105</td>
</tr>
<tr>
<td>Romania</td>
<td>11</td>
<td>1.008</td>
<td>0.987</td>
<td>1.028</td>
</tr>
<tr>
<td>Slovakia</td>
<td>6</td>
<td>0.962</td>
<td>0.908</td>
<td>1.060</td>
</tr>
<tr>
<td>Slovenia</td>
<td>9</td>
<td>0.983</td>
<td>0.973</td>
<td>1.019</td>
</tr>
</tbody>
</table>

Table 4: β-convergence coefficients

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>Overall model</th>
<th>1st stage</th>
<th>2nd stage</th>
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<tbody>
<tr>
<td></td>
<td>β-convergence</td>
<td>β-convergence</td>
<td>β-convergence</td>
</tr>
<tr>
<td>1st-2nd</td>
<td>-0.906***</td>
<td>-1.290***</td>
<td>-0.940***</td>
</tr>
<tr>
<td>2nd-3rd</td>
<td>-1.273***</td>
<td>-0.770***</td>
<td>-1.022***</td>
</tr>
</tbody>
</table>

Note: *** The coefficient is statistically significant at 0.001.
Figure 1: Two-stage bank process

![Diagram showing two-stage bank process](image)

Figure 2: β-convergence and density functions for the overall model.

### Figure 2a
**β-Convergence of productivity growth, 2007-2010**

![Graph showing β-convergence of productivity growth](image)

### Figure 2b
**Density of productivity change, 2007-2010**

![Graph showing density of productivity change](image)

### Figure 2c
**β-Convergence of productivity growth, 2008-2012**

![Graph showing β-convergence of productivity growth](image)

### Figure 2d
**Density of productivity change, 2008-2012**

![Graph showing density of productivity change](image)
Figure 3: $\beta$-convergence and density functions for the first stage.
Figure 4: \( \beta \)-convergence and density functions for the second stage.