Patterns of energy-dependent variability from Comptonization

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ABSTRACT
We study fractional variability as a function of energy from black hole X-ray binaries on time-scales from milliseconds to hundreds of seconds. We build a theoretical model of energy-dependent variability in which the X-ray energy spectrum varies in response to a changing physical parameter. We compare these models to rms spectra obtained from RXTE Proportional Counter Array (PCA) observations of black hole binaries XTE J1550–564 and XTE J1650–500. We show that two main variability models are consistent with the data: variable seed photon input in the hard state and variable power in the Comptonized component in the soft and very high states. The lack of clear reflection features in the rms spectra implies that the reflection and the X-ray continuum, when integrated over Fourier frequencies, are correlated and vary with similar fractional amplitudes. Our models predict two important features of rms spectra, not possible to be clearly seen by the PCA due sensitivity limits. At soft X-rays, \( \lesssim 3 \text{ keV} \), we predict the presence of a break in the rms spectrum at energy directly related to the seed photon temperature. At higher energies, \( \sim 20–30 \text{ keV} \), we predict a peak in the rms spectrum originating from the variability of the spectrum produced by a hybrid thermal/non-thermal electron distribution. If these features are confirmed by broad-band observations, they will impose important constraints on the origin of the seed photons for Comptonization and the electron distribution in the hot plasma.


1 INTRODUCTION
X-ray emission from accreting black holes is commonly thought to originate from inverse Compton scattering of cooler disc photons in a hot optically thin plasma. Depending on the geometry of the accretion flow and the distribution of power between the disc and the hot plasma, a variety of spectral distributions can be produced. This translates into a variety of observed spectral states. One approach to understanding the physics of accretion is by fitting various models to the time-averaged energy spectra. The spectral decomposition of the data is fairly well understood and typically requires a model consisting of disc emission, its Comptonization and Compton reflection of the hard X-ray photons from the disc (see e.g. Zdziarski & Gierliński 2004 and references therein).

Another approach to the X-ray data is by analysing their variability on various time-scales. Fast aperiodic variability on the time-scales from milliseconds to hundreds of seconds is often studied using power density spectra (PDS), in particular, by tracing quasi-periodic oscillations (QPO). For a review, see van der Klis (2004).

Most of this variability occurs on dynamical time-scales in the inner part of the accretion flow, so it makes an excellent probe of the deep gravitational potential around the compact object. However, despite huge amounts of available data, we are still far from understanding how the rapid X-ray variability is produced. Many existing models propose oscillations in the accretion disc (e.g. Cui, Zhang & Chen 1998; Titarchuk, Osherovich & Kuznetsov 1999; Psaltis & Norman 2000) as the origin of variability. Though these models have been successful in explaining observed characteristic frequencies, it is not clear how oscillations of the disc are translated into varying X-rays. Giannios & Spruit (2004) proposed that QPOs in the inner hot flow can be excited by interaction between the flow and the outer cold disc (e.g. by a cooling–heating feedback loop). Bursa et al. (2004) suggested that hard X-rays can be modulated via gravitational lensing of the oscillating accretion flow in the vicinity of the black hole. Another suggestion involves dense cold blobs of material drifting through the inhomogeneous hot inner flow and providing a variable source of seed photons for Comptonization (Böttcher & Liang 1999). An alternative set of models that can directly explain the origin of X-ray modulation invokes propagation of X-ray flares in the accretion flow (e.g. Böttcher & Liang 1998; Poutanen & Fabian 1999; Zycki 2003).

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There are a few possibilities of bridging over the fairly well understood energy spectra and the enigmatic variability. One of them is to study how X-ray energy spectra change with the Fourier frequency, by means of the so-called frequency-resolved spectroscopy. A handful of bright objects have been studied this way, both containing black holes (Revnivtsev, Gilfanov & Churazov 1999b, 2001) and neutron stars (Gilfanov, Revnivtsev & Molkov 2003). A number of important conclusions have been obtained from these data. One particularly interesting result is that the strength of Compton variability depends on the underlying energy spectra and the enigmatic variability. The simplest approach is by looking at the fractional root mean square variability amplitude (integrated over a range of frequencies or time-scales) as a function of energy, \( \text{rms}(E) \), or in other words, a relative variability spectrum. Such a spectrum can tell us about how the spectral components (disc or Comptonization) vary with respect to each other and whether they change their spectral shape on the observed time-scales. Variability spectra have been recently obtained both from Galactic and supermassive black holes (e.g. Revnivtsev, Borozdin & Emel'yanov 1999a; Lin et al. 2000; Wardziński et al. 2002; Vaughan & Fabian 2004). Also, the amplitude of QPOs as a function of energy has also been studied (e.g. Rao et al. 2000; Gilfanov et al. 2003; Rodriguez et al. 2004a,b). However, until recently, little theoretical interpretation was given. Zdziarski (2005) proposed a theoretical model considering radial dependence of the local variability in the disc. By assuming the local rms and disc temperature decreasing with increasing radius, he found that \( \text{rms}(E) \) increasing with energy, consistent, for example, with an ultrasoft state of GRS 1915+105. Zdziarski et al. (2002, hereafter Z02), analysed the spectral variability of Cyg X-1 from the RXTE/All-Sky Monitor (ASM) and Compton Gamma Ray Observatory (CGRO)/BATSE on time-scales of days and months and reported two distinct patterns of \( \text{rms}(E) \) in different spectral states. They proposed that, in the hard state, the variability was driven by changes in the inner radius of the truncated disc, which in turn varied the seed photon input for Comptonization. In the soft state, it was brought about by variations of the power released in a hot corona above the disc.

In this paper, we investigate patterns of \( \text{rms}(E) \) variability generated by variations of the physical properties of the accretion flow. We consider a particular spectral model of a hybrid, thermal/non-thermal Comptonization and study the effects of varying parameters of this model. We compare the results with rms spectra of rapid X-ray variability of two Galactic black holes, XTE J1650–500 and XTE J1550–564.

We would like to point out that our variability spectra have their own limitations. First, we have chosen to integrate them over a wide range of Fourier frequencies (those available to the RXTE/PCA instrument; see Section 2), while the spectral dependence on frequency has been shown to be important in some cases (see, for example, Revnivtsev et al. 1999b). We note that it is entirely possible to create \( \text{rms}(E) \) integrated over a narrow range of frequencies and then that \( \text{rms}(E) \) times the average spectrum will be equal to the corresponding frequency-resolved spectrum. However, such an analysis is beyond the scope of the present work because it would increase the dimension of the parameter space, thus leading to a substantial increase in the complexity of the study. Also, that approach requires data of significantly higher statistics than those required in the case of integration over all available frequencies. Consequently, the results on interpretation of X-ray data presented here are mostly valid for the range of frequencies dominating the power spectrum (typically \( \sim 0.1–10 \text{ Hz} \); see Section 5). On the other hand, our theoretical results are general and can be directly applied to frequency-resolved \( \text{rms}(E) \) in future work.

Furthermore, neither \( \text{rms}(E) \) nor frequency-resolved spectra carry information about either phase/time lags (for a review see Poutanen 2001 and references therein) or coherence (Vaughan & Nowak 1997) between signals at different energies at a given Fourier frequency. Also, our theoretical interpretation of \( \text{rms}(E) \) is based on a one-component model of Comptonization. Thus, this approach does not deal with, for example, propagation effects, likely to be important in actual flows (e.g. Kotov, Churazov & Gilfanov 2001).

2 DATA REDUCTION

We have analysed several observations (with unique identifiers, called obsids) of XTE J1650–500 and XTE J1550–564 from the RXTE PCA and High Energy X-ray Timing Experiment (HEXTE) detectors. For data reduction, we used ftools 5.3. We extracted energy spectra from the top layer of the detector 2 of the PCA and added a 1 per cent systematic error in each channel. We extracted HEXTE spectra from both clusters. For energy spectra, we use the PCA data in 3–20 keV band and the HEXTE data in 20–200 keV band.

We extracted rms spectra from the PCA data using the following approach (see also Zdziarski et al. 2005). First, we extracted light curves with 1/256-s resolution for the PCA absolute energy channels 0–71 (corresponding to energies from \( \sim 2 \) to about 25–30 keV, depending on the PCA epoch). Some of the channels were binned together to improve statistics. Then, we calculated PDS from each of the light curves (over 512-s intervals), subtracted the Poissonian noise and corrected for dead-time effects (Revnivtsev, Gilfanov & Churazov 2000) and background (Berger & van der Klis 1994). The energy-dependent rms was found by integrating the PDS over the (1/512)–128 Hz frequency band.

We would like to stress the importance of background correction of the power spectra. The fractional rms we use in this paper is the standard deviation divided by the mean source count rate, which obviously must exclude the background count rate (see also Berger & van der Klis 1994). The background variability is assumed to be Poissonian and is subtracted from the PDS. At higher energies, \( \gtrsim 30 \text{ keV} \), estimating the PCA background, which dominates most of the spectra, becomes less reliable and so does the calculated rms. We discuss possible effects of the high-energy background estimation is Section 5.3.

3 THE METHOD

Fractional variability spectra are the result of the observed flux varying differently at different energies. In order to devise any theoretical model of energy-dependent variability, one has to make certain assumptions about the energy spectrum. We do so by fitting the observed spectra by a physically motivated model.

3.1 The Comptonization model

In this paper, we use a spectral model consisting of a soft component, modelled by the multicolour blackbody disc emission (Mitsuda et al. 1984), Comptonization of seed photons in the hybrid plasma (eqpair and reflection of the Comptonized photons from the cold disc (Magdziarz & Zdziarski 1995). The model eqpair (Coppi 1999;
Gierliński et al. (1999) calculates self-consistently microscopic processes in a hot plasma with electron acceleration at a power-law rate with an index $\Gamma_{\text{ion}}$, in a background thermal plasma with a Thomson optical depth of ionization electrons, $\tau$. The electron temperature, $T_e$, is calculated from the balance of Compton and Coulomb energy exchange, taking into account pair production as well. The last two processes depend on the plasma compactness, $\ell \equiv L \sigma_T / (R m_e c^2)$, where $L$ is a power supplied to the hot plasma, $R$ is its characteristic size and $\sigma_T$ is the Thomson cross-section. We then define the hard compactness, $\ell_h$, corresponding to the power supplied to the electrons, and the soft compactness, $\ell_s$, corresponding to the power in soft seed photons irradiating the plasma (which are assumed to be emitted by a blackbody disk with the maximum temperature, $T_{\text{disk}}$). The compactness corresponding to the electron acceleration and to a direct heating of the thermal electrons is denoted as $\ell_{\text{nth}}$.

We then use the best-fitting spectral model to build rms($E$) models, which we can compare with variability data. Here, we apply two different approaches to rms($E$) spectra: a simple ‘two-component’ model that has been used in previous works and a novel variable parameter idea, based on the physics of the emission of the accretion flow.

3.2 Two-component variability

The simplest approach to energy-dependent variability is to consider a number of spectral components with different variability amplitudes. We assume in this paper that the energy spectrum consists of two components: the soft (blackbody disc) and the hard (Comptonization). We neglect reflection variability in this model, but discuss its possible effects in Section 7. When the spectral components vary with different variances they create energy-dependent variability, as a fraction of each component in the total spectrum changes as a function of energy. The total variance at a given energy, $E$, is

$$\sigma^2(E) = \sigma^2_s(E) + \sigma^2_h(E) + 2\sigma_{\text{sh}}(E),$$

where $\sigma^2_s(E)$ and $\sigma^2_h(E)$ are the soft and hard variances and $\sigma_{\text{sh}}(E)$ is their covariance. In the following approach, we assume that the soft and hard component variances are fully correlated, so $\sigma_{\text{sh}}(E) = \sigma_s(E)\sigma_h(E)$ and thus

$$\sigma^2(E) = \sigma_s(E) + \sigma_h(E).$$

We define fractional rms variability as $\text{rms}(E) \equiv \sigma(E) / F(E)$, where $F(E)$ is the time-averaged flux. The total fractional rms variability is

$$\text{rms}(E) = \frac{r(N_s) F_s(E)}{F(E)} + \frac{r(N_h) F_h(E)}{F(E)}.$$

The $F(E)$, $F_s(E)$ and $F_h(E)$ are taken from the best-fitting spectral models, while $r(N_s)$ and $r(N_h)$ are free parameters. As fractional variabilities of the normalization, $r(N_s)$ and $r(N_h)$ are energy-independent quantities.

The two-component or multicomponent approach has been used for variability studies before (e.g. Rao et al. 2000; Vaughan & Fabian 2004; Zdziarski et al. 2005). However, there are obvious caveats of this model, as it does not take spectral variability of Comptonization into account. Clearly, where the physical properties of the accretion flow change, we expect, in particular, the spectral slope of Comptonization to vary as well.

3.3 Parameter variability

We have created another model based on variability of physical properties of the accretion flow. Our spectral model (Section 3.1), consisting of the multicolour blackbody disc and Comptonization (again, we neglect the effects of reflection on variability), is a function of several parameters: $F = F(p_1, p_2, \ldots, p_n, E)$. We allow for variation of a given parameter $p_i$ with Gaussian distribution (but we test lognormal distribution as well) around its best-fitting mean value $\overline{p_i}$ with standard deviation $\sigma(p_i)$ or fractional rms $\sigma(p_i) / \overline{p_i}$. This, in turn, causes the whole X-ray spectrum to vary, generating various patterns of energy-dependent variability. We concentrate on varying soft ($\ell_s$) and hard ($\ell_h$) compactness of Comptonization, which correspond to varying luminosity or power released in the seed photons and electrons, respectively. We also check for effects of other parameters varying.

![Figure 1](https://academic.oup.com/mnras/article-abstract/363/4/1349/1047912/fig1)

**Figure 1.** (a) Hard-state (obsid 60113-01-04-00) rms spectrum of XTE J1650–500 observed by RXTE. The dashed curve (red in the colour version of the figure online) represents the two-component variability model, with $r(N_s) = 0.36$ and $r(N_h) = 0.18$, clearly not consistent with the data. The solid (blue in colour) curve shows the model with varying soft photon input (dotted curve, cyan in colour) at $\ell_s = 0.25$ plus an additional hard component normalization variability (dash–dotted curve, green in colour) with $r(N_h) = 0.08$. (b) Energy spectrum of the same observation, with the unfolded PCA and HEXTE data together with the best-fitting model. The model consists of the following components: the soft component modelled by DISKBB (dotted curve, red in colour, showing unscattered seed photons only), its Comptonization in thermal plasma (dashed curve, blue in colour) and reflection (dash–dotted curve, green in colour). The solid curve (magenta in colour) shows their sum.
4 RESULTS

4.1 Hard state

Fig. 1(a) shows an rms spectrum of the black hole binary XTE J1650–500 in the hard X-ray spectral state. This type of spectrum, where fractional rms variability decreases with energy is common in the hard state of black hole binaries. Another common variability type in this state is a flat rms(E), which we discuss later in the paper. We are going to interpret the observed rms(E) making use of the energy spectral model fitted to the PCA/HEXTE data. The energy spectrum of this observation with its best-fitting model is presented in Fig. 1(b). It is well described ($\chi^2_r = 132/129$) by purely thermal Comptonization in a plasma with a hard-to-soft compactness ratio of $\ell_b/\ell_s = 3.8^{+0.2}_{-0.1}$ and an optical depth of $\tau = 1.82^{+0.13}_{-0.06}$. The self-consistently computed temperature of the electrons is 47 keV and the soft component temperature is $kT_s = 0.72^{+0.25}_{-0.19}$ keV. We also found Compton reflection with an amplitude of $\Omega/2\pi = 0.18 \pm 0.03$ and an ionization of the reflector of $\log(\xi/1\text{ erg cm s}^{-1}) = 3.1 \pm 0.2$. This kind of a spectrum is typical of black hole binaries (e.g. Gierliński et al. 1997; Zdziarski et al. 1998; Di Salvo et al. 2001) in the hard state (but see our discussion of the soft component in Section 5.1).

We first try the two-component model, considering soft and Comptonized components varying in luminosity, but not in spectral shape. When the energy spectrum is dominated by one component, the amplitude of variability does not depend on energy. A spectrum varying in normalization only yields the same fractional rms(E). Therefore, varying $\ell_s$ varies the amplitude of variability does not depend on energy. A spectral shape. When the energy spectrum is dominated by one component varying in luminosity, but not in spectral shape.

Fig. 2(a) shows an rms spectrum of XTE J1650–500 in the soft X-ray spectral state. The pattern of variability is distinctly different from the hard state in Fig. 1(a). It rises with energy and saturates above ~10 keV. We have looked through all rms spectra of XTE J1650–500 and found that this pattern is characteristic for all the soft-state spectra.

![Figure 2](https://academic.oup.com/mnras/article-abstract/363/4/1349/1047912)
The soft-state energy spectrum (Fig. 2b) is well described \( \chi^2_{\text{red}} = 115/117 \) by a hybrid Comptonization where electrons are injected into the plasma with a power-law distribution with the index fixed at \( \Gamma_{\text{inj}} = 2.5 \). The fraction of non-thermal power transferred to the electrons (as opposed to thermal heating) is \( \ell_{\text{inj}}/\ell_h = 0.84_{-0.03}^{+0.02} \). The best-fitting hard-to-soft compactness ratio is \( \ell_h/\ell_s = 0.38_{-0.15}^{+0.24} \) and the optical depth is \( \tau = 1.2_{-0.4}^{+0.6} \). The self-consistently computed temperature of the electrons is 11 keV. The disc temperature is \( kT_h = 0.54_{-0.02}^{+0.03} \) keV. We also found Compton reflection with an amplitude of \( \Omega/2\pi \ell_h = 0.22_{-0.12}^{+0.04} \) and a reflector ionization of \( \log(\xi/1 \text{ erg cm}^{-2} \text{ s}^{-1}) = 4.7_{-0.3}^{+0.3} \). This kind of a spectrum is typical of black hole binaries (e.g. Gierliński et al. 1999; Frontera et al. 2001) in the soft state.

Contrary to the hard state (Section 4.1), the rms spectrum is very well described by the two-component model, where disc and Comptonization are allowed to vary in normalization only. We show this model, with dominant variability of Comptonization, in Fig. 2(a), with the dashed (red in colour) curve. A characteristic feature of this model is a break at around 7 keV, above which Comptonization dominates the spectrum (see Fig. 2b) and \( \text{rms}(E) \) becomes flat. The energy of this break is directly related to the disc temperature and we found that \( \text{rms}(E) \) saturates at \( \sim 15kT_h \).

Another model that can explain \( \text{rms} \) increasing with energy is variable hard power input (Z02). This is opposite to the soft photon input variability in the hard state: this time, we kept \( \ell_h \) constant and allowed \( \ell_s \) to vary. This caused the energy spectrum to vary in a way depicted in Section 5 (see Fig. 4b later). The result for \( \ell(\ell_h) = 0.17 \) is shown by the dotted (cyan in colour) curve in Fig. 2(a). The \( \text{rms}(E) \) pattern predicted by this model matches the data well, except for the deficiency in \( \text{rms} \) below \( \sim 4 \) keV. We have accounted for this deficiency by adding little variability in the disc. The total \( \text{rms} \) spectrum (solid curve) matches the data well and is similar in shape to the two-component model. The energy of the break at \( \sim 10 \) keV is also directly related to the seed photon temperature.

As in the hard state, the lognormal distribution of \( \ell_h \) produced very similar results to the Gaussian distribution.

4.3 Very high state

Fig. 3(a) shows \( \text{rms} \) spectrum of another black hole XTE J1550–564 in the very high spectral state. Like in the soft-state spectrum in Fig. 2(a), the fractional variability increases with energy, though there is neither break nor saturation up to at least \( \sim 20 \) keV.

The very high state energy spectrum of XTE J1550–564 (Fig. 3b) is well described by hybrid Comptonization (see also Gierliński & Done 2003) with weak apparent contribution from unscattered disc photons. The best-fitting model \( \chi^2_{\text{red}} = 117/122 \) parameters are: hard-to-soft compactness ratio, \( \ell_h/\ell_s = 0.80_{-0.05}^{+0.08} \), non-thermal fraction, \( \ell_{\text{inj}}/\ell_h = 0.83_{-0.15}^{+0.06} \), an electron injection power-law index of \( \Gamma_{\text{inj}} = 3.3_{-0.2}^{+0.1} \), and an optical depth of \( \tau = 4.73_{-0.10}^{+0.30} \). The electron temperature is 4 keV and the disc temperature is \( kT_h = 0.52_{-0.09}^{+0.06} \) keV. The reflection amplitude is \( \Omega/2\pi \ell_h = 0.20_{-0.11}^{+0.16} \) and its ionization is \( \log(\xi/1 \text{ erg cm}^{-2} \text{ s}^{-1}) = 3.6_{-1.1}^{+0.2} \).

The two-component variability that has successfully described the soft-state \( \text{rms} \) spectrum cannot explain the very high state data, as it predicts a break at \( \sim 6 \) keV and flat \( \text{rms}(E) \) above the break (similar to the model shown in Fig. 2a). The flattening of \( \text{rms}(E) \) occurs always at energies where Comptonization dominates and does not depend on its spectral shape. On the other hand, the \( \ell_h \) variability model matched the data very well, though some additional hard component variability contribution was required, similar to the hard state. Interestingly, in the soft state, the \( \ell_h \) variability created an \( \text{rms}(E) \) pattern with a break at \( \sim 15kT_h \), which is not seen here until about 30 keV; again, using the lognormal distribution of \( \ell_h \) had very little effect on our results.

5 Patterns of variability

In Section 4, we have presented characteristic \( \text{rms} \) spectra in three X-ray spectral states and possible models matching these spectra. Certainly, the two-component model where the disc and Comptonization vary independently is only a zeroth-order approximation. Except for a specific case described later in Section 6, there should be feedback: a change in the disc luminosity changes the supply of the seed photons for Comptonization, which in turn should affect the spectral shape of the Comptonized component. Therefore, the models based on varying a parameter in a physical model where the disc and Comptonization are linked together should provide a better physical picture of variability.

In Fig. 4, we visualize model spectral variability in each spectral state, with varying seed photon input and hard compactness. For simplicity, we do not include additional variability components required by the data (see Figs 1, 2 and 3).
Figure 4. Visualization of model spectral variability (dotted curves, red in the colour version of the figure online) with respect to the best-fitting spectrum in a given state (solid black curve). (a) Hard state: variability of the soft photon input, $\ell_s$. (b) Soft state and (c) very high state: variability of the hard power, $\ell_h$.

Figure 5. Characteristic patterns of rms($E$) variability observed from two Galactic black hole binaries. The upper row shows the evolution XTE J1650–500 during its outburst and transition from the hard (a) through the intermediate (b)–(c) to the soft (d)–(e) spectral state. Spectra from observations 60113-01-X are shown, where X is: (a) 04-00, (b) 08-00, (c) 12-00, (d) 13-01 and (e) 24-00. The lower row shows the evolution of XTE J1550–564 in the beginning of its 1998 outburst and transition from the hard to the very high state. Spectra from observations 30188-06-X are shown, where X is: (f) 01-00, (g) 01-02, (h) 04-00, (i) 06-00 and (j) from 30191-01-00.

In Fig. 5, we show several characteristic rms spectra from XTE J1650–500 and XTE J1550–564 covering hard, soft and very high spectral states. We also show the corresponding PDS in Fig. 6. These PDS were extracted over the same frequency band [(1/512)–128 Hz] as those used for the creation of the rms spectra. To illustrate energy dependence, we show the low- ($\lesssim 13$ keV) and high-energy ($\gtrsim 13$ keV) power spectra. Clearly, in many cases not only the normalization but also the PDS shape changes as a function of energy.

Next, we discuss in detail possible variability models in each of the spectral states shown in Figs 5–6 and give their theoretical interpretation.

5.1 Hard state

In the hard state, the rms spectrum is either flat (Figs 5f and g) or smoothly decreasing with energy (Figs 5a and b). The flat rms($E$) simply corresponds to a situation where the entire spectrum (or Comptonization only, when the disc is not visible in the observed bandwidth) varies in normalization (luminosity) but not in spectral shape (see also Z02). If we assume that luminosity variations are due to changes in the accretion rate, then the only spectral effect we might expect is weak variation in the shape of the high-energy cut-off $\gtrsim 100$ keV, as the optical depth varies following variations in the flow density. We have considered a particular model of the advection dominated flow, where $\tau \propto L^{2/7}$ (Zdziarski 1998). The result is shown in Fig. 7(a) with a solid grey (red in colour) curve. The minimum at $\sim 300$ keV is due to a fortuitous intersection of the spectra resulting from this variability prescription at that energy. Additionally, we have checked the result of replacing the thermal Comptonization with that with fully non-thermal injection ($\ell_{\text{nth}}/\ell_h = 1$), which turns out to be negligible at $\lesssim 100$ keV (dotted curve).

The other pattern observed in the hard state is the rms smoothly decreasing with energy. Z02 explained a similar pattern observed on much longer time-scales by variable seed photon input, $\ell_s$. We found this solution consistent with our observations. The result of this model is plotted in Fig. 7(a) with a solid grey (red in colour) curve. The characteristic feature of this type of variability is the pivoting of the spectrum around $\sim 20–50$ keV (Fig. 4a). Because of that, the rms($E$) reaches a very deep minimum around this energy, so the decline in rms above $\sim 5$ keV is very steep and steeper than that observed (see Fig. 1a). This can be understood when we notice that, in our model, we assumed variations in $\ell_s$ only, with Comptonizing plasma simply responding to these changes in the seed photons,
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Figure 6. The power density spectra corresponding to the rms \(E\) dependencies shown in Fig. 5. The low- and high-energy PDS are shown in black and grey (cyan in the colour version of the figure online), respectively. The low-energy band corresponds to the PCA absolute channels 0–25 (XTE J1650–500) and 0–35 (XTE J1550–564). The high-energy band corresponds to the channels above those up to the channel 71. The energies corresponding to these two bands are \(\sim 2\) to \(\sim 13\) and \(\sim 13\) to \(\sim 25\) keV, respectively. The spectra have been rebinned for clarity. Note the maximum of the frequency times power shown here corresponds to the maximum of the variability power per log of frequency.

Figure 7. Models of rms \(E\) variability in the hard state. (a) Variability of soft photon input, \(r(\ell_s) = 0.2\), and of the total luminosity, \(r(L) = 0.3\), assuming \(r \propto L^{2/7}\). (b) Variability of seed photon temperature, \(r(T_s) = 0.05\), and of optical depth, \(r(\tau) = 0.5\). Solid curves correspond to the best-fitting thermal model from Section 4.1; dotted curves correspond to the same model, but with non-thermal fraction set to 1. The dashed curve in panel (a) represents \(\ell_s\) variability of an alternative (thermal) model with the seed photon temperature set to \(0.1\) keV. While the data required some variation in the power released in the Comptonized component, \(\ell_b\). The full treatment of this problem within our model would require two-dimensional variation in \(\ell_s\) and \(\ell_b\), with some particular relation between the two parameters assumed. For the sake of simplicity, in Section 4.1 we allowed for variations in just one parameter \(\ell_s\) and added a separate rms\(E\) component corresponding to variations in the hard component luminosity. This may correspond to an intermediate case between the flat-rms constant spectral shape model and pure seed photon input variations.

The dotted curves in Fig. 7 show the effect of non-thermal acceleration in the hot plasma. Because it is dominated by the Compton cooling (and often termed as a photon-starved plasma), the electrons in the hard state are efficiently thermalized even when the power provided to them is entirely in the form of non-thermal acceleration (Zdziarski, Coppi & Lamb 1990; Coppi 1999). The effect on the energy spectrum is visible only at high energies and difficult to measure (McConnell et al. 2002). Our PCA/HEXTE spectrum from Section 4.1 can be fitted by Comptonization with thermal and non-thermal electron injection equally well. The rms\(E\) of the thermal spectrum tends to rise steeply at around high-energy cut-off. This is because the spectrum around the cut-off varies in the direction perpendicular to the curve representing the spectrum (Fig. 4a), due to changes in the electron temperature as it adjusts itself to satisfy the energy balance (Z02). However, when a slight non-thermal tail is added to the spectrum, the rms does not increase that dramatically and the high-energy variability pattern resembles that in the soft state (Fig. 4b).

We have also investigated rms spectra resulting from varying seed photon temperature and optical depth of the Comptonizing plasma. The results (Fig. 7b) are highly inconsistent with observations, restricting our models to the variability in \(\ell_s\) and luminosity, as discussed previously.

An interesting feature of the \(\ell_s\) variability in the hard state is its dependence on the seed photon temperature. The rms becomes constant below a certain energy, directly related to the temperature of the seed photons. In our best-fitting model from Section 4.1, the seed photons originate from the soft component of temperature of \(\sim 0.7\) keV and the rms\(E\) flattens below \(\sim 2\) keV (Fig. 4a). We would like to point out that the soft component in this particular model cannot be a standard Shakura–Sunyaev disc (Shakura & Sunyaev 1973), as it is too hot \((\sim 0.7\) keV) and too small \([R_{\infty} \approx 18\) km for a distance of 4 kpc (Tomskic et al. 2003) and inclination of 30° (Sánchez-Fernández et al. 2002)\] for a disc. Instead, we probably see the so-called soft excess, an additional thermal Comptonization component hotter than the disc (see, e.g. Di Salvo et al. 2001; Frontera et al. 2001). The dashed curve in Fig. 7(a) shows an
alternative model in which the seed photon temperature was 0.1 keV and the soft excess was fitted as an additional component (the variability of which was not modelled). Clearly, the slope of $\text{rms}(E)$ below $\sim 2$ keV is distinctly different from the model in which the seed photons came from the observed soft excess. It is arguable whether the flattening of $\text{rms}(E)$ at low energies is present in the hard-state data (Fig. 1a) and additional data from instruments sensitive below 1 keV (e.g. XMM-Newton or Chandra) are required to confirm its veracity. If flattening is real then the seed photons are not from the disc (or at least not entirely from the disc) but from the soft excess. This can yield crucial constraints on the origin of the soft excess and geometry of the accretion flow.

5.2 Soft state

Soft-state rms spectra are distinctly different from hard-state ones. While the hard state typically showed either flat rms or a decrease in power with energy, $\text{rms}(E)$ increases and then saturates at higher energies in the soft state (see also Zdziarski et al. 2005). Similar spectral changes have been also reported in the variability of QPOs (e.g. Rodriguez et al. 2004a). Figs 5(b), (c) and (d) show transition from the hard to the soft state. An interesting feature of these spectra is the break at $\sim 2$–4 keV, present throughout the transition.

Using our best-fitting non-thermal spectral model, we have found in Section 4.2 two similar $\text{rms}(E)$ models that match the observed rms spectrum. The first one assumed variability in the hard component normalization, $N_h$, only. It produces a quick increase in rms and saturation above a few keV, as shown in Fig. 8(a) with the black curve. The grey (cyan in colour) curves show the effect of different $T_s$.

The second model involves constant $\ell_s$ and variable $\ell_h$. Unlike the previous model, it takes into account spectral response of Comptonization to the changing ratio of the power released in the corona to that in the disc. On the other hand, changes in the spectral shape of Comptonization (at least in the non-thermal case) are rather small, so the rms spectrum produced by the $\ell_h$ variability is similar to the one from $N_h$ variability: a quick rise and saturation at higher energies (Fig. 8b).

Our models do not always predict a flat rms above the break energy. In Fig. 8(b), we also show (dashed and dot–dashed curves) the effect of decreasing the fraction of non-thermal acceleration in the total power, $\ell_{\text{nth}}/\ell_h$. As the thermal heating becomes more important, a peak around 20–30 keV in $\text{rms}(E)$ is created. We discuss the origin of this peak in the next section.

A common property of both models is a break in the spectrum followed by flat $\text{rms}(E)$ at higher energies. The formation of the break and flat rms can be seen in the XTE J1650–500 data following its evolution from the intermediate to soft state in Figs 5(b)–(d). As mentioned previously, the flat $\text{rms}(E)$ is due to a spectral component varying in normalization but not in shape. The break corresponds to an energy in the spectrum above which Comptonization dominates, which is roughly at 15$kT_s$ (the best-fitting seed photons temperature was 0.54 keV).

As in the hard state, variability of either the seed photon temperature, $T_s$, or the optical depth of the hot plasma, $\tau$, do not provide $\text{rms}(E)$ patterns consistent with the data (Figs 8a and c). ZO2 considered variability in the hardness of the power-law electrons injected into the hot plasma. This can be done in two ways, either by varying the injection index, $\Gamma_{\text{inj}}$, or by varying the high-energy cut-off in the electron distribution represented by the maximum Lorentz factor, $\gamma_{\text{max}}$. ZO2 concluded that the first pattern was inconsistent with the colour–colour and colour–luminosity correlations, while the last one provided a good fit, though they have not analysed rms spectra emerging from these patterns. Here, we calculate the exact form of $\text{rms}(E)$ both for varying $\Gamma_{\text{inj}}$ and $\gamma_{\text{max}}$. We find that both of them create very similar rms spectra and we show one of them ($\Gamma_{\text{inj}}$) in Fig. 8(d). Both patterns are characterized by a strong rms peak around $\sim 10$ keV and are clearly inconsistent with any rms spectrum found by us so far. Therefore, we conclude that neither $\Gamma_{\text{inj}}$ nor $\gamma_{\text{max}}$ variability can explain the observed $\text{rms}(E)$ patterns.

5.3 Very high state

The very high state energy spectrum is dominated by a strong non-thermal Comptonized tail (Fig. 3b; see also Gierliński & Done 2003). The relative contribution from the disc is much weaker than in the soft state and the Comptonized tail is much softer than in the hard state. The rms spectra are also distinctly different. Figs 5(f)–(j) show evolution of the rms from the hard to the very high state.
The initial flat rms(E) becomes very steep, increasing with energy without any apparent breakup to at least $\sim 20$ keV.

We have investigated the same patterns of variability as in the soft state, using the best-fitting model from Section 4.3. The results are shown in Fig. 9. The simplest model of $N_h$ variability (or, more general, two-component variability) does not work here. As the entire energy spectrum is dominated by Comptonization, the resulting rms(E) is almost flat in this model, with a slight depression below $\sim 3$ keV. The observed strongly increasing rms(E) requires the Comptonized tail to vary in spectral shape, not only in normalization. In Section 4.3, we have found that $\epsilon_h$ variability matches the data well. Fig. 9(b) now shows the dependence of $\epsilon_h$ variability models on the seed photon temperature and non-thermal fraction.

A common feature of $\epsilon_h$ variability patterns in the soft and very high states is the formation of a peak in the rms spectrum at about 20–30 keV when thermal heating is present in the Comptonizing plasma, i.e. when $\epsilon_{\text{nth}}/\epsilon_h$ is less than 1. The origin of the peak can be understood from the decomposition of the hybrid Comptonization spectrum into thermal and non-thermal components (following the method of Hannikainen et al. 2005). This decomposition is shown in Fig. 10(a) for the very high state spectrum where the non-thermal fraction was set to $\epsilon_{\text{nth}}/\epsilon_h = 0.4$, corresponding to rms(E) shown in Fig. 10(b). The peak in rms(E) is due to variations in the high-energy cut-off of the thermal component. A similar kind of variability can be seen in the hard state (Fig. 4a) and it causes a dramatic increase in the rms(E), as seen in Fig. 7(a). However, in the case of the very high state, the increase is suppressed at higher energies, where the non-thermal component begins to dominate. With increasing contribution of thermal heating, the peak becomes stronger. This is an important feature of hybrid Comptonization and it will be interesting to see whether it is detected in future observations.

The very high state rms spectrum of XTE J1550–564 shown in Fig. 3(a) rises with energy until about 25 keV, i.e. until the last PCA energy channel we use in this paper. In Fig. 11, we show the same spectrum computed up to an energy of 67 keV. The last two data points (filled boxes) correspond to the rebinned absolute PCA data channels 72–89 and 90–174. There is a clear increasing trend in the rms, not consistent with the $\epsilon_h$ variability model. However, one should treat the PCA data above $\sim 30$ keV, where background contribution becomes important, with great caution. To test the accuracy of the background estimate, we have looked into our PCA/HEXTE energy spectrum with its best-fitting model from Section 4.3. We extended the PCA energy spectrum up to about 67 keV and compared additional energy channels with the best-fitting model to our standard PCA/HEXTE spectrum. It occurred that the PCA flux in additional high-energy channels was significantly lower than in the model, most likely due to the overestimated PCA background. The model-to-data ratio in the energy bins corresponding to the two additional points in Fig. 11 was 0.93 and 0.82. We have introduced
these corrections to the mean count rate in computed fractional rms (which is rms/mean). The corrected data points are shown in Fig. 11 by open triangles. Now they appear to be consistent with the $\ell_b$ variability model. We would like to stress that, by doing this, we have pushed the PCA data to the limits where systematic uncertainties are not very well known. Therefore, this result should be treated with caution.

Finally, we have calculated model variability patterns for varying $T_\odot$, $\tau$ and $\Gamma_{\infty}$ (Figs 9a, c and d). As it was the case in the soft state, we found them to be not consistent with the data.

### 6 ACCRETION FLOW GEOMETRY

Z02 considered a truncated disc geometry to explain the observed $\text{rms}(E)$ patterns. Within this model, in the hard state the accretion disc is truncated at some radius and replaced by a hot, optically thin, inner flow (e.g. Esin, McClintock & Narayan 1997; Poutanen, Krolik & Ryde 1997). Variations in the inner disc radius provide the necessary change in the soft photon input that can explain the observed $\text{rms}(E)$. However, Z02 analysed the long-term variability of Cyg X-1 on time-scales of days and months, much longer than the viscous time-scale, so significant variations in the inner disc radius are possible. This is not the case on the time-scales of milliseconds and seconds, observed here. The radial drift velocity in the accretion disc at 0.05 of the Eddington luminosity truncated at $20GM/c^2$ around a 10$M_\odot$ black hole is $\sim 4$ km s$^{-1}$ in the radiation pressure dominated zone (Shakura & Sunyaev 1973). The time-scale required to change the truncation radius by, for example, 10$GM/c^2$, is $\sim 20$ s. When the disc is truncated farther away, this time-scale is even longer. Clearly, there must be another mechanism for varying soft photon input.

Most of the rapid X-ray variability models assume some kind of oscillations in the accretion flow. It is not yet clear how these oscillations can be converted into the observed modulation of X-ray flux. Our result suggests that in the hard state this happens via modulation of the seed photons for Comptonization. In the model of Giannios & Spruit (2004), oscillations in the hot inner flow are excited by variations in the Compton cooling rate. During oscillations, the inner flow changes its Compton y parameter, which results in the pivoting of the Comptonized spectrum. This is in agreement with observations and our $\text{rms}(E)$ models. The drifting-blob model of Böttcher & Liang (1999), where the local seed photon input varies as the blob travels though inhomogeneous hot flow, predicts increase of the rms with energy, contrary to what is observed.

According to the truncated disc model in the soft (and probably very high) state, the cold disc extends down to the marginally stable orbit and the Comptonized emission originates from the active regions or corona above the disc (e.g. Gierliński et al. 1999; Poutanen & Fabian 1999). The $\text{rms}(E)$ patterns quickly increasing with energy are consistent with the stable disc and variable corona (see also Churazov, Gilfanov & Revnivtsev 2001). coronal flares produce most of the power at higher energies (Böttcher, Jackson & Liang 2003), so naturally we expect most variability at photon energies $\geq 10$ keV.

The two $\text{rms}(E)$ models we considered in Section 4.2 can be explained within the disc–corona geometry. The model with varying Comptonized normalization (but not spectral shape) can correspond to a scenario in which the covering fraction of the (patchy) corona varies, due to, for example, new flares being formed, and the $\ell_c/\ell_b$ ratio for each flare is roughly the same, so the spectral shape of Comptonization does not vary. However, with a changing number of flares (covering fraction), the luminosity of Comptonization would change; hence, the observed high-energy variability. Since the uncovered fraction of the disc changes as well, we should expect some variability in the disc. The very small disc variability found in this model (Fig. 2a) requires a small covering fraction of the corona, $\lesssim 0.1$.

The model with varying $\ell_b$ may correspond to changing power in the corona without changing the covering fraction. In the soft state of XTE J1650–500, it required $r(\ell_b) \approx 0.17$ and additional 0.07 variability in the disc (Fig. 2a). A quick estimate shows that this level of variability is expected from reprocessing of hard Comptonized photons in the disc. In the energy spectrum shown in Fig. 2(b), the Comptonized component luminosity, $L_c$, is roughly 0.4 of the disc luminosity, $L_d$. If we consider a corona or active regions above the disc, then we would expect less then a half of $L_b$ to be absorbed and re-emitted by the disc, which constitutes $\lesssim 0.2$ of $L_b$. Because the coronal variability is about 0.2, the expected variability of the disc from reprocessing is $\lesssim 0.04$, in rough agreement with the observed rms of 0.07. The remaining fraction of variability might be intrinsic to the disc.

### 7 REFLECTION

We have neglected the effects of Compton reflection in our variability models so far. Here, we perform a simple test of possible effects reflection variability can have on the rms spectra. To do this, we take our best-fitting model to the hard state from Section 4.1 as a template. This time, we take into account Compton reflection. We also assume the simplest possible two-component variability model, where the continuum (with the soft and hard components added) and the reflection vary only in normalization, not in spectral shape. When the continuum and reflection variabilities are correlated (i.e. the covariance $\sigma_{\text{c,r}} = \sigma_{\text{c}} \sigma_{\text{r}}$) and vary with the same relative amplitude, the resulting $\text{rms}(E)$ is obviously energy-independent (solid curve in Fig. 12). However, strong reflection-related features appear in the rms spectrum (dotted curve in Fig. 12), when continuum and reflection are uncorrelated ($\sigma_{\text{c,r}} = 0$). Similar features can be seen when they are correlated, but vary with different relative amplitudes (0.3 and 0.15 per cent for the continuum and reflection, respectively; dashed curve in Fig. 12). Certainly, this is a very simplified model, but similar features in $\text{rms}(E)$ are generated in our...
parameter-variability models in all spectral states when we allow for the reflection variability to be detached from the continuum variability, i.e. either uncorrelated, of different amplitude, or entirely independent.

We do not see any obvious such features in the observed rms spectra (Fig. 5) in any of the spectral states. This provides an important constraint on the relation between the irradiating X-ray continuum and Compton reflection. As they are well correlated, the reflected component must originate from Compton reprocessing of the observed continuum. This also means that the reflection variability (at the frequencies dominating the power spectra used in the analysis; see Fig. 6) is due to variability in the irradiating X-ray continuum and not due to changes in the properties of the reflector (e.g. waves or warps in the disc).

We have pointed out in Section 1 that the rms(E) spectra presented here have been integrated over a rather wide frequency range, (1/512)–128 Hz, whereas spectral dependence on frequency is possible. It has been known that the amplitude of Compton reflection can strongly decrease at high frequencies, \( \gtrsim 10 \) Hz, e.g. in the hard state of Cyg X-1 (Revnivtsev et al. 1999b; Gilfanov, Churazov & Revnivtsev 1999). However, those frequencies contribute relatively little to the integrated PDS used by us (Fig. 6). On the other hand, we note that reflection amplitude changing with frequency would still produce featureless rms(E) spectrum as long as the fractional variability of the reflection remains roughly constant with changing frequency.

8 CONCLUSIONS

We have explained various patterns of the observed energy-dependent X-ray variability in black hole binaries with a model in which the energy spectrum varies in response to a changing physical parameter. Our spectral model consisted of the disc emission and its hybrid (thermal/non-thermal) Comptonization. In the hard state, we found the decreasing rms(E) consistent with variations in the seed photon input (together with some variation in \( \epsilon_0 \)). In the soft and very high states, the data were consistent with varying power released in the hot Comptonizing plasma. Another model of the soft state involved varying coronal luminosity, without changing spectral shape.

Our models predict a few important features of the rms(E) spectra. The break in the rms(E) observed in the soft (and perhaps hard) state is directly related to the seed photon temperature. We estimate it to occur at \( \sim 15kT_e \). Thus, rms spectra extending to lower energies than those used in this paper might yield an important constraint on the origin of the soft excess observed in the hard state. In the very high state (and perhaps sometimes in the soft state), our models predict a strong peak in rms(E) at \( \sim 30 \) keV, related to the temperature of the thermal electrons in the hybrid plasma. If this peak is confirmed with high-energy data, it would strongly support the presence of hybrid electrons in the hot plasma. We stress that the presence of the peak requires hybrid, not just power-law electrons.

Lack of clear reflection features in the rms spectra implies that the reflection and the X-ray continuum are well correlated and vary with the same amplitude. Therefore, the reflected component originates from the reflection of the observed continuum indeed, and its rapid variability is due to changes in the irradiating continuum and not due to changes in the reflector properties. However, we stress that this result applies to the range of frequencies dominating the PCA power spectra of the studied objects, i.e. \( \lesssim 10 \) Hz.

Z02 studied energy-dependent patterns of variability from Cyg X-1, based on RXTE/ASM and CGRO/BATSE light curves on time-scales from days to months. In this work, we have extended this study to other black hole candidates and to much shorter time-scales, from milliseconds to hundreds of seconds. The observed dependence of fractional rms variability on energy is very similar in both cases, despite very different time-scales. Moreover, similar patterns have been observed from PDS components, like QPOs (e.g. Rodriguez et al. 2004a,b; Zdziarski et al. 2005), also in the case of neutron-star binaries (e.g. Gilfanov et al. 2003). Clearly, there is a common physical mechanism behind those rms(E) patterns. Their universality indicates their fundamental nature and importance for understanding the physics of accretion.

On the other hand, a significant difference in time-scales implies different physics. While Z02 explained hard-state variability by changing inner disc radius, it cannot operate on time-scales of milliseconds, which are much shorter than the viscous time-scale. In both cases, the common underlying mechanism is modulation of hard X-rays by the varying seed photon input, however the dynamical link between the cold disc and hot Comptonizing region must be different.

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REFERENCES

Mitsuda K. et al., 1984, PASJ, 36, 741

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