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# Dundee Discussion Papers in Economics

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## Moment Matching in the Present Value identity, and a New Model

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# Moment Matching in the Present Value identity, and a New Model.

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## **Abstract**

The constrained Vector Autoregression and the fairly recent state space approach are commonly used in the asset pricing literature to estimate present value models. They are used to model time series dynamics of discount rates and expected dividend growth, with the objective of understanding predictability and stock market movements. This paper shows that an ARMA(1,1) structure of price-dividend ratio and realized dividend growth nests an AR(1) specification for expected returns and expected dividend growth. A simpler model is proposed which involves estimating realized dividend growth and the price-dividend ratio as an ARMA(1,1), and matching the variance and autocorrelation of the estimated models to those of the present value to estimate parameters. Monte Carlo results show that the state space model has larger standard errors. Expected returns is persistent in both models, unlike expected dividend growth in the ARMA(1,1). A modest application of the model to the predictability literature shows stronger evidence towards dividend growth predictability.

**JEL-Classification:** G12, G17, C32

**Keywords:** Present Value, VAR, State Space, Moment Matching

## 1 Introduction

The present value model is particularly important for examining return and dividend growth predictability and stock market movements, for it allows both sign and size restrictions on estimated parameters. In its log-linear form, this model, popularized by Campbell and Shiller (1988), has given rise to an impressive literature geared towards asking two important questions in empirical finance: Are returns and dividend growth predictable by the price-dividend ratio ? And secondly, how much of the variation in stock prices corresponds to news about the discount rate and expected future dividend growth ? Recent literature in the field suggests three main findings: Both return and dividend growth are predictable. Discount rates are the main driver of movements in stock prices. Expected dividend growth and expected returns are positively correlated.

One of the most common procedures used to estimate present value parameters empirically is the Vector Autoregression (VAR), which models returns and dividend growth and the price-dividend ratio jointly. This model has been widely used to defend return or dividend growth predictability, and also to compute variance decompositions. An interesting but incomplete list of related papers include Campbell and Shiller (1988), Campbell and Ammer (1993), Vuolteenaho(2002) Campbell and Vuolteenaho (2004), Cochrane (2008), Chen and Zhao (2009), Engsted and Pedersen (2010), Engsted et. al(2012), Maio and Philip(2015). Besides providing an estimate for expected returns and expected dividend growth, it imposes the identity that returns and dividend growth cannot be *jointly* unpredictable from dividend yield.

Another approach to estimate present value is the fairly recent state space approach, which allows the derivation of time series of dividend growth and expected returns given the availability of information on realized dividend growth and the price-dividend ratio at a particular point of time. Koijen and Binsbergen (2010) and Rytchkov (2012) consider the

case where expected returns and expected dividend have an AR(1) specification. Expected returns and expected dividend growth are filtered from realized dividend growth and the price dividend ratio. When applied to US data, the corresponding time series for expected returns is very persistent and close to a unit root, while that of expected dividend growth rate showed moderate persistence. This approach has been used in other studies. For instance, Golinski et. al.(2015) show the case where return predictability regressions are misleading in the presence of a fractional order of integration. Similarly, Piatti and Trojani (2014) show that dividend growth unpredictability cannot be ruled out when proper inference by bootstrap is considered. Ma and Wohar (2014) compares the state space to the VAR in the UK market, to show that estimates from both models can vary across both models. Rambaccussing(2015) considers the case for the housing markets in US and UK.

One of the practical issues with the state space is that it is computationally demanding to estimate a global optima with a tight criterion imposed on the optimizer. In an unconstrained optimization problem, local optima are common especially given the number of parameters estimated compared to the sample size. Inference in such a model needs to be addressed properly by correct bootstrap methods. Another issue related to inference is the problem persistence of expected returns. Expected returns is found have a root close to unity. Asymptotic behaviour of those parameters are not well known for larger sample sizes. As shown in Ma and Wohar (2014), problems identification can lead to erroneous estimates. On the other hand, under conditions of stationarity, the state space can nest a VAR of infinite order, as shown in Cochrane(2008).

In this paper, a model to estimate the parameters from the state space present value is introduced to the literature. If the latent variables-expected returns and expected dividend growth are modelled as an AR(1), it is shown that the corresponding observable: realized dividend growth rate should follow an ARMA(1,1) process. In order to estimate expected returns components, the price-dividend ratio is modelled as an ARMA(1,1) with lagged realized dividend growth as an additional regressor. The autocovariance, autocorrelation and variance are computed and matched with their theoretical components. The model requires

important assumptions such as stationarity and setting the correlation between shocks from realized and expected dividend growth equal to zero, and shocks between expected returns and realized dividend growth.

The paper proceeds as follows. Section 2 introduces the present value in the context of the state space model. Section 3 illustrates the ARMA(1,1) counterpart. Section 4 illustrates evidence from the Monte Carlo. An application is considered for the US and UK market. Section 5 provides concluding remarks.

## 2 The Present Value Model

This section makes use of the Campbell-Shiller equation to show the relationship between returns, dividend growth and the price-dividend ratio :

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}, \quad (2.1)$$

where  $r_{t+1}$  and  $\Delta d_{t+1}$  denote returns from holding an asset and the corresponding dividend growth from period  $t$  to  $t + 1$ .  $pd_t$  is the price-dividend ratio from in period  $t$ .  $\kappa$  is a linearisation parameter defined as  $\kappa = \log(1 + \exp(\overline{pd})) - \rho \overline{pd}$ , and  $\rho = \frac{\exp(\overline{pd})}{1 + \exp(\overline{pd})}$ . In exante, the corresponding expectations of returns are defined as  $\mu_t = E_t(r_{t+1})$  and  $g_t = E_t(\Delta d_{t+1})$ . The present value in exante can be written in terms of expected returns and expected dividend growth over the lifetime of an asset. Assuming that there is no bubble, the price-dividend ratio is composed of the expectations of future returns and future dividend growth:

$$pd_t = \frac{\kappa}{1 - \rho} + \frac{1}{1 - \rho} (g_{t+1} - \mu_{t+1}). \quad (2.2)$$

(2.2) shows that any movements in the price dividend ratio can be due to changes in either expected dividend growth or expected returns, which are both unobservable between  $t$  and  $t + 1$ . Kojien and Binsbergen(2010) impose the case where expected returns and expected dividend growth rate follow an autoregressive process of order one. The specifications are

shown in 2.3 and 2.4 :

$$\widehat{\mu}_{t+1} = \delta_1 \widehat{\mu}_t + \varepsilon_{\mu,t+1}, \quad (2.3)$$

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon_{g,t+1}, \quad (2.4)$$

Equation 2.3 and 2.4 are demeaned autoregressive expected returns and expected dividend growth rate, where  $\widehat{\mu}_{t+1} = \mu_{t+1} - \delta_0$  and  $\widehat{g}_{t+1} = g_t - \gamma_0$ .  $\delta_1$  and  $\gamma_1$  are the autoregressive parameters, and are assumed to be less than one. There are two latent variable ( $\mu_{t+1}$  and  $g_{t+1}$ ) and one observed variable ( $pd_t$ ),. Dividend growth is chosen as the other observable. The realized growth rate is defined as the expected dividend growth rate plus the unobserved dividend shock  $\varepsilon_{d,t+1}$ :

$$\Delta d_{t+1} = g_t + \varepsilon_{d,t+1}. \quad (2.5)$$

The shocks to the expected returns, expected dividend growth, and realized dividend growth are assumed to be iid normally distributed:

$$\begin{pmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{g,t+1} \\ \varepsilon_{d,t+1} \end{pmatrix} \sim \text{i.i.d } \mathbf{N} \left( \mathbf{0}, \begin{pmatrix} \sigma_{\mu}^2 & \sigma_{g\mu} & \sigma_{d\mu} \\ \sigma_{g\mu} & \sigma_g^2 & \sigma_{dg} \\ \sigma_{d\mu} & \sigma_{dg} & \sigma_d^2 \end{pmatrix} \right).$$

In the state space framework, the measurement equations, are given by 2.2 and 2.5, and the transition equations are given by 2.3 and 2.4. The model is solved by optimizing the likelihood function from a Kalman Filter. Identification of the model requires that  $\sigma_{dg} = 0$ . The vector of parameters to be estimated is the following

$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_{\mu}, \sigma_x, \rho_{g\mu}, \rho_{\mu d})$$

where  $\rho_{g\mu}$  and  $\rho_{\mu d}$  denote the correlation between expected dividend growth and expected returns, and the correlation between expected returns and dividend growth respectively. Using estimated  $\Theta$ , series of expected returns and dividend growth can also be constructed.

## 2.1 ARMA (1,1) representation

The parameters from the present value model summarized by 2.2, 2.5, 2.3 and 2.4 can be estimated by matching the moments of the empirical regression where dividend growth is modelled as an ARMA(1,1) and the price-dividend ratio is modelled as an ARMA(1,1) with lagged dividend growth. The correlation between expected returns and realized dividend growth is set to zero for identification purposes. The latter assumption does not need to be imposed in the state space model. Equation 2.4 which shows expected dividend growth as an autoregression can be written as follows:

$$(1 - \gamma_1 L)\widehat{g}_{t+1} = \varepsilon_{g,t+1} \quad (2.6)$$

where  $\widehat{g}_{t+1} = g_{t+1} - \gamma_0$  and  $L$  is the lag operator. Lagging 2.6 by one period and rewriting in terms of  $\widehat{g}_t$  :

$$\widehat{g}_t = \frac{\varepsilon_{g,t}}{(1 - \gamma_1 L)} \quad (2.7)$$

Replacing (2.7) into (2.5) realized dividend growth may be rewritten as (2.8):

$$\begin{aligned} \Delta d_t &= \gamma_0(1 - \gamma_1) + \gamma_1 \Delta d_{t-1} + \varepsilon_{g,t-1} + \varepsilon_{d,t} \\ &\quad - \gamma_1 \varepsilon_{d,t-1} \end{aligned} \quad (2.8)$$

$$= \bar{\gamma} + \gamma_1 \Delta d_{t-1} + \eta_{dg,t} \quad (2.9)$$

where  $\bar{\gamma} = \gamma_0(1 - \gamma_1)$  and  $\eta_{dg,t} = \varepsilon_{g,t-1} + \varepsilon_{d,t} - \gamma_1 \varepsilon_{d,t-1}$ . (2.8) has an ARMA (1,1) structure.  $\eta_{dg,t}$  is the residual of the theoretical model which is a moving average of shocks

to expected and realized dividend growth. The autocovariance, variance and autocorrelation of  $\eta_{dg,t}$  are:

Autocovariance at first lag:

$$\lambda_0(1) = E(\eta_{dg,t}\eta_{dg,t-1}) = -\gamma_1\sigma_d^2$$

Variance: 2.6

$$var(\eta_{dg,t}) = \sigma_g^2 + (1 + \gamma_1^2)\sigma_d^2 \quad (2.10)$$

Autocorrelation at first lag:

$$\rho_{\eta_{dg,t}}(1) = \frac{-\gamma_1\sigma_d^2}{\sigma_g^2 + (1 + \gamma_1^2)\sigma_d^2} \quad (2.11)$$

Moreover, the variance of the observed dividend growth in terms of the autoregressive parameter  $\gamma_1$  and the variance of the shocks  $\sigma_g^2$  and  $\sigma_d^2$  is :

$$var(\Delta d_t) = \frac{\sigma_g^2 + (1 - \gamma_1^2)\sigma_d^2}{(1 - \gamma_1^2)} \quad (2.12)$$

Properties of  $\eta_{dg,t}$  can be estimated. Let us define the empirical counterpart of  $\eta_{dg,t}$  as  $v_{d,t}$  which is made up of a residual and its lag:

$$\eta_{dg,t} = v_{d,t} = u_t - \theta u_{t-1}$$

where  $u_t \sim N(0, \sigma_u^2)$ . The empirical counterpart to (2.8) is 2.13:

$$\Delta d_t = \gamma_0(1 - \gamma_1) + \gamma_1\Delta d_{t-1} + u_t - \theta u_{t-1} \quad (2.13)$$

The variance, autocovariance and autocorrelation of  $v_{d,t}$  are respectively equal to:

$$\text{var}(v_t) = (1 + \theta^2)\sigma_u^2, \quad (2.14)$$

$$\begin{aligned} E(v_{t-1}v_t) &= -\theta\sigma_u^2 \\ \rho_v(1) &= \frac{-\theta}{(1 + \theta^2)} \end{aligned} \quad (2.15)$$

$\gamma_1$  can be estimated directly from the slope coefficient of lagged dividend growth, while  $\gamma_0$  can be computed from the intercept term. To estimate  $\sigma_d$  and  $\sigma_g$ , the moving average components from the present value must be matched with the empirical components. The autocorrelation from the theoretical model is matched to (2.15) which yields the following:

$$\frac{-\gamma_1\sigma_d^2}{\sigma_g^2 + (1 + \gamma_1^2)\sigma_d^2} = \frac{-\theta}{(1 + \theta^2)}.$$

The signal to noise ratio  $R = \frac{\sigma_g^2}{\sigma_d^2}$ , is :

$$R = \gamma_1 \left( \frac{1}{\theta} + \theta \right) - (\gamma_1^2 + 1)$$

The variance of expected dividend growth is therefore equal to  $R\sigma_d^2$  :

$$\sigma_g^2 = [\gamma_1 \left( \frac{1}{\theta} + \theta \right) - \gamma_1^2 - 1]\sigma_d^2 \quad (2.16)$$

Replacing (2.16) into the variance equation (2.10), and equating it to the estimated variance of  $v_t$  (2.14) yields:

$$[\gamma_1 \left( \frac{1}{\theta} + \theta \right) - \gamma_1^2 - 1]\sigma_d^2 + (1 + \gamma_1^2)\sigma_d^2 = (1 + \theta^2)\sigma_{u,dg}^2$$

Variance of dividend growth is hence equal to:

$$\sigma_d^2 = \frac{\theta\sigma_{u,dg}^2}{\gamma_1} \quad (2.17)$$

(2.17) implies that  $\sigma_d^2$  can be computed from the estimated values of  $\theta$  and  $\gamma_1$ .  $\sigma_g^2$  is therefore equal to :

$$\sigma_g^2 = [\gamma_1 \left( \frac{1}{\theta} + \theta \right) - \gamma_1^2 - 1] \frac{\theta \sigma_u^2}{\gamma_1} \quad (2.18)$$

(2.6) - (2.18) show that parameters  $\gamma_0, \gamma_1, \sigma_g^2, \sigma_d^2$  can be derived from simply estimating 2.13. For the remaining parameters  $\delta_0, \delta_1, \sigma_\mu^2$  and  $\rho_{g\mu}$ , the model suggests estimating an alternative regression with the price-dividend ratio as the dependent variable. The reduced form links next period's price dividend ratio with the expected dividend growth and the shock terms. Equation 2.3 is lagged and rewritten in terms of the shock term only, and replaced in 2.2. The expected dividend growth 2.4 is replaced into 2.2 to derive the reduced form of the price dividend ratio:

$$pd_{t+1} = (1 - \delta_1)B_0 - B_2(\gamma_1 - \delta_1)\hat{g}_t + \delta_1 pd_t - B_1 \varepsilon_{\mu,t+1} + B_2 \varepsilon_{g,t+1}, \quad (2.19)$$

where  $B_0 = \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho}$ ,  $B_1 = \frac{1}{1-\rho\delta_1}$  and  $B_2 = \frac{1}{1-\rho\gamma_1}$ .

Lagging 2.19 and replacing the expected dividend growth term by the realized dividend growth from 2.13, the price dividend ratio is therefore equal to :

$$pd_t = Z_0 + Z_1 \Delta d_{t-1} + \delta_1 pd_{t-1} + Z_2 \varepsilon_{g,t-1} + Z_3 \varepsilon_{d,t-1} + B_2 \varepsilon_{g,t} - B_1 \varepsilon_{\mu,t},$$

where :

$$\begin{aligned} Z_0 &= (1 - \delta_1)B_0 + B_2 \gamma_0 \gamma_1 (\gamma_1 - \delta_1) \\ Z_1 &= -\gamma_1 B_2 (\gamma_1 - \delta_1), \\ Z_2 &= -B_2 (\gamma_1 - \delta_1), \\ Z_3 &= \gamma_1 B_2 (\gamma_1 - \delta_1), \end{aligned}$$

Define  $\eta_{pd,t} = Z_2 \varepsilon_{g,t-1} + Z_3 \varepsilon_{d,t-1} + B_2 \varepsilon_{g,t} - B_1 \varepsilon_{\mu,t}$ .

For the model to be fully identified, the covariance between dividend growth and expected returns, and dividend growth and expected returns must be equal to zero.  $\sigma_{\mu d} = \sigma_{gd} = 0$ . The variance, autocovariance and autocorrelation of  $\eta_{pd,t}$  are:

$$\begin{aligned} Var(\eta_{d,t}) &= (Z_2^2 + B_2^2)\sigma_g^2 + Z_3^2\sigma_d^2 + B_1^2\sigma_\mu^2 - 2B_1B_2\sigma_{\mu g} \\ \lambda_{pd}(1) &= Z_2B_2\sigma_g^2 - Z_2B_1\sigma_{\mu g} \\ \rho_{\eta_{pd,t}}(1) &= \frac{Z_2B_2\sigma_g^2 - Z_2B_1\sigma_{\mu g}}{(Z_2^2 + B_2^2)\sigma_g^2 + Z_3^2\sigma_d^2 + B_1^2\sigma_\mu^2 - 2B_1B_2\sigma_{\mu g}} \end{aligned}$$

The empirical model estimated for the price-dividend ratio is in ARMA(1,1) form with lagged dividend growth as an additional regressor:

$$pd_t = Z_0 + Z_1\Delta d_{t-1} + \delta_1pd_{t-1} - \phi u_{pd,t-1} + u_{pd,t}, \quad (2.20)$$

where the moving average components are:  $v_{pd,t} = u_{pd,t} - \theta u_{pd,t-1}$ .

The variance, autocovariance and autocorrelation for the estimated model is defined as follows:

$$\begin{aligned} Var(v_{pd,t}) &= (1 + \phi^2)\sigma_{u,pd}^2, \\ \lambda_{pd}(1) &= -\phi\sigma_{u,pd}^2 \\ \rho_{v_{pd,t}}(1) &= \frac{-\phi}{(1 + \phi^2)} \end{aligned}$$

$\delta_1$  is directly estimated from 2.20, as the coefficient of lagged price-dividend ratio on the price-dividend ratio.  $\delta_0$  is computed from the estimated intercept term  $Z_0$ . Under the condition that  $\delta_1 \neq 1$ :

$$\delta_0 = \frac{1 - \rho}{\delta_1 - 1} (Z_0 - \gamma_0\gamma_1B_2(\gamma_1 - \delta_1)) + \kappa + \gamma_0$$

Setting the empirical and theoretical autocovariance lag to each other,  $\sigma_{\mu g}$  can be computed as follows:

$$\sigma_{\mu g} = \frac{1}{Z_2 B_1} (Z_2 B_2 \sigma_g^2 - \phi \sigma_{u, pd}^2)$$

for  $Z_2 B_1 \neq 0$ .

Replacing  $\sigma_{\mu g}$  inside the autocovariance equation and setting it equal to empirical autocovariance, the variance of expected returns is :

$$\sigma_\mu^2 = \frac{1}{B_1^2} (a - \sigma_g^2 B_2^2 - \sigma_d^2 Z_3^2 - \sigma_g^2 Z_2^2 + a\phi^2)^2$$

### 3 Monte Carlo Experiment

A Monte Carlo is considered where present value parameters are simulated by a constrained VAR for two sample sizes. The model is estimated using the state space and the new ARMA(1,1) model. The data generating process for returns, dividend growth and price dividend ratio follows a VAR(1):

$$r_t = a_r + b_r pd_{t-1} + \epsilon_{r,t}, \quad (3.1)$$

$$\Delta d_t = a_d + b_d pd_{t-1} + \epsilon_{d,t}, \quad (3.2)$$

$$pd_t = a_{pd} + b_{pd} pd_{t-1} + \epsilon_{pd,t}, \quad (3.3)$$

where  $a_r, a_d$  and  $a_{pd}$  are intercept terms.  $b_r, b_d$  and  $b_{pd}$  are slope parameters for the price-dividend ratio. The choice of simulated parameters are estimated from the constrained VAR. According to the present value, the identity  $b_r = 1 - \rho b_{pd} + b_d$ , must be respected. Only marginal deviations from the identity are to be noted in the estimated parameters. In the data generating process, the parameters set are as follows:  $b_r = -0.0898, b_d = 0.0062$ , and  $b_{pd} = 0.9341$ . The intercept terms are equal to 0.049, 0.015 and 3.394. The residuals of 3.1,

3.2 and 3.3 are drawn from a normal distribution with standard errors of 0.034, 0.012 and 0.025 respectively. The sample sizes considered are 90 and 300 observations respectively. The number of replications in the experiment is 5000. The experiment performed for the state space is slightly different. It takes into account the selection problem for initial values, where it is possible to end up with a local optima. Hence, Initial values are randomly generated 5 times in each Monte Carlo run. In each Monte Carlo run, given the generated dividend growth and price dividend ratio, a vector of 5 optimal parameters are selected. Hence in each run, the final figure reported is the median. The results from Monte Carlo are illustrated in table 1.

Sample	$T = 90$		$T = 300$	
	SS	ARMA(1,1)	SS	ARMA(1,1)
$\gamma_0$	0.021 (0.026)	0.015 (0.024)	0.009 (0.177)	0.015 (0.023)
$\delta_0$	0.054 (0.154)	0.048 (0.152)	0.084 (0.656)	0.048 (0.155)
$\gamma_1$	0.690 (0.569)	0.001 (0.509)	0.977 (0.971)	0.002 (0.487)
$\delta_1$	0.886 (0.312)	0.883 (0.065)	0.997 (0.642)	0.910 (0.034)
$\sigma_g$	0.003 (0.025)	0.034 (0.012)	0.056 (0.117)	0.033 (0.011)
$\sigma_d$	0.001 (0.021)	0.034 (0.007)	0.079 (0.202)	0.034 (0.004)
$\sigma_\mu$	0.007 (0.654)	0.005 (0.017)	0.084 (0.617)	0.005 (0.011)
$\rho_{g\mu}$	0.151 0.527	0.931 0.263	0.184 0.719	0.929 0.138
$\rho_{d\mu}$	-0.113 0.535	- -	0.024 0.745	- -

Table 1: The table reports estimates of the present value parameters from the state space and the ARMA(1,1) structures. The sample sizes are  $T = 90$  and 300. Figures expressed in brackets are standard errors of the Monte Carlo estimates.

Table (1) reports the Monte Carlo estimates from both models. The smaller sample, ( $T = 90$ ) is a scenario which closely reflects sample size of most empirical studies. No significant differences are to be noted in most of the parameters for the smaller size. The

notable exception is the autoregressive coefficient for expected dividend growth.

The unconditional mean of expected dividend growth is 2.1% in the State Space Model, while that of the ARMA(1,1) is 1.5%. Similar estimates are present in the case of unconditional expected returns at 5%. Persistence parameters for expected returns are closer to unity at 0.88. However the standard error is much higher for the state space than the case of the ARMA(1,1). The standard deviation of shocks to expected dividend growth, realized dividend growth and expected return is much smaller for the state space than in ARMA(1,1) case. However, the standard errors of these estimates are slightly higher in the state space model. Interestingly, the positive correlation between expected returns is higher in the ARMA(1,1) which may be due to setting the correlation between expected returns and realized dividend growth to zero. Interestingly, these parameters will have important implications for variance decompositions. It is mostly likely that stock movements are due to both dividend growth and discount rate news in the ARMA(1,1) model, unlike the state space, where the autoregressive component of expected tends to dominate the weaker correlation levels.

The autoregressive parameter for expected dividend growth is equal to 0.690 in the state space, which is roughly similar to values shown by other studies, namely Kojien and Binsbergen (2010), Golinski et al.(2015) and Ma and Wohar (2014). In most applied studies, this parameter varies across definitions of dividend growth and price-dividend ratio. For example in the case of Kojien and Binsbergen)(2010), there is evidently a difference in the persistence parameters based on whether dividends are re-invested at the risk-free rate or at market re-invested rates. However, the persistence in expected dividend growth is nonexistent for the ARMA(1,1).

There appears to be considerable variation for the larger sample size ( $T = 300$ ), where the persistence of dividend growth increases in the state space, while that of expected returns remains stable. Some of the persistence in the price-dividend ratio is shared by the expected dividend growth process for a larger sample size. The unconditional mean log dividend growth goes down from 2.1% to 1%. The unconditional mean return increases

from 5.4% to 8.4%. Interestingly, the state space also shows higher standard deviation of shocks to dividend growth, realized dividend growth and expected returns. On the other hand, the ARMA(1,1) shows more robustness for the larger sample size. The distribution for each parameter for both sample sizes are reported in the appendix. The parameters for ARMA(1,1) show little dispersion, except in the case of the autoregressive coefficient for expected dividend growth rate.

## 4 Application

### 4.1 Estimation

In this section, an application to US data is considered. Annual time series on dividend growth and the price dividend ratio from 1926 to 2014 are downloaded from Shiller's website and the S&P500. Dividend growth is computed from real series of dividends, and the price dividend ratio is computed from the logarithm of price/dividend. The optimal values from both models are shown in table 2.

Sample	1946 – 2014	
	SS	ARMA(1,1)
$\gamma_0$	0.022	0.008
$\delta_0$	0.051	0.084
$\gamma_1$	0.357	-0.622
$\delta_1$	0.922	0.855
$\sigma_g$	0.059	0.024
$\sigma_d$	0.018	0.029
$\sigma_\mu$	0.026	0.003
$\rho_{g\mu}$	0.451	0.516
$\rho_{d\mu}$	-0.186	0

Table 2: The table reports estimates of the present value parameters for the SP500. The second and third columns respectively report the parameters for the state space and ARMA(1,1) process.

From table 2, it can be seen that there are significant differences across state space and ARMA(1,1). The mean of the expected dividend growth, standard deviation of shocks to realized dividend growth, and standard deviation to expected returns are similar in both models. The unconditional expected log returns from the state space equals 5.1 %, contrary to that of ARMA(1,1) at 8.4 %. The ARMA(1,1) model shows that the mean of expected returns is closer to the mean of equity premium.

Expected returns tend to be persistent in both models with autoregressive parameters at 0.922 and 0.855 respectively. This is consistent with findings from Fama and French (1988), Campbell and Cochrane (1999) and Golinski et. al (2015). However, the autoregressive coefficient for dividend growth tends vary across both models. In both models  $\gamma_1$  tends to be lower than  $\delta_1$ . They tend to differ in terms of magnitude. The state space model has an inflated autoregressive coefficient for expected dividend growth (0.56) which exceeds the -0.62 value for dividend growth. Expected dividend growth shocks are closer to realized dividend growth, according to the ARMA(1,1) model. As shown by the Monte Carlo estimates in the

previous section, the state space model is biased upwards in this particular example.

While both models show that expected returns are persistent over time, the state space model shows that expected returns are closer to a unit root process. On the other hand, expected returns from the ARMA(1,1) has a slightly lower autoregressive coefficient. Shocks between expected returns and expected dividend growth are positively correlated at 0.45 and 0.52 for the state space and the ARMA(1,1) respectively. Positive correlation is consistent with the findings of Kojen and Van Binsbergen (2010) and Lettau and Ludvigson(2005). The standard deviation of expected returns is very low in the ARMA(1,1) at 0.003. The shock to expected returns is twice as high in the state space model. The time series plot of expected returns and dividend growth are illustrated in figure 1 and 2.

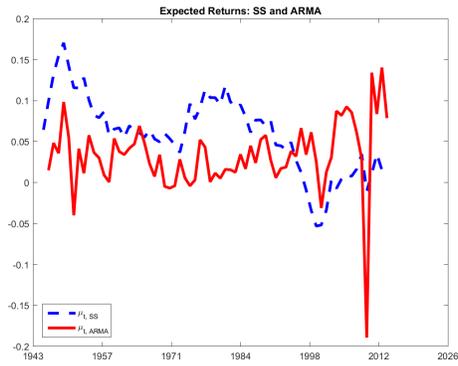


Figure 1: Time Series of Expected Returns. The red line shows the expected returns derived from the ARMA(1,1) while the ragged blue line shows the expected returns from the State Space Model.

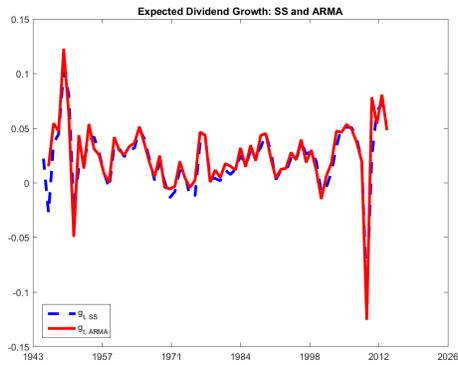


Figure 2: Time Series of Expected Dividend Growth. The red line shows the expected dividend growth derived from the ARMA(1,1) while the ragged blue line shows the expected dividend growth from the State Space Model.

Time series plots illustrate strong correlation for dividend growth in both models. Both time series are volatile and tend to exhibit swings which are procyclical with macroeconomic conditions. Expected returns are countercyclical. The aftermath of the financial crash of 2007 is illustrated by a massive drop in expected future dividend growth. However the time series of expected returns shows a rapid fall in expected returns just before the financial crash. The ARMA(1,1) tends to exhibit higher variance than the state space model. In the aftermath of the crash, it can be seen that expected returns turn out to be higher, which coincides with lower prices.

## 4.2 Implications for predictability

As a modest application to the finance literature, we consider predictability from the state space and dividend growth. The  $R^2$  is computed as:

$$R_{Ret}^2 = 1 - \frac{\widehat{var}(r_{t+1} - \mu_t)}{\widehat{var}(r_{t+1})}$$

$$R_{div}^2 = 1 - \frac{\widehat{var}(\Delta d_{t+1} - g_t)}{\widehat{var}(\Delta d_{t+1})}$$

where  $\widehat{var}$  is the sample variance. The state space states that returns can be predicted at 8.31% while that from the ARMA(1,1) shows a negative  $R_{Ret}^2$  at  $-8.10\%$ . On the other hand, the ARMA(1,1) illustrates better dividend growth predictability at 23.6% compared to 16.5%. Interestingly, despite the fact that expected returns is not predictable from the ARMA(1,1) process, there is enough evidence of dividend growth predictability.

Furthermore, standard predictive regression models are considered of the form:

$$r_{t+1} = \theta_0 + \theta_1 pd_t + \zeta_{\mu,t+1} \tag{4.1}$$

$$r_{t+1} = \theta_0 + \theta_1 \mu_{ss,t} + \zeta_{\mu,t+1} \tag{4.2}$$

$$r_{t+1} = \theta_0 + \theta_1 \mu_{ARMA,t} + \zeta_{\mu,t+1} \tag{4.3}$$

$$r_{t+1} = \Psi_0 + \Psi_1 pd_t + \zeta_{g,t+1} \quad (4.4)$$

$$r_{t+1} = \Psi_0 + \Psi_1 g_{ss,t} + \zeta_{g,t+1} \quad (4.5)$$

$$r_{t+1} = \Psi_0 + \Psi_1 g_{ARMA,t} + \zeta_{g,t+1} \quad (4.6)$$

4.1, 4.2 and 4.3 forecasts insample returns from the price-dividend ratio, expected returns from the state space and expected returns from the ARMA(1,1) respectively. 4.4, 4.5 and 4.6 forecasts dividend growth from the price-dividend ratio, expected dividend growth from the state space and expected dividend dividend growth from the ARMA(1,1) respectively. Estimates and the goodness of fit are reported in table 3:

Predictive Regression			
Panel A			
regressor	$\theta_0$	$\theta_1$	$R^2$
$pd_t$	0.419	-0.102	0.082
$\mu_{ss,t}$	0.009	0.962	0.083
$\mu_{ARMA,t}$	0.070	-0.098	0.001
Panel B			
regressor	$\Psi_0$	$\Psi_1$	$R^2$
$pd_t$	0.068	-0.132	0.007
$g_{ss,t}$	0.001	0.989	0.168
$g_{ARMA,t}$	-0.002	1.051	0.238

Table 3: The table reports the intercept, slope coefficient and goodness of fit parameters for predictive regressions for returns and dividend growth.

According to the results, the price-dividend ratio and the expected returns from the state space model are good predictors of realized returns based on  $R^2$ . The expected returns is marginally better than the price-dividend ratio. The ARMA(1,1) has zero predictability for realized returns. However the ARMA(1,1) has very good predictability for realized dividend growth. This contrasts with the state space model, which still has very good predictive accuracy and unlike the price-dividend ratio which has none. The results lead to the conclusion that the choice of model needs to be chosen selectively depending on the variable to be predicted.

## 5 Conclusion

This paper proposes a model for extracting and modelling unobserved expected returns and expected dividend growth in the present value. If expected returns and dividend growth are specified as an AR(1), it is shown that the observed realized dividend growth has an ARMA(1,1) structure. The price-dividend ratio has an ARMA(1,1) structure with lagged dividend growth as an additional regressor. The variance, autocovariance and autocorrelation components of both empirical models must be equal to that of the latent structure. For the model to be identified, the correlation of shocks between expected returns and realized dividend growth, and the correlation of shocks between expected dividend growth and realized dividend growth must be equal to zero.

The new model performs well for large sample sizes, with small and robust standard errors. On the other hand, an increase in persistence in expected dividend growth for larger sample sizes is witnessed for the state space model. The Monte Carlo replications show that the standard errors are higher for the larger sample size. Unlike the case of the state space model, there is no persistence in expected dividend growth. Expected returns are persistent but are further from the unit circle in the ARMA(1,1) model.

The model is applied to US data, where it is found that expected dividend growth rate has a negative autoregressive coefficient, unlike the state space. Persistence levels in expected

returns were similar for both models. Shocks to expected returns are much smaller. There is a high correlation for expected dividend growth from both models unlike in the case of expected returns. A basic application of the model was used in assessing predictability of both models. The state space fared equally well as the price-dividend ratio in predicting returns. Dividend growth predictability was stronger using the ARMA(1,1).

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# 6 Appendix

## 6.1 Monte Carlo Distribution

The following graphs illustrate the distribution for  $T = 250$ .

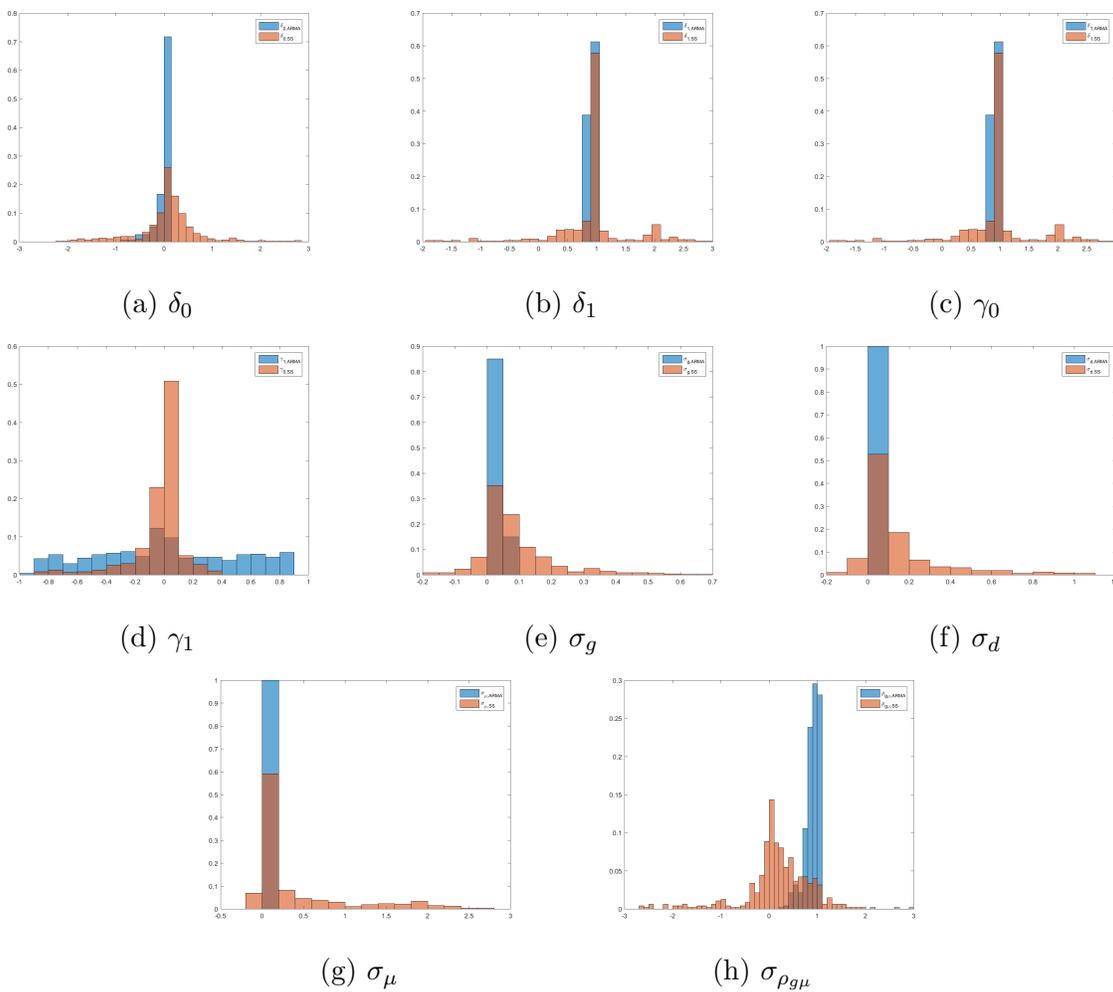
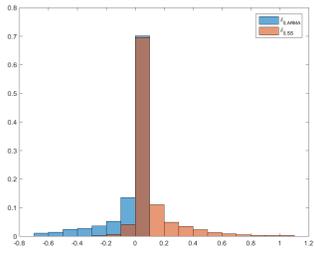
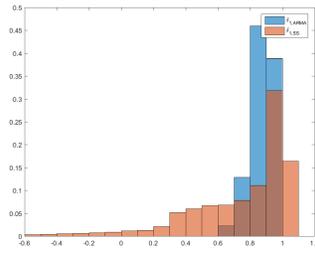


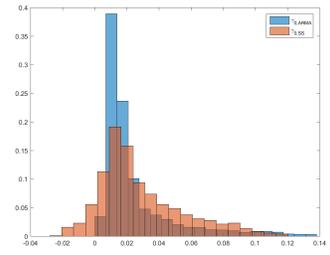
Figure 3: Distribution of Monte Carlo for  $T = 250$



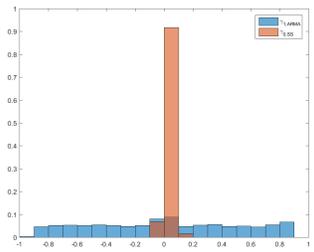
(a)  $\delta_0$



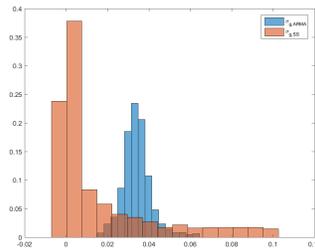
(b)  $\delta_1$



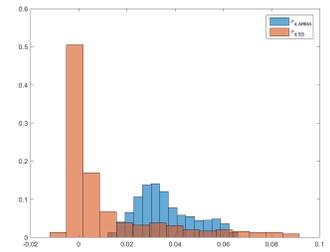
(c)  $\gamma_0$



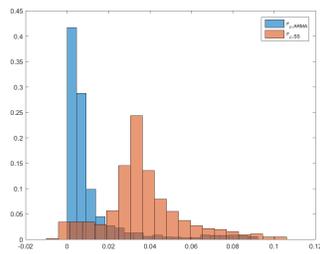
(d)  $\gamma_1$



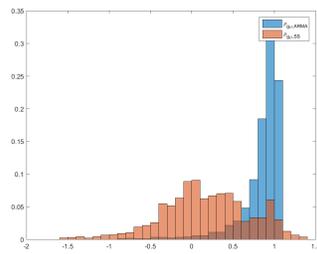
(e)  $\sigma_g$



(f)  $\sigma_d$



(g)  $\sigma_\mu$



(h)  $\sigma_{\rho_g\mu}$

Figure 4: Distribution of Monte Carlo for  $T = 90$